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Daniel
Henry
Tombs,

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# EUCLIDIS ELEMENTORUM

Libri xv. breviter demonſtrati,

*Opera*

IS. BARROW, *Cantabrigienſis*, *Coll.* TRIN. *Soc.*

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Καθαρμοὶ ψυχῆς λογικῆς εἰσιν αἱ μαθηματικαὶ ἐπιστῆμαι. HIEROCL.

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*LONDINI,*

Excudebat R. DANIEL, Impenſis GUIL. NEALAND Bibliopolæ *Cantabrig.* CIↃ IↃC LIX.

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*Nobiliſsimis & Generoſiſsimis*

*Adoleſcentibus*,

Dno EDOUARDO CECILIO,

*Illuſtriſſ. Comitis* Sarisburienſis *Filio*;

Dno *IOHANNI KNATCHBVL*,

Et

D. *FRANCIS. WILLOVGHBY*,

ARMIGERIS.

Unicuique veſtrum (Optimi Adoleſcentes) tantum me debere reputo, quantum homo homini debere poteſt. Mea enim ſententia, ultra ſincerum amorem non eſt quod

quiſpiam de alio bene mereri poſſit. Hunc autem jamdiu eſt quo ex ſingulari veſtra bonitate mihi indultum experior ; ejuſque ſenſus, intimis animi medullis inhærens, ipſi ardens ſtudium impreſſit quovis honeſto modo reciprocos affectus prodendi. Quandoquidem vero ea fortunarum mearum tenuitas, ea veſtrarum amplitudo, exiſtit, ut nec ego alia quam gratæ alicujus agnitionis ſignificatione uti queam, nec vos aliam admittere velitis ; ea propter haud illibenter hanc occaſionem arripio, honoris & benevolentiæ, quibus vos proſequor, publicum hoc & durabile

bile μνημόσυνον edendi. Etſi cum oblati anathematis exilitatem, & libellum veſtris nominibus conſecratum; quam is longe infra veſtrorum meritorum dignitatem ſubſidat, attentius conſidero, timor ſubinde aliquis & dubitatio animum inceſſant, ne hoc ſtudium erga vos meum vobis dehoneſtamento ſit potius quam ornamento; ſcilicet memor cum ſim, ut malæ cauſæ, ſic & mali libri patrocinium in patroni contumeliam magis quam in gloriam cedere. Sed quum veſtrarum virtutum id robur, eam fore ſoliditatem, recognoſcerem, quæ veſtrum decus, meo quantumvis labefa-

ctato, inconcuſſum ſuſtinere poſſint; idcirco non dubitavi vos in aliquatenus commune mecum periculum induere. Virtutes illas intelligo, quibus nemo unquam in veſtra ætate aut in veſtro ordine, ſaltem me judice, majores deprehendit; quæ vos inſigniter gratos omnibus & amabiles reddunt; eximiam modeſtiam, ſobrietatem, benignitatem animi, morum comitatem, prudentiam, magnanimitatem, fidem, præclaram inſuper ingenii indolem, quæ vos ad omnem ingenuam ſcientiam non tantum excellenti captu, ſed & appetitu forti ac ſincero, inſtruxit. Quas veſtras præclariſſimas dotes

prout nemo eſt fortaſſis qui me melius novit, aut pro conſuetudine, quam jamdudum vobiſcum dulciſſimam coluiſſe ex veſtro favore mihi contigit, penitus introſpexit, ita nemo eſt qui impenſius miratur & ſuſpicit; aut qui ipſas libentius prædicare ac celebrare vellet, ſi non cum eloquii mei vires ſupergrederentur, tum etiam quæ in ſingulis vobis elucent, prolixi alicujus commentarii aut panegyricæ orationis libertatem, potius quam præſtitutas hujuſmodi ſalutationibus anguſtias, expoſcerent. Quin potius divinam clementiam imploro, ut vos earundem virtutum ſancto tramiti inſi-

ſtere, atque hos egregios fructus vernæ veſtræ ætatis felicibus incrementis matureſcere concedat; vitamque vobis in hoc ſeculo ingenuam, innocentem, jucundam, & in futuro beatam ac ſempiternam tranſigere largiatur. Minime autem dubito, ne pro conſueto veſtro in me candore hoc ultimum fortaſſis quod vobis præſtare potero, benevolentiæ erga vos & obſervantiæ teſtimonium, alacriter accepturi ſitis; quod vobis propenſiſſimo affectu offert

*Veſtri in æternum amantiſſimus,*

*& obſervantiſſimus,*

I. B.

# Benevolo LECTORI.

*SI quid in hac elementorum editione præſtitum ſit, ſcire deſideras, amice Lector, accipe, pro genio operis, breviter. Ad duos præcipue fines conatus meos direxi. Primum, ut cum requiſita perſpicuitate ſummam demonſtrationum brevitatem conjungerem, quo eam libello molem compararem, quæ commode abſque moleſtia circumferri poſſet. Id quod aſſecutus videor, ſi abſentem Typographi cura non fruſtretur. Concinnius enim quiſpiam meliori ingenio aut majori peritia excellens, at nemo forſan brevius pleraſque propoſitiones demonſtraverit; præſertim cum in numero & ordine propoſitionum ipſe nihil immutarim, nec licentiam mihi aſſumpſerim quamcunque propoſitionem Euclideam procul ablegandi tanquam minus neceſſariam, aut quaſdam faciliores in axiomatum cenſum referendi; quod nonnulli fecerunt: inter quos peritiſſimus Geometra Andr. Tacquetus, quem ideo etiam nomino, quod quædam ex eo deſumpta agnoſcere honeſtum duco; poſt cujus elegantiſsimam editionem, ipſe nihil atten-*

*tare voluiſſem, ſi non viſum fuiſſet doctiſſimo viro non niſi octo* Euclidis *libros ſuâ curâ adornatos publico communicare, reliquis ſeptem, tanquam ad elementa Geometriæ minus ſpectantibus, omnino quaſi ſpretis atque poſthabitis. Mihi autem jam ab initio alia provincia demandata fuit, non elementa Geometriæ utcunque pro arbitrio conſcribendi, verum* Euclidem *ipſum, eumque totum, quam poſſem breviſſime, demonſtrandi. Quod enim quatuor libros ſpectat, ſeptimum, octavum, nonum, decimum, quamvis illi ad Geometriæ planæ & ſolidæ elementa, ut ſex præcedentes & duo ſubſequentes, non tam prope pertineant; quod tamen ad res Geometricas admodum utiles ſint, tam propter Arithmeticæ & Geometriæ valde propinquam cognationem, quam ob notitiam commenſurabilium & incommenſurabilium magnitudinum ad figurarum tam planarum quam ſolidarum intellectum apprime neceſſariam, nemo eſt è peritioribus Geometris qui ignorat. Quæ vero in tribus ultimis libris continetur, 5 corporum regularium nobilis contemplatio, illa non niſi injuria prætermitti potuit; quando nempe illius gratia noſter* στοιχειωτὴς, *Platonicæ familiæ philoſophus, hoc elementorum ſyſtema univerſum condidiſſe perhibetur;*

*uti*

*uti teſtis eſt* * Proclus, *iis verbis*, Ὅθεν δὴ καὶ τῆς συμπάσης στοιχειώσεως τέλος προεστήσατο τὴν τῶν καλουμένων πλατωνικῶν σχημάτων σύστασιν. *Præterea facile in animum induxi ut opinarer, nemini harum ſcientiarum amanti non futurum eſſe cordi penes ſe habere integrum* Euclidæum *opus, quale paſsim ab omnibus citatur & celebratur. Quare nullum librum nullamque propoſitionem negligere volui earum quæ apud* P. Herigonium *habentur; cujus veſtigiis preſſe inſiſtere neceſſe habui, quoniam ejuſce libri ſchematiſmis maxima ex parte uti ſtatutum erat, quod præviderem mihi ad novas deſcribendas tempus non ſuppetere; etſi nonnunquam id facere præoptaſſem. Eadem de cauſa nec alias pleraſque quam* Euclidæas *demonſtrationes adhibere volui, ſuccinctiori forma expreſſas, niſi forte in* 2, *&* 13, *& parce in* 7, 8, 9 *libris; ubi ab eo nonnihil deflectere operæ pretium videbatur. Bona igitur ſpes eſt ſaltem in hac parte cum noſtris conſiliis, tum ſtudioſorum votis, aliquo modo ſatisfactum iri. Nam quæ adjecta ſunt in Scholiis problemata quædam & theoremata, ſive ob ſuum frequentem uſum ad naturam elementarem accedentia, ſive ad eorum quæ ſequuntur expeditam demonſtrationem conducentia, ſeu quæ regula-*

* lib. 2.

rum

*rum practicæ Geometriæ quarundam præcipuarum rationes innuunt ad ſuos fontes relatas, per ea, ut ſpero, libellus ultra deſtinatam molem magnopere non intumeſcet.*

*Alter ſcopus ad quem collineatum eſt, eorum deſideriis conſuluit qui demonſtrationibus ſymbolicis potius quam verbalibus delectantur. In quo genere cum plerique apud nos* Guilielmi Oughtredi *ſymbolis aſſueti ſint, ea plerumque uſurpare conſultius duximus. Nam qui* Euclidem *hâc viâ tradere & interpretari aggreſſus ſit, hactenus, quod ego ſciam, præter unum* P. Herigonium, *repertus eſt nemo. Cujus viri longe doctiſsimi methodus, ſane in multis egregia, ac ejus peculiari propoſito admodum accomodata, duplici tamen defectu laborare mihi viſa eſt. Primo, quod cum Propoſitionum ad unius alicujus theorematis aut problematis probationem adductarum poſterior à priori non ſemper dependeat; quando tamen illæ inter ſe cohærent, quando non, nec ex ordine ſingularum, nec ullo alio modo, ſatis prompte innoteſcere poteſt: unde ob defectum conjunctionum & adjectivorum* (ergo, rurſus, *&c.*) *non raro difficultas & dubitandi occaſio, præſertim minus exercitatis, inter legendum oboriri ſolent. Deinde ſæpenumero evenit, ut prædicta methodus ſupervacaneas repetitiones effugere nequeat, à quibus demonſtrationes eſt quando prolixæ, aliquando*

*& magis intricatæ, evadunt. Quibus vitiis noſter modus facile per verborum ſignorumq; arbitrariam mixturam medetur. Atque hæc de opellæ hujus intentione & methodo dicta ſufficiant. Cæterum quæ in laudem Matheſeos in genere, aut Geometriæ ipſius; & quæ de hiſtoria harum ſcientiarum, ideoque de* Euclide *horum elementorum digeſtore, dici poſſent, & reliqua hujuſmodi* ἐξωτερικὰ, *cui hæc placent, apud alios interpretes conſulere poteſt. Neque nos anguſtias temporis quod huic operi impendi potuit, nec interpellationes negotiorum, nec adjumentorum ad hæc ſtudia apud nos egeſtatem, & quædam alia, ut liceret non immerito, in excuſationem obtendemus; metu ſcilicet inducti, ne hæc noſtra omnibus minus ſatisfaciant. Verum quæ ingenui Lectoris uſibus elaboravimus, eadem in ſolidum ipſius cenſuræ ac judicio ſubmittimus; probanda ſi utilia ſibi compererit; ſin omnino ſecus, rejicienda.*

I. B.

## Ad amiciſſimum Virum, *I. B.* de *EUCLIDE* contracto

Εὐφημισμός.

*FActum bene! didicit Laconice loqui*
*Senex profundus, & aphoriſmos induit.*
*Immenſa dudum margo commentarii*
*Diagramma circuit minutum; utque Inſula*
*Problema breve natabat in vaſto mari.*
*Sed unda jam detumuit; & gloſſa arctior*
*Stringit Theoremata: minoris anguli*
*Lateribus ecce totus* Euclides *jacet,*
*Incluſus olim velut Homerus in nuce;*
*Pluteoque ſarcina modo qui incubuit, levis*
*En fit manipulus. Pelle in exigua latet*
*Ingens Matheſis, matris ut in utero Hercules,*
*In glande quercus, vel Ithaca Eurus in pila.*
*Nec mole dum decreſcit, uſu fit minor;*
*Quin auctior jam evadit, & cumulatius*
*Contracta prodeſt erudita pagina.*
*Sic ubere magis liquor è preſſo effluit;*
*Sic pleniori vaſa inundat ſanguinis*
*Torrente cordis Syſtole; ſic fuſius*
*Procurrit æquor ex Abylæ anguſtiis.*
*Tantilli operis ars tanta referenda unice eſt*
*BAROVIANO nomini, ac ſolertiæ.*
*Sublimis euge mentis ingenium potens!*
*Cui invium nil, arduum eſſe nil ſolet.*
*Sic uſque pergas proſpero conamine,*
*Radiuſque multum debeat ac abacus tibi;*
*Sic creſcat indies feracior ſeges,*
*Simili colonum germine aſsiduo beans.*
*Specimen futuræ meſsis hic ſiet laber,*
*Magnæque famæ illuſtria hæc præludia.*
*Juvenis dedit qui tanta, quid dabit ſenex?*

Car. Robotham, *CANTAB.*
*Coll. Trin. Sen. Soc.*

In novam *Elementorum*

*EUCLIDIS*

Editionem à D. *IS. BARROW*, *Collegii SS.* TRIN. *Socio*, viro opt. & erudititiſſimo, adornatam.

*BEnigne Lector! ſi uſpiam auditum eſt tibi,*
*Quantus tenella Nix Geometres ſiet;*
*Quæ mille radiis, mille ludit angulis,*
*Totumque puro ducit Euclidem ſinu:*
*Am abi s ultro candidiſsimum Virum,*
*Cui plena nivium eſt indoles, ſed quas tamen*
*Præclarus ardor mentis urget Enthea;*
*Et uſque blandis temperat caloribus:*
*Quo ſuavius nil vivit, & melius nihil.*
*Is, dum liquentes pectore excutit nives,*
*Et inde & inde ſpargit, en aliam tibi,*
*Lector benigne, è nivibus Geometriam!*

G.C. *A.M.* *C.E.S.*

# *Notarum explicatio.*

$=$ æqualitatem.
⊏ majoritatem.
⊐ minoritatem.
$+$ plus, vel addendum eſſe.
$-$ minus, vel ſubtrahendum eſſe.
$-\!:$ differentiam vel exceſſum; item quantitates omnes, quæ ſequuntur, ſubtrahendas eſſe, ſignis non mutatis.
$\times$ multiplicationem, vel ductum lateris rectanguli in aliud latus.
Idem denotat conjunctio literarum, ut $AB = A \times B$.
$\surd$ Latus, vel radicem quadrati, vel cubi, &c.
Q. & q quadratum. C. & c cubum.
Q. Q. rationem quadrati numeri ad quadratum numerum.

} ſignificat.

*Reliquas, quæ ubicunque occurrunt, vocabulorum abbreviationes ipſe Lector per ſe facile intelliget; exceptis iis, quas tanquam minus generalis uſus, ſuis locis explicandas relinquimus.*

# LIB. I.

## *Definitiones.*

I. PUnctum eſt cujus pars nulla eſt.

II. Linea vero longitudo latitudinis expers.

III. Lineæ autem termini ſunt puncta.

IV. Recta linea eſt , quæ ex æquo ſua interjacet puncta.

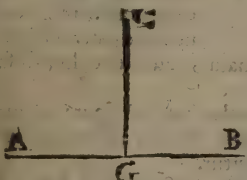
V. Superficies eſt , quæ longitudinem, latitudinemque tantum habet.

VI. Superficiei autem extrema ſunt lineæ.

VII. Plana ſuperficies eſt, quæ ex æquo ſuas interjacet lineas.

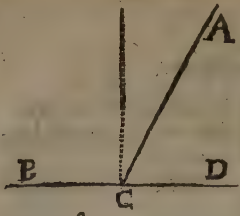
VIII. Planus vero angulus eſt, duarum linearum in plano ſe mutuo tangentium, & non in directum jacentium alterius ad alteram inclinatio.

IX. Cum autem quæ angulum continent, lineæ , rectæ fuerint , rectilineus ille angulus appellatur.




X. Cum vero recta linea C G ſuper rectam lineam A B conſiſtens , eos qui ſunt deinceps angulos C G A ; C G B æquales inter ſe fecerit, rectus eſt uterque æqualium angulorum , & quæ inſiſtit recta linea C G, perpendicularis vocatur ejus (A B) cui inſiſtit.

Not. *Cum plures anguli ad unum punctum : (ut ad* G) *exſiſtunt , deſignatur quilibet angulus tribus literis , quarum media ad verticem eſt illius de quo agitur : ut angulus quem rectæ* C G , A G *efficiunt ad partes* A *vocatur* C G A, *vel* A G C.



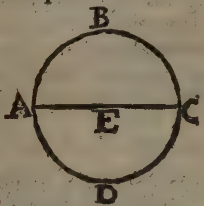

XI. Obtuſus angulus eſt, qui recto major eſt, ut A C B.

XII. Acutus vero, qui minor eſt recto, ut A C D.

XIII. Terminus eſt, quod alicujus extremum eſt.

XIV. Figura eſt, quæ ſub aliquo, vel aliquibus terminis comprehenditur.

XV. Circulus eſt figura plana, ſub una linea comprehenſa, quæ peripheria appellatur, ad quam ab uno puncto eorum, quæ intra figuram ſunt poſita, cadentes omnes rectæ lineæ inter ſe ſunt æquales.




XVI. Hoc vero punctum centrum circuli appellatur.

XVII. Diameter autem circuli eſt recta quædam linea per centrum ducta, & ex utraque parte in circuli peripheriam terminata, quæ circulum bifariam ſecat.

XVIII. Semicirculus vero eſt figura, quæ continetur ſub diametro, & ſub ea linea, quæ de circuli peripheria aufertur.

*In circulo* E A B C D. E *eſt centrum,* A C *diameter,* A B C *ſemicirculus.*

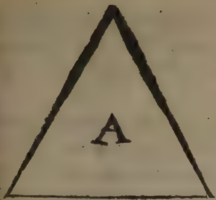
XIX. Rectilineæ figuræ ſunt, quæ ſub rectis lineis continentur.

XX. Trilateræ quidem, quæ ſub tribus.

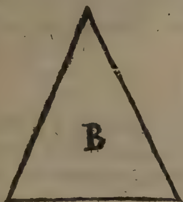
XXI. Quadrilateræ vero, quæ ſub quatuor.

XXII. Multilateræ autem, quæ ſub pluribus, quam quatuor rectis lineis comprehenduntur.

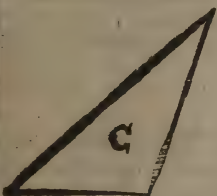
XXIII.



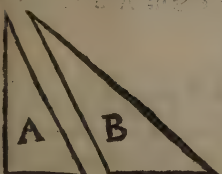

XXIII. Trilaterarum autem figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia, ut triangulum A.




XXIV. Iſoſceles autem, quod duo tantum æqualia habet latera, ut triangulum B.

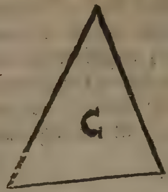



XXV. Scalenum vero, quod tria inæqualia habet latera, ut C.



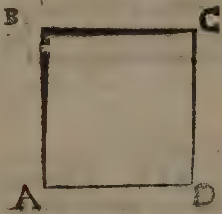

XXVI. Adhæc etjam trilaterarum figurarum, rectangulum quidem triangulum eſt, quod rectum angulum habet, ut triangulum A.

XXVII. Amblygonium autem, quod obtuſum angulum habet, ut B.

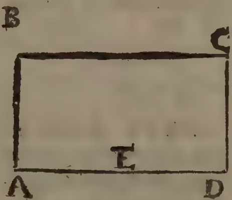



XXVIII. Oxygonium vero, quod tres habet acutos angulos, ut C.

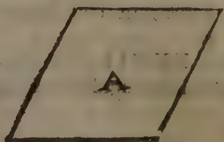
Figura æquiangula eſt, cujus omnes anguli inter ſe æquales ſunt. Duæ vero figuræ æquiangulæ ſunt; ſi ſinguli anguli unius ſingulis angulis alterius ſint æquales. Similiter de figuris æquilateris concipe.



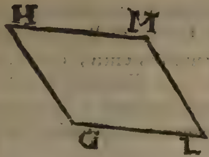

XXIX. Quadrilaterarum autem figurarum, quadratum quidem eſt; quod & æquilaterum, & rectangulum eſt, ut A B C D.



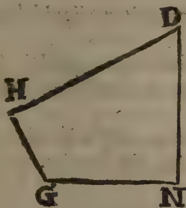

XXX. Altera vero parte longior figura eſt, quæ rectangula quidem, at æquilatera non eſt, ut A B C D.



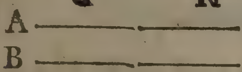

XXXI. Rhombus autem, quæ æquilatera, ſed rectangula non eſt, ut A.



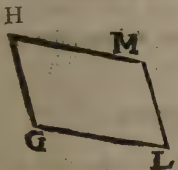

XXXII. Rhomboydes vero, quæ adverſa & latera, & angulos habens inter ſe æquales, neque æquilatera eſt, neque rectangula, ut G L M H.



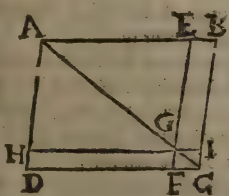

XXXIII. Præter has autem reliquæ quadrilateræ figuræ trapezia appellentur; ut G N D H.




XXXIV. Parallelæ rectæ lineæ ſunt, quæ cum in eodem ſint plano, & ex utraque parte in infinitum producantur, in neutram ſibi mutuo incidunt, ut A, & B.




XXXV. Parallelogrammum eſt figura quadrilatera, cujus bina oppoſita latera ſunt parallela, ſeu æquidiſtantia, ut G L H M.




XXXVI. Cum vero in parallelogrammo A B C D diameter A C ducta fuerit, duæque lineæ E F, H I, lateribus parallelæ ſecantes diametrum in uno eodemque puncto G, ita ut parallelogrammum ab hiſce

 paral-

parallelis in quatuor diſtribuatur parallelogramma; appellantur duo illa D G, G B, per quæ diameter non tranſit, Complementa; duo vero reliqua H E, F I, per quæ diameter incedit, circa diametrum conſiſtere dicuntur.

Problema eſt, *cum proponitur aliquid efficiendum.*

Theorema eſt, *cum proponitur aliquid demonſtrandum.*

Corollarium eſt *conſectarium, quod è facta demonſtratione tanquam lucrum aliquod colligitur.*

Lemma eſt *demonſtratio præmiſſæ alicujus, ut demonſtratio quæſiti evadat brevior.*

## *Poſtulata.*

1. POſtuletur, ut à quovis puncto ad quodvis punctum rectam lineam ducere concedatur.

2. Et rectam lineam terminatam in continuum recta producere.

3. Item, quovis centro, & intervallo circulum deſcribere.

## *Axiomata.*

1. Quæ eidem æqualia, & inter ſe ſunt æqualia.

ut A = B = C. ergo A = C, vel ergo omnes A, B, C, æquantur inter ſe.

Nota, *Cum plures quantitates hoc modo conjunctas invenias, vi hujus axiomatis primam ultimæ & quamlibet earum cuilibet æquari. Quo in caſu ſæpe, brevitatis cauſa, ab hoc axiomate citando abſtinemus; etſi vis conſecutionis ab eo pendeat.*

2. Et ſi æqualibus æqualia adjecta ſunt, tota ſunt æqualia.

3. Et

3. Et ſi ab æqualibus æqualia ablata ſunt, quæ relinquuntur ſunt æqualia.

4. Et ſi inæqualibus æqualia adjecta ſint, tota ſunt inæqualia.

5. Et ſi ab inæqualibus æqualia ablata ſint, reliqua ſunt inæqualia.

6. Et quæ ejuſdem vel æqualium ſunt duplicia, inter ſe ſunt æqualia. Idem puta de triplicibus, quadruplicibus, &c.

7. Et quæ ejuſdem, vel æqualium ſunt dimidia, inter ſe ſunt æqualia. Idem concipe de ſubtriplis, ſubquadruplis, &c.

8. Et quæ ſibi mutuo congruunt, ea inter ſe ſunt æqualia.

*Hoc axioma in rectis lineis, & angulis valet converſum, ſed non in figuris, niſi illæ ſimiles fuerint.*

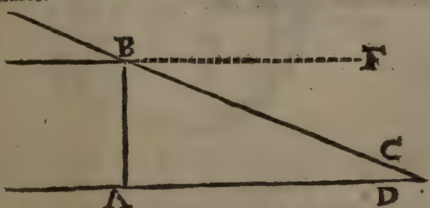
*Cæterum, magnitudines congruere dicuntur, quarum partes applicatæ partibus, æqualem vel eundem locum occupant.*

9. Et totum ſua parte majus eſt.

10. Duæ rectæ lineæ non habent unum & idem ſegmentum commune.

11. Duæ rectæ in uno puncto concurrentes, ſi producantur ambæ, neceſſario ſe mutuo in eo puncto interſecabunt.

12. Item omnes anguli recti ſunt inter ſe æquales.




13. Et ſi in duas rectas lineas AD, CB, altera recta BA incidens, internos ad eaſdemque partes

 angu-

angulos B A D, A B C duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ ſibi mutuo incident ad eas partes, ubi ſunt anguli duobus rectis minores.

14. Duæ rectæ lineæ ſpatium non comprehendunt.

15. Si æqualibus inæqualia adjiciantur, erit totorum exceſſus adjunctorum exceſſui æqualis.

16. Si inæqualibus æqualia adjungantur, erit totorum exceſſus exceſſui eorum, quæ á principio, æqualis.

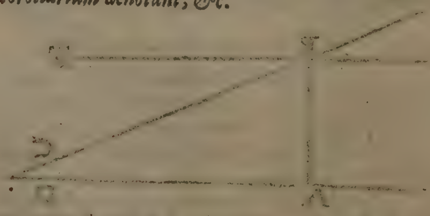
17. Si ab æqualibus inæqualia demantur, erit reſiduorum exceſſus, exceſſui ablatorum æqualis.

18. Si ab inæqualibus æqualia demantur, erit reſiduorum exceſſus exceſſui totorum æqualis.

19. Omne totum æquale eſt omnibus ſuis partibus ſimul ſumptis.

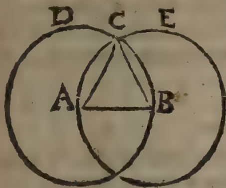
20. Si totum totius eſt duplum, & ablatum ablati, erit & reliquum reliqui duplum. Idem de reliquis multiplicibus intellige.

*Citationes intellige ſic. Cum duo numeri occurrunt, prior deſignat propoſitionem, poſterior librum.* Ut per 4. 1. intelligitur quarta propoſitio primi libri, atque ita de reliquis. *Cæterum, ax. axioma, poſt. poſtulatum, def. definitionem, ſch. ſcholium, cor. corollarium denotant, &c.*



# LIB. I.

## PROP. I.




*Super data recta linea terminata* A B, *triangulum æquilaterum* A B C *conſtituere.*

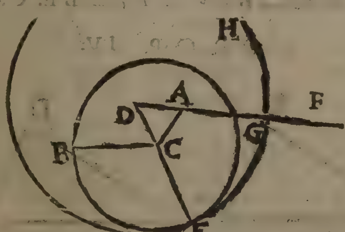
Centris A & B, eodem intervallo A B, vel B A *a* deſcribe duos circulos ſe interſecantes in puncto C; ex quo *b* duc rectas C A, C B. Erit A C *c* = A B *c* = B C *d* = A C. *e* Quare triangulum A C B eſt æquilaterum. Quod Erat Faciendum.

a 3. *poſt.*
b 1. *poſt.*
c 15. *def.*
d 1. *ax.*
e 23. *def.*

### *Scholium.*

Eodem modo ſuper A B deſcribetur triangulum Iſoſceles, ſi intervalla æqualium circulorum majora ſumantur, vel minora, quam A B.

## PROP. II.




*Ad datum punctum* A *datæ rectæ lineæ* B C *æqualem rectam lineam* A G *ponere.*

Centro C, intervallo C B *a* deſcribe circulum C B E. *b* Iunge A C, ſuper qua *c* fac triangulum æquilaterum A D C. *d* produc D C ad E. cen-

a 3. *poſt.*
b 1. *poſt.*
c 1. 1.
d 2. *poſt.*

centro D,ſpatio D E, deſcribe circulum D E H: cujus circumferentiæ occurrat D A e protracta ad G. Erit A G = C B.

e 2. poſt.
f 15. def.
g conſtr.
h 3. ax.
k 15. def.
l 1. ax.

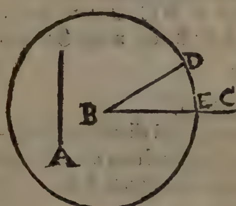
Nam D G f = D E, & D A g = D C, quare A G h = C E k = B C l = A G. Q. E. F.

Poſitio puncti A, intra vel extra datam B C, caſus variat, ſed ubique ſimilis eſt conſtructio, & demonſtratio.

*Scholium.*

Poterat A G circino ſumi, ſed hoc facere nulli poſtulato reſpondet, ut bene innuit Proclus.

PROP. III.



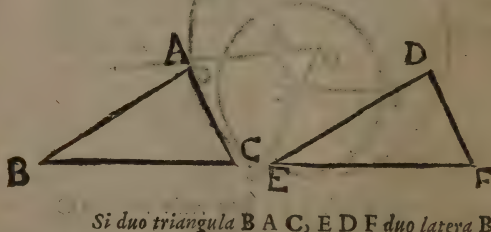

*Duabus datis rectis lineis* A, *&* B C, *de majore* BC *minori* A *æqualem rectam lineam* B E *detrahere.*

a 2. 1.

Ad punctum B a pone rectam B D = A. Circulus centro B, ſpatio B D deſcriptus auferet B E b = B D c = A d = B E. Q. E. F.

b 15. def.
c conſtr.
d 1. ax.

PROP. IV.



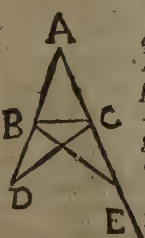

*Si duo triangula* B A C, E D F *duo latera* B A, A C *duobus lateribus* E D, D F *æqualia habeant, utrumque utrique* (hoc eſt B A = E D, *&* A C = D F) *habeant vero angulum* A, *angulo* D *æqualem,*

*lem, ſub æqualibus rectis lineis contentum, & baſim* B C *baſi* E F *æqualem habebunt; eritque triangulum* B A C *triangulo* E D F *æquale, ac reliqui anguli* B, C *reliquis angulis* E, F *æquales erunt, uterque utrique, ſub quibus æqualia latera ſubtenduntur.*

Si punctum D puncto A applicetur, & recta D E rectæ A B ſuperponatur, cadet punctum E in B, quia D F *a* = A B. Item recta D F cadet in A C, quia ang. A *a* = D. Quinetiam punctum F puncto C coincidet, quia A C *a* = D F. Ergo rectæ E F, B C, cum eoſdem habeant terminos, *b* congruent, & proinde æquales ſunt. Quare triangula B A C, E D F; & anguli B, E; itemque anguli C, F etiam congruunt, & æquantur. Quod erat Demonſtrandum.

a *hyp.*
b 14. *ax.*

## PROP. V.



*Iſoſcelium triangulorum* ABC *qui ad baſim ſunt anguli* A B C, A C B *inter ſe ſunt æquales. Et productis æqualibus rectis lineis* A B, A C *qui ſub baſe ſunt anguli* C B D, B C E *inter ſe æquales erunt.*

*a* Accipe A F = A D, & *b* junge C D, ac B F. a 3. 1.

b 1. *poſt.*

Quoniam in triangulis A C D, A B F, ſunt A B *c* = A C, & A F *d* = A D, anguluſq; A communis, *e* erit ang. A B F = A C D; & ang. A F B *e* = A D C, & baſ. B F *e* = D C; item F C *f* = D B. ergo in triangulis B F C, B D C *g* erit ang. F C B, = D B C. Q.E.D. Item ideo ang. F B C = D C B. atqui ang. A B F *h* = A C D. ergo ang. A B C *k* = A C B. Q. E. D.

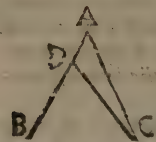
c *hyp.*
d *conſtr.*
e 4. 1.
f 3. *ax.*
g 4. 1.
h *pr.*
k 3. *ax.*

*Corollarium.*

Hinc, Omne triangulum æquilaterum eſt quoque æquiangulum.

PROP.

PROP. VI.




*Si trianguli* A B C *duo anguli* A B C, A C B *æquales inter ſe fuerint, & ſub æqualibus angulis ſubtenſa latera* A B, A C *æqualia inter ſe erunt.*

Si fieri poteſt, ſit utravis B A ⊏ C A, *a* Fac igitur B D = C A, & *b* duc C D.

a 3. 1.
b 1. *poſt.*

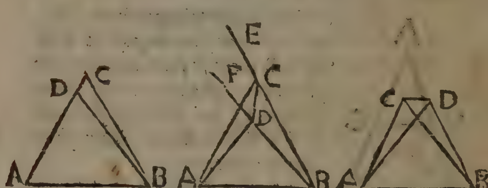
In triangulis DBC, ACB, quia BD *c* = CA, & latus BC commune eſt, atque ang. DBC *d* = A C B, *e* erunt triangula D B C, A C B æqualia inter ſe, pars & totum, *f* Quod Fieri Nequit.

c *ſuppoſ.*
d *hyp.*
e 4. 1.
f 9. *ax.*

*Coroll.*

Hinc, Omne triangulum æquiangulum eſt quoque æquilaterum.

PROP. VII.




*Super eadem recta linea* A B *duabus eiſdem rectis lineis* A C, B C, *aliæ duæ rectæ lineæ æquales* A D; B D, *utraque utrique* (*hoc eſt,* A D = AC, & B D = B C) *non conſtituentur ad aliud punctum* C, *atque aliud* D, *ad eaſdem partes* C, *eoſdemque terminos* A, B *cum duabus initio ductis rectis lineis habentes.*

a 9. *ax.* 1. *Caſ.* Si punctum D ſtatuatur in A C. *a* liquet non eſſe A D = A C.

2. *Caſ.* Si punctum D dicatur intra triangulum A C B, duc C D, & produc B D F, ac B C E. Iam vis A D = A C. ergo ang. ADC *b* = ACD; item quia BD *c* = BC, erit ang. FDC *b* = ECD.

b 5. 1.
c *ſuppoſ.*

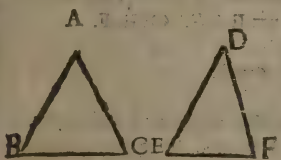
ergo

ergo ang. F D C *d* ⊏ A C D, id eſt ang. F D C ⊏ A D C *d* Q. E. N. d 9. *ax.*

3. *Caſ.* Sin D cadat extra triangulum A C B, jungatur C D.

Rurſus, ang. B C D *e* = BDC, & BCD *e* = B D C. *f* ergo ang. A C D ⊐ B D C. & proinde multo magis ang. B C D ⊐ B D C. Sed erat ang. B C D = B D C. Quæ repugnant. Ergo, &c. e 5. 1. f 9. *ax.*

## PROP. VIII.




*Si duo triangula* A B C, D E F *habuerint duo latera* A B, A C *duobus lateribus* D E, D F, *utrumque utrique æqualia; habuerint vero & baſim* B C, *baſi* E F, *æqualem: angulum* A *ſub æqualibus rectis lineis contentum angulo* D *æqualem habebunt.*

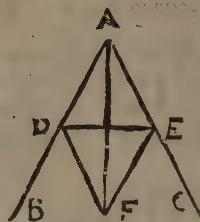
Quia B C *a* = E F, ſi baſis B C ſuperponatur baſi EF, illæ *b* congruent. ergo, cum AB *c* = DE, & AC *c* = D F, cadet punctum A in D. (nam in aliud punctum cadere nequit, per præcedentem) ergo angulorum A, & D latera coincidunt. *d* quare anguli illi pares ſunt. Q. E. D. a *hyp.* b 8. *ax.* c *hyp.* d 8. *ax.*

### *Coroll.*

1. Hinc triangula ſibi mutuo æquilatera, etiam mutuo *x* æquiangula ſunt. x 4. 1.

2. Triangula ſibi mutuo æquilatera *y* æquentur inter ſe. y 4. 1.

## PROP. IX.




*Datum angulum rectilineum* B A C *bifariam ſecare.*

a 3. 1. *a* Sume A D = A E;
b 1. 1. duc D E, ſuper qua *b* fac triang. æquilat. D F E.

Ducta A F angulum B A C biſecabit.

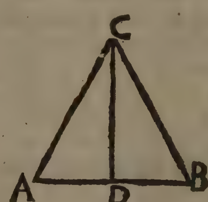
Nam A D *c* = A E,
c conſtr. & latus A F commune eſt, & baſ. D F *c* = F E.
d 8. 1. *d* ergo ang. D A F = E A F. Q. E. F.

### *Coroll.*

Hinc patet quomodo angulus ſecari poſſit in æquales partes 4, 8, 16, &c. Singulos nimirum partes iterum biſecando.

Methodus vero regula & circino angulos ſecandi in æquales quotcunque hactenus Geometras latuit.

## PROP. X.



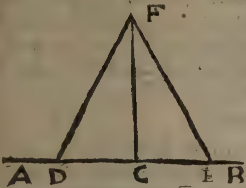

*Datam rectam lineam* A B *bifariam ſecare.*

a 1. 1. Super data A B *a* fac triang. æquilat. A B C.
b 9. 1. ejus angulum C *b* biſeca recta C D. Eadem datam A B biſecabit.

c conſtr. Nam A C *c* = B C, & latus C D eſt commune; & ang. A C D *c* =
d 4. 1. B C D, *d* ergo A D = B D. Q. E. F. Praxin hujus & præcedentis, conſtructio primæ hujus libri ſatis indicat.

PROP.

PROP. XI.



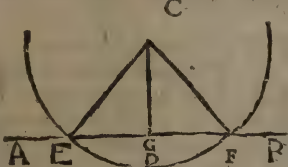

*Data recta linea AB, & puncto in ea dato C, rectam lineam CF ad angulos rectos excitare.*

a Accipe hinc inde CD = CE. Super DE b fac triang. æquilat. DFE. Ducta FC perpendicularis eſt. a 3. 1. b 1. 1.

Nam triangula DFC, EFC ſibi mutuo c æquilatera ſunt. d ergo ang. DCF = ECF. e ergo FC perpendicularis eſt. Q. E. F. c conſtr. d 8. 1. e 10. def.

Praxis tam hujus, quam ſequentis expeditur facillime ope normæ.

PROP. XII.

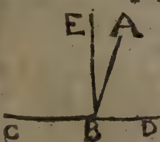



*Super datam rectam lineam infinitam AB, à dato puncto C quod in ea non eſt, perpendicularem rectam CG deducere.*

Centro C a deſcribe circulum, qui ſecet datam AB in punctis E & F b biſeca EF in G. ducta CG perpendicularis eſt. a 3. poſt. b 10. 1.

Ducantur enim CE, CF. Triangula EGC, FGC, ſibi mutuo c æquilatera ſunt. d ergo anguli EGC, FGC, æquales, & e proinde recti ſunt. Q. E. F. c conſtr. d 8. 1. e 10. def.

PROP. XIII.




*Cum recta linea AB, ſuper rectam lineam CD conſiſtens, facit angulos ABC, ABD; aut duos rectos, aut duobus rectis æquales efficiet.*

a 10. *def.* Si anguli ABC, ABD pares ſint, *a* liquet illos
b 11. 1. rectos eſſe; ſin inæquales ſint, ex B *b* excitetur
c 19. *ax.* perpendicularis B E. Quoniam ang. A B C *c* =
d 3. *ax.* Rect. + ABE; & ang. ABD *d* = Rect. − ABE;
e 2. *ax.* erit ABC + ABD *e* = 2 Rect. + ABE − ABE = 2 Rect. Q. E. D.

*Coroll.*

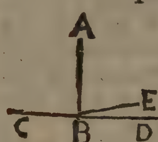
1. Hinc, ſi unus ang. A B D rectus ſit, alter A B C etiam rectus erit; ſi hic acutus, ille obtuſus erit, & contra.

2. Si plures rectæ quam una ad idem punctum eidem rectæ inſiſtant, anguli ſient duobus rectis æquales.

3. Duæ rectæ invicem ſecantes efficiunt angulos quatuor rectis æquales.

4. Omnes anguli circa unum punctum conſtituti conficiunt quatuor rectos. patet ex Coroll. 2.

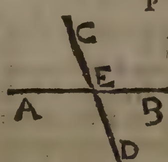
PROP. XIV.




*Si ad aliquam rectam lineam* A B, *atque ad ejus punctum* B *duæ rectæ lineæ* C B, B D *non ad eaſdem partes ductæ, eos qui ſunt deinceps angulos* A B C, A B D *duobus rectis æquales fecerint, in directum erunt inter ſe ipſæ rectæ lineæ* C B, B D.

a 13. 1. Si negas, faciant C B, B E unam rectam. ergo
b *hyp.* ang. A B C + ABE *a* = 2 Rect. *b* = ABC +
c 9. *ax.* A B D. *c* Quod Eſt abſurdum.

PROP. XV.

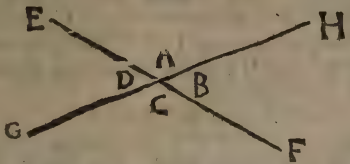



*Si duæ rectæ lineæ* A B, C D *ſe mutuo ſecuerint, angulos ad verticem* C E B, A E D *æquales inter ſe efficient.*

Nam ang. A E C + C E B
a 13. 1. *a* = 2 Rect. *a* = A E C +
b 3. *ax.* A E D. *b* Ergo C E B = A E D. Q. E. F.

*Schol.*

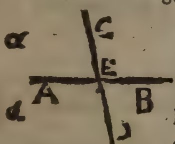
*Schol.*




Si ad aliquam rectam lineam G H, atque ad ejus punctum, A duæ rectæ lineæ E A, A F non ad eaſdem partes ſumptæ, angulos ad verticem D, & B æquales fecerint, ipſæ rectæ lineæ E A, A F in directum ſibi invicem erunt.

Nam 2 Rect. = *a* D + A *a* = B + A. *b* ergo EA, AF ſunt in directum ſibi invicem. Q.E.D. a 13. 1. b 14. 1.

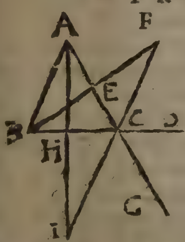
*Schol. 2.*




Si quatuor rectæ lineæ E A, E B, E C, E D ab uno puncto E exeuntes, angulos oppoſitos ad verticem æquales inter ſe fecerint, erunt quælibet duæ lineæ A E, E B, & C E, E D in directum poſitæ.

Nam quia ang. AEC + AED + CEB + DEB *a* = 4 Rect. erit A E C + A E D *b* = CEB + DEB = 2 Rect. *c* ergo CED, & AEB ſunt rectæ lineæ. Q. E. D. a 4 Cor. 13. 1 b 13. 1. & 2. ax. c 14. 1.

## PROP. XVI.



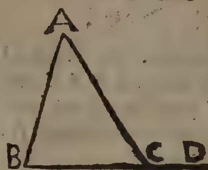

*Cujuſcunque Trianguli* A B C *uno latere* B C *producto, externus angulus* A C D *utrolibet interno & oppoſito* C A B, C B A, *major eſt.*

Latera A C, B C *a* biſecent rectæ A H, B E, è quibus productis *b* cape EF = B E, *b* & H I = A H, a 10. 1. & 1. poſt. b 3. 1.

Conjuganturque F C, I. B Quo-

c conſtr. Quoniam C E *c* = E A, & E F *c* = E B, &
d 15.1. ang. F E C *d* = BEA; *e* erit ang. ECF = EAB.
e 4.1. Simili argumento ang. ICH (*f* FCD) = ABH.
f 15.1. ergo totus ACD *g* major eſt utrovis CAB, &
g 9.ax. ABC. Q. E. D.

PROP. XVII

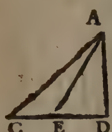



*Cujuſcunque trianguli* ABC *duo anguli duobus rectis ſunt minores, omnifariam ſumpti.*

Producatur latus B C. Quoniam ang. ACD +
a 13.1. ACB *a* = 2 Rect. & ang,
b 16.1. ACD *b* ⊏ A, *c* erit A + ACB ⊐ 2 Rect. Eo-
c 4.ax. dem modo erit ang. B + ACB ⊐ 2 Rect. Denique producto latere A B, erit ſimiliter ang. A + B ⊐ 2 Rect. Quæ E. D.

*Coroll.*

1. Hinc, in omni triangulo, cujus unus angulus fuerit rectus, vel obtuſus, reliqui acuti ſunt.

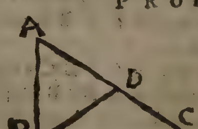



2. Si linea recta A E cum alia recta C D angulos inæquales faciat, unum AED acutum, & alterum A E C obtuſum, linea perpendicularis A D ex quovis ejus puncto A ad aliam illam CD demiſſa, cadet ad partes anguli acuti AED.

Nam ſi AC ad partes anguli obtuſi ducta, dicatur perpendicularis; in triangulo AFC erit ang.
x 17.1. AEC + ACE ⊏ 2 Rect. *x* Q. F. N.

3. Omnes anguli trianguli æquilateri, & duo anguli trianguli Iſoſcelis, ſupra baſim, acuti ſunt.

PROP. XVIII.



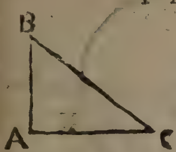

*Omnis trianguli* A B C *majus latus* A C *majorem angulum* ABC *ſubtendit.*

Ex A C *a* aufer A D =
a 3.1. A B, & junge D B. *b* ergo
b 5.1. ang. A D B = A B D. Sed

*c* ADB

*e* ADB $\sqsubset$ C. ergo ABD $\sqsubset$ C. *d* ergo totus ang. ABC $\sqsubset$ C. Eodem modo erit ABC $\sqsubset$ A. Q. E. D. c 16. 1. d 9 ax.

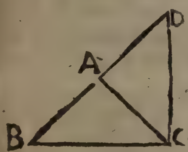
PROP. XIX.



*Omnis trianguli* A B C *major angulus* A *majori lateri* BC *ſubtenditur.*

Nam ſi dicatur A B = BC, *a* erit ang. A = C. contra Hypoth. & ſi A B $\sqsubset$ BC, *b* erit ang. C $\sqsubset$ A, contra hyp. quare potius B C $\sqsubset$ A B. & eodem modo B C $\sqsubset$ A C. Q. E. D. a 5. 1. b 18. 1.

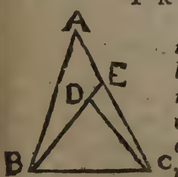
PROP. XX.



*Omnis trianguli* A B C *duo latera* B A, A C *reliquo* B C *ſunt majora quomodocunque ſumpta.*

Ex B A producta *a* cape A D = A C, & duc D C. *b* ergo ang. D = A C D. *c* ergo totus B C D $\sqsubset$ D *d* ergo B D (*e* B A + A C) $\sqsubset$ B C. Q. E. D. a 3. 1. b 5. 1. c 9. ax. d 19. 1, e conſtr. & 2. ax.

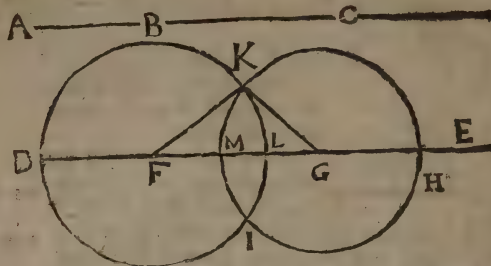
PROP. XXI.



*Si ſuper trianguli* A B C *uno latere* BC, *ab extremitatibus duæ rectæ lineæ* B D, C D, *interius conſtitutæ fuerint, hæ conſtitutæ reliquis trianguli duobus lateribus* B A, C A *minores quidem erunt, majorem vero angulum* BDC *continebunt.*

Producatur BD in E. eſtque CE + ED *a* $\sqsubset$ CD adde commune B D, *b* erit BE + EC $\sqsubset$ BD + DC. Rurſus BA + AE *a* $\sqsubset$ BE; *b* ergo BA + AC $\sqsubset$ BE + EC. quare B A + AC $\sqsubset$ B D + D C. Q. E. D. 2. Ang. B D C *c* $\sqsubset$ DEC *c* $\sqsubset$ A. ergo ang. BDC $\sqsubset$ A. Q. E. D. a 20 1. b 4. ax. c 16. 1.

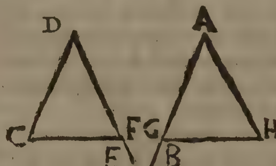
## PROP. XXII.




*Ex tribus rectis lineis* F K, F G, G K, *quæ ſint tribus datis rectis lineis* A, B, C, *æquales, triangulum* F K G *conſtituere. Oportet autem duas reliqua eſſe majores omnifariam ſumptas; quoniam uniuſcujuſque trianguli duo latera omnifariam ſumpta reliquo ſunt majora.*

a 3. 1. Ex infinita DE *a* ſume DF, FG, GH datis
b 3. poſt. A, B, C ordine æquales. Tum ſi *b* centris F, & G, intervallis F D, & G H ducantur circuli ſe interſecantes in K; junctis rectis K F, K G con-
c 15. def. ſtituetur triangulum F K G, *c* cujus latera F K,
d 1. ax. F G, G K tribus DF, F G, G H, *d* id eſt tribus datis A, B, C æquantur. Q. E. F.

## PROP. XXIII.



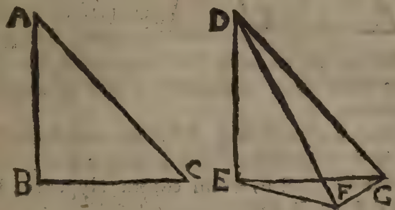

*Ad datam rectam lineam* A B, *datumque in ea punctum* A, *dato angulo rectilineo* D *æquale angulum rectilineum* A *conſtituere.*

*a* Duc rectam C F ſecantem dati anguli latera
a 1. poſt. utcunque. *b* Fac AG = CD. Super AG *c* con-
b 3. 1.
c 22. 1. ſtitue triangulum alteri C D F æquilaterum, ita

ut

ut AH = DF, & GH = CF; & habebis ang. A $^{d}$ = D. Q. E. F. d 8. 1.

## PROP. XXIV.




*Si duo triangula* A B C, D E F *duo latera* A B, A C *duobus lateribus* D E, D F *æqualia habuerint, utrumque utrique; angulum vero* A *angulo* EDF *majorem ſub æqualibus rectis lineis contentum, & baſim* BC, *baſi* EF, *majorem habebunt.*

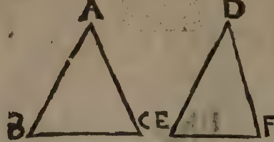
*a* Fiat ang. EDG = A, & DG *b* = D F *c* = A C, connectanturque E G, F G. a 23. 1. b 3. 1. c hyp.

1. *Caſ.* Si E G cadit ſupra E F. Quia A B *d* = D E, & A C = *e* DG, & ang. A *e* = EDG, *f* erit B C = E G. Quia vero D F *e* = D G, *g* erit ang. DFG = DGF. *h* ergo ang. DFG $\sqsubset$ EGF; *h* & proinde ang. EFG $\sqsubset$ EGF. *k* quare E G (B C) $\sqsubset$ E F. Q. E. D. d hyp. e conſtr. f 4. 1. g 5. 1. h 9. ax. k 19. 1.

2. *Caſ.* Si baſis E F baſi E G coincidat, *l* liquet E G (B C) $\sqsubset$ E F. l 9 ax.

3. Sin E G Cadat infra E F. Quoniam D G + G E *m* $\sqsubset$ D F + F E, ſi hinc inde auferantur D G, D F, æquales, manet E G (B C) *n* $\sqsubset$ E F. Q. E. D. m 21. 1. n 5. ax.

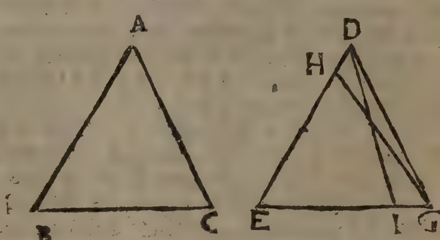
## PROP. XXV.




*Si duo triangula* ABC, DEF *duo latera* AB, AC *duobus lateribus* DE, DF *æqualia habuerint, utrumq; utrique, baſim vero* BC *baſi* EF *majorem; & angulum* A *ſub æqualibus rectis lineis contentum angulo* D *majorem habebunt.*

a 4. 1. Nam ſi dicatur ang. A = D. a erit baſis BC = EF, contra Hyp. Sin dicatur ang. A $\sqsupset$ D,
b 24. 1. b erit BC $\sqsupset$ EF, etiam contra Hyp. ergo BC $\sqsubset$ EF. Q.E.D.

## PROP. XXVI.




*Si duo triangula* BAC EDG, *duos angulos* B, C, *duobus angulis* E, DGE, *æquales habuerint, utrumque utrique, unumque latus uni lateri æquale, ſive quod æqualibus adjacet angulis, ſeu quod uni æqualium angulorum ſubtenditur: reliqua latera reliquis lateribus æqualia, utrumque utrique, & reliquum angulum reliquo angulo æqualem habebunt.*

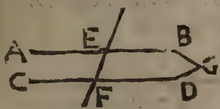
1. *Hyp.* Sit BC = EG. Dico BA = ED, & AC = DG, & ang. A = EDG. Nam ſi dicatur
a 3. 1. ED $\sqsubset$ BA, a fiat EH = BA, ducaturque GH.

Quoniam

Quoniam AB *b* = HE, & BC *c* = EG; & ang. B *c* = E, erit ang. EGH *d* = C *e* = DGE. *f* Q. E. A. ergo AB = ED. Eodem modo AC = DG. *d* quare etiam ang. A = EDG. b *ſuppoſ.* c *hyp.* d 4. 1. e *hyp.* f 9. *ax.*

2. *Hyp.* Sit AB = ED. Dico BC = EG; & AC = DG & ang. A = EDG. Nam ſi dicatur EG ⊏ BC, fiat EI = BC, & connectatur DI. Quia AB *g* = ED, & BC *h* = EI, & ang B *g* = E, erit ang. EID *k* = C *m* = EGD. *n* Q. E. A. ergo BC = EG. ergo ut prius, AC = DG, & ang. A = EDG. Q. E. D. g *hyp.* h *ſuppoſ.* k 4. 1. m *hyp.* n 16. 1.

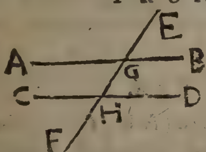
PROP. XXVII.




*Si in duas rectas lineas AB, CD recta incidens linea EF alternatim angulos AEF, DFE, æquales inter ſe fecerit, parallelæ erunt inter ſe illæ rectæ lineæ AB, CD.*

Si AB, CD dricantur non eſſe parallelæ; conveniant productæ, nempe in G. quo poſito angulus externus AEF interno DFE *a* major erit, cui tamen ponitur æqualis. Quæ repugnant. a 16. 1.

PROP. XXVIII.

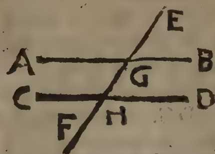



*Si in duas rectas lineas AB, CD recta incidens linea EF externum angulum AGE interno & oppoſito, & ad eaſdem partes CHG æqualem fecerit, aut internos & ad eaſdem partes AGH, CHG duobus rectis æquales; parallelæ erunt inter ſe ipſæ rectæ lineæ AB, CD.*

1. *Hyp.* Quia per hyp. ang. AGE = CHG, *a* erit altern. BGH = CHG. *b* parallelæ igitur ſunt AB, CD. Q. E. D. a 15. 1. b 27. 1.

2. *Hyp.* Quia ex hyp. Ang. AGH + CHG = 2 Rect. *a* = AGH + BGH; *b* erit CHG = BGH. Ergo *c* AB, CD parallelæ ſunt. Q. E. D. a 13. 1. b 3. *ax.* c 27. 1.

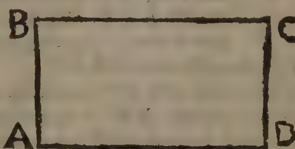
## PROP. XXIX.




*In parallelas rectas lineas* A B, C D, *recta incidens linea* E F, *& alternatim angulos* D H G, A G H *æquales inter ſe efficit; & externum* B G E *interno, & oppoſito, & ad eaſdem partes* DHE *æqualem; & internos & ad eaſdem partes* A G H, C H G *duobus rectis æquales facit.*

a 13. ax. Liquet A G H, + C H G = 2 Rect. *a* alias A B, C D non eſſent parallelæ, contra hyp. Sed
b 13. 1. & ang. DHG + CHG *b* = 2 Rect. ergo DHG
c 13. ax. d 15. 1. *c* = A G H *d* = B G E. Q. E. D.

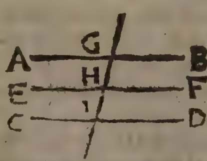
*Coroll.*




Hinc omne Parallelogrammum A C habens unum angulum rectum A, eſt rectangulum.

29. 1. Nam A + B *a* = 2 Rect. ergo cum A re-
a 3. ax. ctus ſit, *b* etiam B rectus erit. Eodem argumento D, & C recti ſunt.

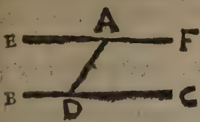
## PROP. XXX.




*Quæ* (A B, C D) *eidem rectæ lineæ* E F *parallelæ, & inter ſe ſunt parallelæ.*

Tres rectas ſecet utcunque recta G I. Quoniam A B, E F parallelæ ſunt,
a 29. 1. *a* erit ang. AGI = EHI, Item propter CD, EF
b 1. ax. parallelas, *a* erit ang. EHI = DIG. *b* ergo ang.
c 27. 1. AGI = D I G. *c* quare AB, CD parallelæ ſunt. Q. E. D.

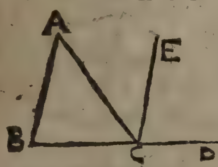
## Prop. XXXI.




*A dato puncto* A *datæ rectæ lineæ* B C *ducere parallelam rectam lineam* A E.

Ex A ad datam BC duc rectam utcunque AD. ad quam, ejuſque punctum a 23. 1.
A *a* fac ang. D A E = A D C. *b* erunt A E, BC b 27. 1.
parallelæ. Q. E. F.

## Prop. XXXII.




*Cujuſcunque trianguli* A B C *uno latere* B C *producto, externus angulus* A C D *duobus internis, & oppoſitis,* A B *eſt æqualis. Et trianguli tres interni anguli,* A, B, A C B *duobus ſunt rectis æquales.*

Per C *a* duc C E parall. B A. Ang. A *b* = a 31. 1.
ACE. & ang. B *b* = ECD. ergo A + B *c* = b 29. 1.
A C E + E C D *d* = A C D. Q. E. D. Pono c 2. ax. d 19. ax.
ACD + ACB *e* = 2 Rect. *f* ergo A + B + e 13. 1. f 1. ax.
A C B = 2 Rect. Q. E. D.

*Corollaria.*

1. Tres ſimul anguli cujuſvis trianguli æquales ſunt tribus ſimul cujuſcunque alterius. Unde

2. Si in uno triangulo duo anguli (aut ſinguli, aut ſimul) æquales ſint duobus angulis (aut ſingulis, aut ſimul) in altero triangulo, etiam reliquus reliquo æqualis eſt. Item, ſi duo triangula unum angulum uni æqualem habeant, reliquorum ſummæ æquantur.

3. In triangulo ſi unus angulus rectus ſit, reliqui unum rectum conficiunt. Item, angulus, qui duobus reliquis æquatur, rectus eſt.

4. Cum in Iſoſcele angulus æquis cruribus contentus rectus eſt, reliqui ad baſim ſunt ſemirecti.

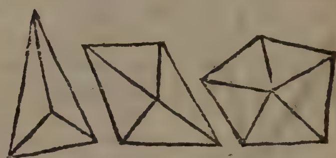
5. Tri-

5. Trianguli æquilateri angulus facit duas tertias unius recti, nam $\frac{1}{3}$ 2 Rect. $=$ $\frac{2}{3}$ Rect.

*Schol.*

Hujus propoſitionis beneficio, cujuſlibet figuræ rectilineæ tam interni quam externi anguli quot rectos conficiant, innoteſcet per duo ſequentia theoremata.

THEOREMA 1.



*Omnes ſimul anguli cujuſcunque figuræ rectilineæ conficiunt bis tot rectos demptis quatuor, quot ſunt latera figuræ.*

Ex quovis puncto intra figuram ducantur ad omnes figuræ angulos rectæ, quæ figuram reſolvent in tot triangula quot habet latera. Quare cum ſingula triangula conficiant duos rectos, omnia ſimul conficient bis tot rectos, quot ſunt latera. Sed anguli circa dictum punctum conficiunt quatuor rectos. Ergo, ſi ab omnium triangulorum angulis demas angulos circa id punctum, anguli reliqui qui componunt angulos figuræ conficient bis tot rectos demptis quatuor, quot ſunt latera figuræ. Q. E. D.

Hinc *Coroll.* Omnes ejuſdem ſpeciei rectilineæ figuræ æquales habent angulorum ſummas.

THEOREMA 2.

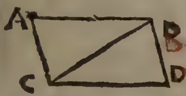
*Omnes ſimul externi anguli cujuſcunque figuræ rectilineæ conficiunt quatuor rectos.*

Nam ſinguli figuræ interni anguli cum ſingulis externis conficiunt duos rectos. Ergo interni

terni ſimul omnes, cum omnibus ſimul externis conficiunt bis tot rectos, quot ſunt latera figuræ. Sed (ut modo oſtenſum eſt,) interni ſimul omnes etiam cum quatuor rectis efficiunt bis tot rectos, quot ſunt latera figuræ. Ergo externi anguli quatuor rectis æquantur. Q. E. D.

*Coroll.* Omnes cujuſcunque ſpeciei rectilineæ figuræ æquales habent externorum angulorum ſummas.

## PROP. XXXIII.

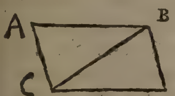


*Rectæ lineæ* A C, B D, *quæ æquales & parallelas lineas* AB, CD, *ad partes eaſdem conjungunt, & ipſæ æquales ac parallelæ ſunt.*

Connectatur C B. Quoniam ob A B; C D parallelas. ang. ABC *a* = BCD, & per hyp. AB = CD, & latus CB commune eſt, *b* erit AC = BD, *b* & ang. A C B = D B C. *c* ergo A C, B D etiam parallelæ ſunt. Q. E. D.

a 29. 1. b 4. 1. c 27. 1.

## PROP. XXXIV.



*Parallegrammorum ſpatiorum* ABDC *æqualia ſunt inter ſe quæ ex adverſo latera* A B, CD; ac A C, B D; *angulique* A, D, & ABD, ACD, *& illa bifariam ſecat diameter* C B.

Quoniam AB, CD *a* parallelæ ſunt, *b* erit ang. ABC = BCD. Item ob AC, DB *a* parallelas, *b* erit ang. ACB = CBD. *c* ergo toti anguli A C D, A B D æquantur. Similiter ang. A = D. Porro, cum communi lateri CB adjaceant anguli A B C, A C B, ipſis B C D, C B D pares *d*, erunt AC = BD, *d* & AB = CD. adeoque etiam triang. ABC = CBD. Quæ E. D.

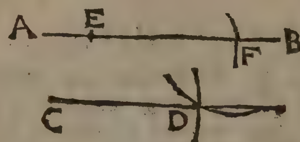
a hyp. b 29. 1. c 2. ax. d 26. 1.

*SCHOL.*

SCHOL.

*Omne quadrilaterium* A B D C *habens latera oppoſita æqualia, eſt parallelogrammum.*

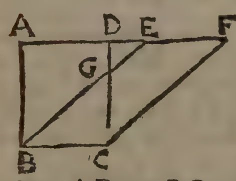
a 27. 1. Nam per 8. 1. ang. A B C = B C D. a ergo A B, C D parallelæ ſunt. Eadem ratione ang. BCA = CBD; a quare A C, B D etiam paralle-
b 35. def. 1. læ ſunt. b Ergo ABDC eſt parallelogrammum. Q. E. D.




Hinc expeditius per datum punctum C datæ rectæ A B ducetur parallela C D.

Sume in A B quodvis punctum E. centris E. & C ad quodvis intervallum duc æquales circulos E F, C D. centro vero F, ſpatio E C duc circulum F D, qui priorem C D ſecet in D. Erit ducta C D parall. A B. Nam ut modo demonſtratum eſt, C E F D eſt parallelogrammum.

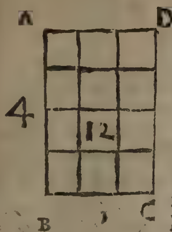
PROP. XXXV.




*Parallelogramma* BCDA, BCFE *ſuper eadem baſi* B C, *& in eiſdem parallelis* AF, BC *conſtituta, inter ſe ſunt æqualia.*

a 34. 1. Nam A D a = B C a = E F. adde commu-
b 2. ax. nem DE, b erit AE = DF. Sed & AB a = DC;
c 29. 1. & ang. A c = C D F. d ergo triang. A B E =
d 4. 1. D C F. aufer commune D G E, e erit Trapez.
e 3. ax. ABGD = EGCF. adde commune BGC, f erit
f 2. ax. Pgr. ABCD = EBCF, Q. E. D. Reliquorum caſuum non diſſimilis, ſed ſimplicior & facilior eſt demonſtratio.

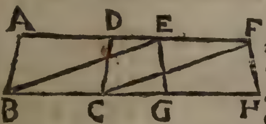
*Scholium.*




Si latus A B parallelogrammi rectanguli A B C D ferri intelligatur perpendiculariter per totam B C, aut B C per totam A B, producetur eo motu area rectanguli A B C D. Hinc rectangulum fieri dicitur ex ductu ſeu multiplicatione duorum laterum contiguorum. Sit exempl. gr. B C pedum 3, A B 4. Duc 3 in 4; proveniunt 12 pedes quadrati pro area rectanguli.

Hoc ſuppoſito, ex hoc theoremate cujuſcunq; parallelogrammi (* EBCF) habetur dimenſio. Illius enim area producitur ex altitudine B A ducta in baſim B C. Nam area rectanguli A C parallelogrammo E B C F æqualis, fit ex B A in B C. ergo, &c. * v. fig. propoſ. 35.

## PROP. XXXVI.

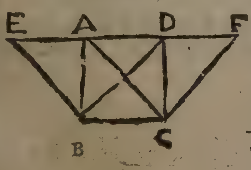



*Parallelogramma* B C D A, G H F E *ſuper æqualibus baſibus* BC, G H, *& in eiſdem parallelis* AF, BH *conſtituta, inter ſe ſunt æqualia.*

Ducantur BE, CF. Quia BC *a* = GH *b* = EF, *c* erit BCFE parallelogrammum. ergo Pgr. BCDA *d* = BCFE *d* GHFE. Q. E. D.

a hyp.
b 34. 1.
c 33. 1.
d 35. 1.

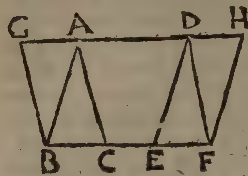
## PROP. XXXVII.




*Triangula* BCA, B G D *ſuper eadem baſi* BC *conſtituta, & in eiſdem paral-ſlis* B C, E F *inter ſe ſunt æqualia.*

a 31. 1. b 34. 1 c 35. 1. & 7. ax. a Duc B E parall. C A, a & C F parall. B D. Erit triang. B C A b = ½ Pgr. B C A E = c ½ B D F C b = B C D. Q. E. D.

## PROP. XXXVIII.



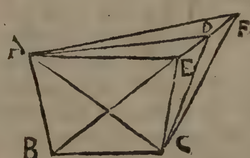

*Triangula* BCA, EFD *ſuper æqualibus baſibus* B C, E F *conſtituta, & in eiſdem parallelis* G H, B F, *inter ſe ſunt æqualia.*

Duc B G parall. C A. & F H parall. E D.
a 34. 1. b 36. 1 & 7. ax. c 34. 1. erit triang. B C A a = ½ Pgr. B C A G b = ½ E D H F c = E F D. Q. E. D.

### Schol.

Si baſis B C ⊏ E F, liquet triang. B A C ⊏ EDF. & ſi BC ⊐ EF, erit BAC ⊐ E D F.

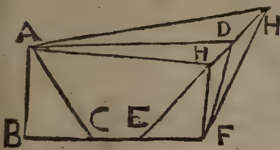
## PROP. XXXIX.




*Triangula æqualia* B C A, B C D, *ſuper eadem baſi* B C, *& ad eaſdem partes conſtituta, etiam in eiſdem ſunt parallelis* AD, B C.

Si negas, ſit altera A F parall. B C; & ducatur
a 37. 1. b hyp. c 9. ax. CF. ergo triang. C B F a = C B A b = C B D. c Q. E. A.

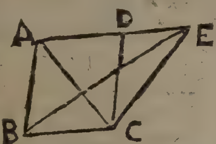
## PROP. XL.




*Triangula æqualia* B C A, E F D *ſuper æqualibus baſibus* B C, E F, *& ad eaſdem partes conſtituta, & in eiſdem ſunt parallelis* A D, B F.

Si negas, ſit altera AH parall. BF. & ducatur F H. ergo triang. E F H a = B C A b = EFD. c Q. E. A. a 38. 1. b hyp. c 9. ax.

## PROP. XLI.




*Si parallelogrammum* A B C D *cum triangulo* B C E *eandem baſim* B C *habuerit, in eiſdemque fuerit parallelis* A E, B C, *duplum erit parallelogrammum* ABCD *ipſius trianguli* BCE.

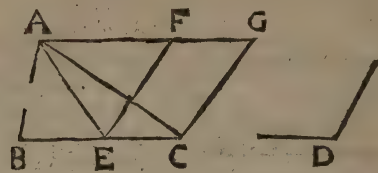
Ducatur AC. Triang. BCA a = BCE. ergo Pgr. ABCD b = 2 BCA c = 2 BCE. Q. E. D. a 37. 1. b 34. 1. c 6 ax.

### *Scholium.*

Hinc habetur area cujuſcunq; trianguli BCE. Nam cum area parallelogrammi ABCD producatur ex altitudine in baſim ducta; producetur area trianguli ex dimidia altitudine in baſim ducta, vel ex dimidia baſi in altitudinem. ut ſi baſis B C ſit 8, & altitudo 7; erit trianguli B C E area, 28.

PROP.

## PROP. XLII.

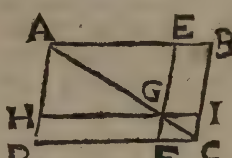



*Dato triangulo* A B C *æquale parallelogrammum* E C G F *conſtituere in dato angulo rectilineo* D.

a 31. 1. Per A *a* duc A G parall. B C. *b* fac ang. B C G
b 23. 1. = D. baſim B C *c* biſeca in E. *a* duc E F parall.
c 10. 1. C G. Dico factum.

Nam ducta A E. erit ex conſtr. ang. E C G
d 38. 1. = D, & triang. B A C *d* = 2 A E C *e* = Pgr.
e 41. 1. E C G F. Q. E. F.

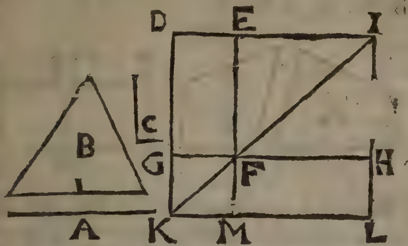
## PROP. XLIII.




*In omni parallelogrammo* ABCD *complementa* D G, G B *eorum quæ circa diametrum* AC *ſunt parallelogrammorum* HE, F I *inter ſe ſunt æqualia.*

a 34. 1. Nam Triang. ACD, = *a* ACB. & triang.
b 3. ax. AGH *a* = AGE. & triang. GCF *a* = GCI.
*b* ergo Pgr. D G = G B. Q. E. D.

## PROP. XLIV.

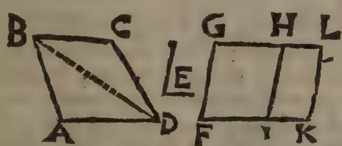



*Ad datam rectam lineam* A, *dato triangulo* B, *æquale parallelogrammum* FL *applicare in dato angulo rectilineo* C.

*a* Fac Pgr. F D = triang. B, ita ut ang. G F E = C. & pone lateri G F in directum F H = A. Per H *b* duc I L parall. E F; cui occurrat D E producta ad I. per I F ductæ rectæ occurrat D G protracta ad K. Per K *b* duc K L parall. G H; cui occurrant E F, & I H prolongatæ ad M, & L. Erit F L. Pgr. quæſitum. a 42. 1. b 31. 1.

Nam Pgr. FL *c* = F D = B *d* & ang. M F H = G F E = C. Q. E. F. c 43. 1. d 15. 1.

## PROP. XLV.

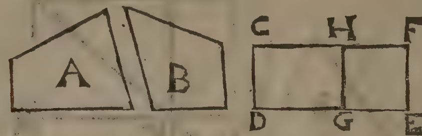



*Ad datam rectam lineam* F G *dato rectilineo* ABCD *æquale parallelogrammum* F L *conſtituere, in dato angulo rectilineo* E.

Datum rectilineum reſolve in triangula BAD, BCD. *a* = Fac Pgr. FH = BAD ita ut ang. F = E. producta F I *a* fac (ad H I) Pgr. a 44. 1.

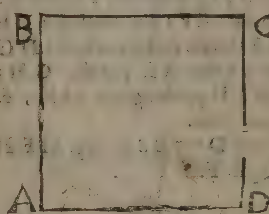
b 19. ax. c conſtr. IL = BCD. erit Pgr. FL = b FH + IL c = ABCD. Q. E. F.

*Schol.*




Hinc facile invenitur exceſſus HE, quo rectilineum aliquod A ſuperat rectilineum minus B; nimirum ſi ad quamvis rectam C D applicentur Pgr. DF = A. & DH = B.

PROP. XLVI.




*A data recta linea A D quadratum A C deſcribere.*

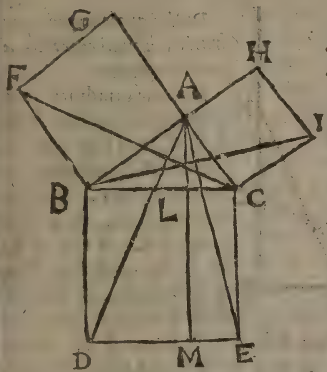
a 11. 1. b 3. 1. a Erige duas perpendiculares A B, D C b æquales datæ A D; & junge B C. dico factum.

c conſtr. d 28. 1. e conſtr. f 34. 1. g Sch. 29. 1. h 29. def. Cum enim ang. A + D c = 2 Rect. d erunt AB, DC parallelæ. Sunt vero etiam e æquales, f ergo A D, B C pares etiam ſunt, & parallelæ. ergo Figura AC eſt parallelogramma, & æquilatera. Anguli quoque omnes recti ſunt, g quoniam unus A eſt rectus. h ergo AC eſt quadratum. Q. E. F.

Eodem modo facile deſcribes rectangulum, quod ſub datis duabus rectis contineatur.

PROP.

## PROP. XLVII.




*In rectangulis triangulis* BAC *quadratum* BE, *quod à latere* BC *rectum angulum* BAC *ſubtendente deſcribitur, æquale eſt eis,* BG, CH, *quæ à lateribus* AB, AC *rectum angulum continentibus deſcribuntur.*

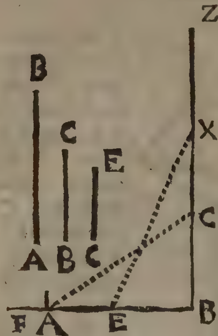
Iunge AE, AD; & duc AM. parall. CE.

Quoniam ang. DBC *a* = FBA; adde com- a 12.ax.
munem ABC, erit ang. ABD = FBC. Sed &
AB *b* = FB, & BD *b* = BC. *c* ergo triang. b 29.def.
ABD = FBC. atqui Pgr. BM. *d* = 2 ABD; & c 4. 1. d 41. 1.
Pgr. BG *d* = 2 FBC (nam GAC eſt una recta e 6.ax.
per hyp. & 14. 1.) *e* ergo Pgr. BM = BG. Simili diſcurſu Pgr. CM = CH. Totum igitur BE = BG + CH. Q. E. D.

*Schol.*

Hoc nobiliſſimum, & utiliſſimum theorema ab inventore Pythagora, Pythagoricum dici meruit. Ejus beneficio quadratorum additio, & ſubſtractio perficitur; quo ſpectant duo ſequentia problemata.

PROBL. I.

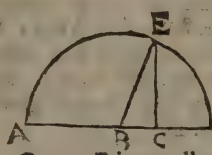



Andr. Tacq.

*Datis quotcunque quadratis, unum omnibus æquale conſtruere.*

Dentur quadrata tria, quorum latera ſint A B, B C, C E. a 11. 1. *a* Fac ang. rectum F B Z infinita habentem latera, in eaque transfer B A, & B C, & junge A C, b 47. 1. *b* erit A Cq = A Bq + B Cq. Tum A C transfer ex B in X; & C E tertium latus datum transfer ex B in E, & junge E X, *b* erit c 2 ax. EXq = EBq (CEq) + BXq (ACq) *c* = CEq + ABq + BCq. Q. E. F.

PROBL. 2.

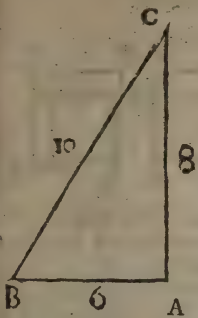



*Datis duabus rectis inæqualibus* A B, B C, *exhibere quadratum, quo quadratum majoris* A B *excedit quadratum minoris* B C.

Centro B intervallo BA deſcribe circulum. ex C erige perpendicularem C E occurrentem peripheriæ in E. & ducatur B E. a 47. 1. b 3. ax. *a* Erit B Eq (BAq) = BCq + CEq. *b* ergo BAq − BCq = CEq. Q. E. F.

PROBL.

PROBL. 3.

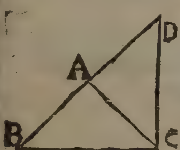



*Notis duobus quibuſcunque lateribus trigoni rectanguli* A B C, *reliquum invenire,*

Latera rectum angulum ambientia ſint A C, A B, hoc 6. pedum, illud 8. ergo cum A C q + A Bq = 64 + 36 = 100 = B Cq. erit BC = $\sqrt{}$ 100 = 10. 47. 1.

Nota ſint deinde latera A B, B C, hoc 10. pedum, illud 6. ergo cum B Cq − A Bq = 100 − 36 = 64 = A Cq. erit Acq = $\sqrt{}$ 64 = 8. 47. 1.

PROP. XLVIII.




*Si quadratum quod ab uno latere* B G *trianguli deſcribitur, æquale ſit eis quæ à reliquis trianguli lateribus* A B, A C *deſcribuntur quadratis, angulus* B A C *comprehenſus ſub* A B, A C *reliquis duobus trianguli lateribus, rectus eſt.*

Duc ad AC perpendicularem D A = A B, & junge C D.

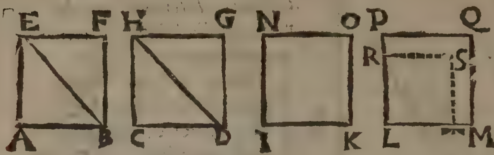
Iam C Dq *a* = A Dq + A Cq = A Bq + A Cq = B Cq. * ergo C D = B C. ergo triangula C AB, CAD, ſibi mutuo æquilatera ſunt; quare ang. CAB *b* = CAD *c* = Rect. Q.E.D.

a 47. 1.
* *Vide ſeq. Theor.*
b 8. 1.
c *hyp.*

*Schol.*

Aſſumpſimus exinde quod CDq. = BCq, ſequi CD = BC. Hoc vero manifeſtum fiet ex ſequenti theoremate.

*THEOREMA.*




*Linearum æqualium* A B, C D, *æqualia ſunt quadrata* A F, C G; *& quadratorum æqualium* N K, P M *æqualia ſunt latera* I K, L M.

Pro 1 Hyp. Duc diametros E B, H D. Liquet A F = *a* 2 triang. E A B = *b* 2 triang. H C D = *a* C G. Q. E. D.

a 34. 1.
b 4. 1. & 6. ax.

2. Hyp. Si fieri poteſt, ſit L M ⊏ I K. fac L T = I K; *a* ſitque L S = L Tq. ergo L S *b* = NK *c* = LQ. *d* Q. E. A. ergo LM = IK.

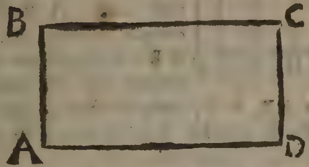
a 46. 1.
b 1. part.
c hyp.
d 9. ax.

*Coroll.*

Eodem modo quælibet rectangula inter ſe æquilatera æqualia oſtendentur.

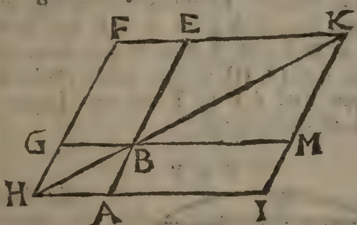
# LIB. II.

## Definitiones.



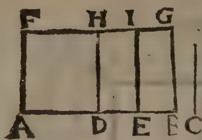

I. OMne parallelogrammum rectangulum A B C D contineri dicitur ſub rectis duabus A B & A D, quæ rectum comprehendunt angulum.

*Quando igitur dicitur rectangulum ſub* B A, A D; *vel brevitatis cauſa, rectangulum* B A D, *vel* B A x AD, (*vel* Z A *pro* Z x A) *deſignatur rectangulum, quod continetur ſub* B A, & A D *ad rectum angulum conſtitutis.*




II. In omni parallelogrammo ſpatio FHIK unumquodq; eorum, quæ circa diametrum illius ſunt, parallelogrammorum, cum duobus complementis Gnomon vocetur. *ut Pgr* FB + BI + GA (EHM) *eſt Gnomon. item* Pgr. FB + BI + EM (GKA) *eſt Gnomon.*

PROP. I.




*Si fuerint duæ rectæ lineæ* A B, A F, *ſeceturque ipſarum altera* A B *in quotcunque ſegmenta* AD, DE, EB: *rectangulum comprehenſum ſub illis duabus rectis lineis* A B, A F, *æquale eſt eis, quæ ſub inſecta* A F, *& quolibet ſegmentorum* A D, D E, E B *comprehenduntur rectangulis.*

a 11. 1. *a* Statue A F, perpendicularem ad A B. *a* per F duc infinitam F G perpendicularem ad A F. *a* Ex D, E, B erige perpendiculares D H, E I, B G. erit A G rectangulum ſub A F, A B, &

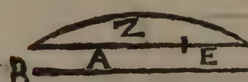
b 19 ax. 1. *b* eſt æquale rectangulis A H, D I, E G, hoc eſt

c 34. 1. (quia D H, E I, A F *c* pares ſunt) rectangulis ſub A F, A D; ſub A F, D E; ſub A F, E B. Q. E. D.

*Schol.*

Propoſitiones decem primæ hujus libri valent etiam in numeris. Reliquas quilibet tyro examinet. pro hac, ſit A F 6, & A B 12, ſectus in A D 5, D E 3, & E B 4. Eſtque 6x12 (A G) = 72. 6x5 (A H) = 30. 6 in 3 (D I) = 18. denique 6x4 (E G) = 24. Liquet vero 30 + 18 + 24 = 72.

PROP. II.



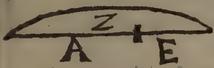

*Si recta linea* Z *ſecta ſit utcunque; rectangula, quæ ſub tota* Z, *& quolibet ſegmentorum* A, E *comprehenduntur, æqualia ſunt ei, quod à tota* Z *fit, quadrato.*

Dico Z A + Z E = Zq. Nam ſume B = Z.

a 1. 2. *a* Eſtque BA + BE = BZ; hoc eſt (ob B = Z) ZA + ZE = Zq. Q. E. D.

PROP.

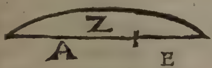
## PROP. III.




*Si recta linea Z ſecta ſit utcunque; rectangulum ſub tota Z, & uno ſegmentorum E comprehenſum, æquale eſt illi, quod ſub ſegmentis A, E comprehenditur, rectangulo, & illi quod à prædicto ſegmento E deſcribitur, quadrato.*

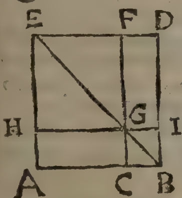
Dico. $ZE = AE + Eq$. *a* Nam $EZ = EA + EE$. a 1. 2.

## PROP. IV.




*Si recta linea Z ſecta ſit utcunque; Quadratum, quod à tota Z deſcribitur, æquale eſt, & illis quæ à ſegmentis A, E deſcribuntur, quadratis, & ei, quod bis ſub ſegmentis A, E comprehenditur, rectangulo.*

Dico $Zq = Aq. + Eq. 2\,AE$. *a* Nam $ZA = Aq + AE$. *a* & $ZE = Eq + AE$. quum igitur $ZA + ZE$ *b* $= Zq$, *c* erit $Zq = Aq + Eq + 2\,AE$. Q. E. D. a 3. 2. b 2. 2. c 1. ax.




*Aliter.* Super AB fac quadratum AD, cujus diameter EB. per diviſionis punctum C duc perpendicularem CF; & per G duc HI parall. AB.

Quoniam ang. $EHG = A$ rectus eſt, & AEB *d* ſemirectus, *e* erit reliquus HGE etiam ſemirectus. Ergo HE *f* $=$ HG *g* $=$ EF *g* $=$ AC. *h* proinde HF quadratum eſt rectæ AC. eodem modo CI eſt CBq. ergo AG, GD rectangula ſunt ſub AC, CB. Quare totum quadratum AD *k* $= ACq + CBq + 2\,ACB$. Q. E. D. d 4. Cor. 32. 1. e 32. 1. f 6. 1. g 34. 1. h 19. def. 1. k 19. ax. 1.

Coroll.

*Coroll.*

1. Hinc liquet parallelogramma circa diametrum quadrati eſſe quadrata.

2. Item diametrum cujuſvis quadrati ejus angulos biſecare.

3. Si $A = \frac{1}{2} Z$; erit $Zq = 4\, Aq$, & $Aq = \frac{1}{4} Zq$. item è contra, ſi $Zq = 4\, Aq$. erit $A = \frac{1}{2} Z$.

## PROP. V.




*Si recta linea* A B *ſecetur in æqualia* A C *&* CB, *& non æqualia* A D, D B, *rectangulum ſub inæqualibus ſegmentis* A D, D B *comprehenſum una cum quadrato, quod fit ab intermedia ſectionum* C D, *æquale eſt ei, quod à dimidia* C B *deſcribitur, quadrato.*

Dico $C\,Bq = A\,D\,B + C\,Dq$.

a 4. 2.
b 3. 2.
c hyp.
d 1. 2.

$$\text{Æquantur enim iſta} \begin{cases} C\,Bq. \\ {}^{a}\,CDq + CDB + DBq + CDB \\ CDq + {}^{b}CBD\,({}^{c}AC \times BD) + CDB \\ CDq + {}^{d}\,ADB. \end{cases}$$

*Scholium.*




Si A B aliter dividatur, propius ſcilicet puncto biſectionis, in E; dico $AEB \sqsubset ADB$.

a 5. 2 & 3. ax.

Nam $AEB\,{}^{a} = CBq - CEq$. & $ADB\,{}^{a} = CBq - CDq$. ergo quum $CDq \sqsubset CEq$, erit $AEB \sqsubset ADB$. Q. E. D.

*Coroll.*

b 4. 2.

Hinc $ADq + DBq \sqsubset AEq + EBq$. Nam $ADq + DBq + 2\,ADB\,{}^{b} = ABq\,{}^{b} = AEq + EBq + 2\,AEB$. ergo quum $2\,AEB \sqsubset 2\,ADB$, erit $ADq + DBq \sqsubset AEq + EBq$. Q. E. D.

c 3. ax.

Unde 2. $ADq + DBq - AEq\,{}^{c} + EBq = 2\,AEB - 2\,ADB$.

PROP.

## PROP. VI.



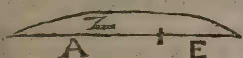

*Si recta linea* A *bifariam ſecetur, & illi recta quæpiam linea* E *in directum adjiciatur; rectangulum comprehenſum ſub tota cum adjecta* (ſub. A + E), *& adjecta* E, *una cum quadrato, quod à dimidia* $\frac{1}{2}$ A, *æquale eſt quadrato à linea, quæ tum ex dimidia, tum ex adjecta componitur, tanquam ab una* $\frac{1}{2}$ A + E *deſcripto.*

Dico $\frac{1}{4}$Aq (a Q. $\frac{1}{2}$A) + AE + Eq = Q. $\frac{1}{2}$ A + E. a Nam Q. $\frac{1}{2}$ A + E = $\frac{1}{4}$ Aq + Eq + AE. a 4. & 3. Car. 4 2.

*Coroll.*

Hinc ſi tres rectæ E, E + $\frac{1}{2}$ A, E + A ſint in proportione Arithmetica, rectangulum ſub extremis E, E + A contentum, una cum quadrato exceſſus $\frac{1}{2}$ A, æquale erit quadrato mediæ E + $\frac{1}{2}$ A.

## PROP. VII.




*Si recta linea* Z *ſecetur utcunque; Quod à tota* Z, *quodque ab uno ſegmentorum* E, *utraque ſimul quadrata, æqualia ſunt illi, quod bis ſub tota* Z, *& dicto ſegmento* E *cemprehenditur, rectangulo, & illi, quod à reliquo ſegmento* A *fit, quadrato.*

Dico Zq + Eq = 2 ZE + Aq. Nam Zq a = Aq + Eq + 2 AE. & 2 ZE b = 2 Eq. + 2 AE. a 4. 2. b 3. 2.

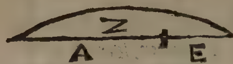
*Coroll.*

Hinc, quadratum differentiæ duarum quarumcumque linearum Z, E, æquale eſt quadratis utriuſque minus duplo rectangulo ſub ipſis.

c Nam Zq + Eq − 2 ZE = Aq = Q. Z − E. c 7. 2. & 3. ax.

PROP.

## PROP. VIII.




*Si recta linea Z ſecetur utcunque; rectangulum quater comprehenſum ſub tota Z, & uno ſegmentorum E, cum eo, quod à reliquo ſegmento A ſit, quadrato, æquale eſt ei, quod à tota Z, & dicto ſegmento E, tanquam ab una linea Z+E deſcribitur, quadrato.*

a 7. 2. & 3. ax. Dico 4 ZE+Aq=Q. Z+E. Nam 2 ZE *a* = Zq+Eq− Aq. ergo 4 ZE+Aq=Zq+Eq+2
b 4. 2. ZE *b* = Q. Z+E. Q. E. D.

## PROP. IX.




*Si recta linea* AB *ſecetur in æqualia* AC, CB, *& non æqualia* AD, DB. *quadrata, quæ ab inæqualibus totius ſegmentis* AD, DB *fiunt, ſimul duplicia ſunt, & ejus, quod à dimidia* AC, *& ejus, quod ab intermedia ſectionum* CD *fit, quadrati.*

Dico ADq+DBq = 2 ACq+2 CDq. Nam
a 4. 2. ADq+DBq *a* = ACq+CDq+2ACD+DBq:
b hyp. atqui 2 ACD (*b* 2 BCD) + DBq *c* = Cq
c 7. 2. (ACq) + CDq. *d* ergo ADq+DBq = 2 ACq
d 2. ax. + 2 CDq. Q. E. D.

## PROP. X

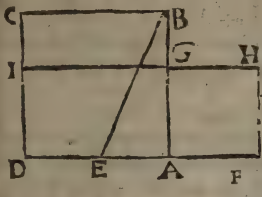



*Si recta linea* A *ſecetur bifariam, adjiciatur autem ei in rectum quæpiam linea; Quod à tota* A *cum adjuncta* E, *& quod ab adjuncta* E, *utraque ſimul quadrata, duplicia ſunt & ejus, quod à dimidia* $\frac{1}{2}$ A; *& ejus, quod à compoſita ex dimidia, & adjuncta, tanquam ab una* $\frac{1}{2}$ A + E, *deſcriptum eſt, quadrati.*

a 4. 2. Dico Eq+Q. A+E, hoc eſt *a* Aq+2 Eq+2
b Cor. 4. 2. AE=2 Q. $\frac{1}{2}$ A+2 Q. $\frac{1}{2}$ A+E. Nam 2 Q. $\frac{1}{2}$ A *b*
c 4. 2. = $\frac{1}{2}$ Aq. & 2 Q. $\frac{1}{2}$ A+E *c* = $\frac{1}{2}$ Aq+2Eq+2 AE.

PROP.

## PROP. XI.




*Datam rectam lineam* A B *ſecare in* G, *ut comprehenſum ſub tota* A B, *& altero ſegmentorum* B G *rectangulum, æquale ſit ei, quod à reliquo ſegmento* AG *fit, quadrato.*

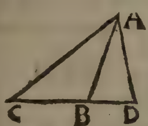
Super A B [a] deſcribe quadratum A C. latus AD [b] biſeca in E. duc E B. ex E A producta cape $EF = EB$. ad A F [a] ſtatue quadratum A H. Erit $AH = AB \times BG$. a 46. 1. b 10. 1.

Nam protracta H G ad I; Rectang. $DH + EAq$ [c] $= EFq$ [d] $= EBq$ [e] $= BAq + EAq$. ergo D H [f] $= BAq$ [d] $=$ quad. AC. ſubtrahe commune A I; [f] remanet quad. $AH = GC$; [d] id eſt $AGq = AB \times BG$. Q. E. F. c 6. 2. d conſtr. e 47. 1. f 3. ax.

*Scholium.*

Hæc Propoſitio numeris explicari nequit; * neque enim ullus numerus ita ſecari poteſt, ut productum ex toto in partem unam æquale ſit quadrato partis reliquæ. * vid. 6. 13.

## PROP. XII.




*In amblygoniis triangulis* ABC *quadratum, quod fit à latere* A C *angulum obtuſum* A B C *ſubtendente, majus eſt quadratis, quæ fiunt à lateribus* A B, B C *obtuſum angulum* A B C *comprehendentibus, rectangulo bis comprehenſo, & ab uno laterum* B C, *quæ ſunt circa obtuſum angulum* ABC, *in quod, cum protractum fuerit, cadit perpendicularis* AD, *& ab aſſumpta exterius linea* BD *ſub perpendiculari* A D *prope angulum obtuſum* A B C.

Dico

Dico $ACq = CBq + ABq + 2\,CB \times BD$.

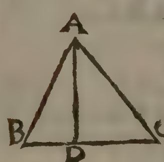
Nam iſta æqualia ſunt inter ſe:
$ACq$.
a 47. 1. — $^{a}\,CDq + ADq$.
b 4. 2. — $^{b}\,CBq + 2\,CBD + BDq + ADq$
c 47. 1. — $CBq + 2\,CBD \;^{c}\!+ ABq$.

*Schol.*

Hinc, *cognitis lateribus trianguli obtuſanguli* ABC, *facile invenientur tum ſegmentum* BD *inter perpendicularem* A D, *& obtuſum angulum* ABC *interceptum, tum ipſa perpendicularis* A D.

Sic; Sit AC 10, AB 7, CB 5; unde ACq 100, ABq 49, CBq 25. Proinde $ABq + CBq = 74$. hunc deme ex 100, manet 26 pro 2 CBD. unde C B D erit 13. hunc divide per C B 5, provenit $2\frac{3}{5}$ pro BD. quare AD invenitur per 47. 1.

## PROP. XIII.




*In oxygoniis triangulis* ABC *quadratum à latere* A B *angulum acutum* A C B *ſubtendente, minus eſt quadratis, quæ fiunt à lateribus* A C, C B *acutum angulum* A C B *comprehendentibus, rectangulo bis comprehenſo, & ab uno laterum* B C, *quæ ſunt circa acutum angulum* ACB, *in quod perpendicularis* AD *cadit, & ab aſſumpta interius linea* D C *ſub perpendiculari* A D, *prope angulum acutum* A C B.

Dico $ACq + BCq = ABq + 2\,BCD$.

Nam æquantur iſta:
$ACq + BCq$.
a 47. 1. — $^{a}\,ADq + DCq + BCq$.
b 7. 2. — $^{b}\,ADq + BDq + 2\,BCD$.
c 47. 1. — $^{c}\,ABq + 2\,BCD$.

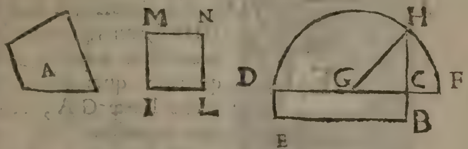
*Coroll.*

Hinc etiam *cognitis lateribus trianguli* A B C, *invenire eſt tam ſegmentum* D C *inter perpendicula-*

*rem* A D, *& acutum angulum* A B C *interceptum, quam ipſam perpendicularem* A B.

Sit AB 13, A C 15, B C 14. Detrahe A Bq (169) ex A Cq + B Cq hoc eſt ex 225 + 196 = 421; remanet 252 pro 2 BCD; unde BCD erit 126. hunc divide per B C 14, provenit 9 pro D C. unde A D = $\sqrt{:}$ 225 − 81 = 12.

## PROP. XIV.




*Dato rectilineo* A *æquale quadratum* M L *invenire.*

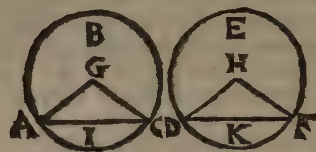
*a* Fac rectangulum D B = A; cujus majus latus D C produc ad F, ita ut C F = C B. *b* Biſeca D F in G, quo centro ad intervallum G F deſcribe circulum F H D, producatur C B, donec occurrat circumferentiæ in H. Erit C Hq = *M L = A  a 45. 1. b 10. 2. * 46. 1. c conſtr.

Ducatur enim G H. Eſtque A *c* = D B *c* = D C F *d* = G Fq − G Cq *e* = H Cq *c* = M L Q. E. F.  d 5. 2. & 3. ax. e 47. 1. & 3. ax.

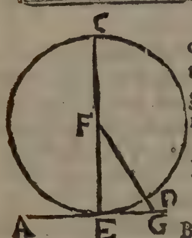
L I B.

# LIB. III.

## *Definitiones.*

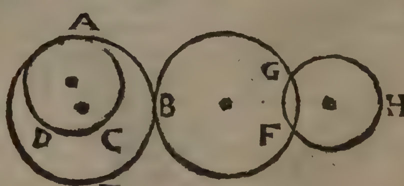



I. ÆQuales circuli (G A B C, HDEF) ſunt, quorum diametri ſunt æquales, vel quorum quæ ex centris rectæ lineæ G A, H D, ſunt æquales.



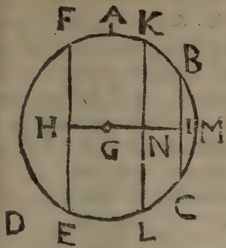

II. Recta linea A B circulum F E D tangere dicitur, quæ cum circulum tangat, ſi producatur circulum non ſecat.

Recta F G ſecat circulum F E D.

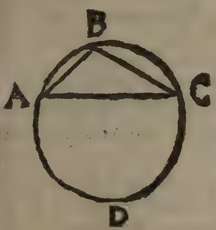



III. Circuli D A C, A B E (item F B G, A B E) ſe mutuo tangere dicuntur, qui ſe mutuo tangentes ſeſe mutuo non ſecant.

Circulus BFG ſecat circulum FGH. In




IV. In circulo G A B D æqualiter diſtare à centro dicuntur rectæ lineæ F E K L, cum perpendiculares GH, GN quæ à centro G in ipſas ducuntur, ſunt æquales. Longius autem abeſſe illa B C dicitur, in quam major perpendicularis G I cadit.

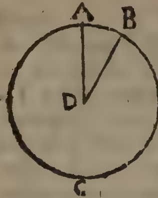



V. Segmentum circuli (A B C) eſt figura, quæ ſub recta linea A C, & circuli peripheria A B C comprehenditur.

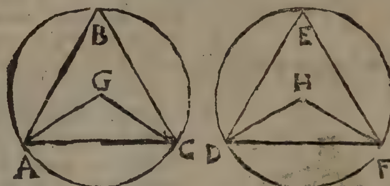
VI. Segmenti autem angulus (C A B) eſt, qui ſub recta linea CA, & circuli peripheria AB comprehenditur.

VII. In ſegmento autem (A B C) angulus (A B C) eſt, cum in ſegmenti peripheria ſumptum fuerit quodpiam punctum B, & ab illo in terminos rectæ ejus lineæ A C, quæ ſegmenti baſis eſt, adjunctæ fuerint rectæ lineæ A B, C B, is inquam angulus ABC ab adjunctis illis lineis A B, C B comprehenſus.

VIII. Cum vero comprehendentes angulum A B C, rectæ lineæ A B, B C aliquam aſſumunt peripheriam ADC, illi angulus ABC inſiſtere dicitur.

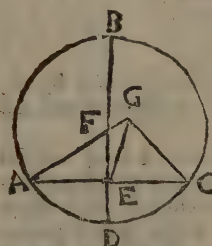



IX. Sector autem circuli (ADB) eſt, cum ad ipſius circuli centrum D conſtitutus fuerit angulus ADB; comprehenſa nimirum figura A D B. & à rectis lineis A D, B D angulum continentibus, & à peripheria A B ab illis aſſumpta.




X. Similia circuli ſegmenta (ABC, DEF) ſunt, quæ angulos (A B C, D E F) capiunt æquales; aut in quibus anguli ABC, DEF inter ſe ſunt æquales.

P R O P. I.




*Dati circuli* A B C *centrum* F *reperire.*

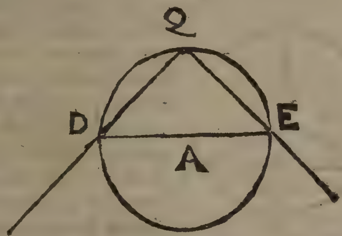
Duc in circulo rectam AC utcunq;,quam
a 10. biſeca in *a* E. per E duc perpendicularẽ DB.hanc biſeca in F.erit F centrũ.

Si negas, centrum eſto G, extra rectam DB (nam in ea eſſe non poteſt, cum ubique extra F dividatur inæqualiter) ducanturque GA,
a 15.def.1. GC, GE. Vis G centrum eſſe; *a* ergo GA = GC; & per conſtr. AE = EC, latus vero GE
b 8.1. commune eſt; *b* ergo anguli GEA, GEC pares,
c 10.def.1. & *c* proinde recti ſunt. *d* ergo ang. GEC = FEC
d 12.ax. e 9.ax. rect. *e* Q. E. A.

*Coroll.*

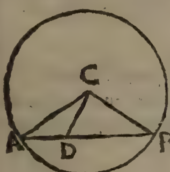
*Coroll.*

Hinc, ſi in circulo recta aliqua linea B D aliquam rectam lineam A C biſariam & ad angulos rectos ſecet, in ſecante B D erit centrum.




*Facillime per normam invenitur centrum vertice Q ad circumferentiam applicato.* Si enim recta D E jungens puncta D, & E, in quibus normæ latera Q D, Q E peripheriam ſecant, biſecetur in A, erit A centrum. Demonſtratio pendet ex 31. hujus. *Andr. Tacq.*

PROP. II.




*Si in circuli* C A B *peripheria duo quælibet puncta*, A, B *accepta fuerint; recta linea* A B, *quæ ad ipſa puncta adjungitur, intra circulum cadet.*

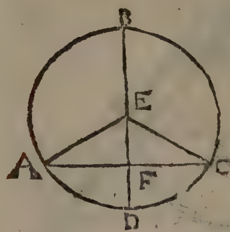
Accipe in recta A B quodvis punctum D, & ex centro C duc C A, C D, C B. & quoniam C A *a* = C B, *b* erit ang. A = B. Sed ang. C D B *c* ⊏ A; ergo ang. C D B ⊏ B. *d* ergo C B ⊏ C D. atqui C B tantum pertingit ex centro ad circumferentiam; ergo C D eouſque non pertingit. ergo punctum D eſt intra circulum. Idemque oſtendetur de quovis alio puncto rectæ A B. Tota igitur A B cadit intra circulum. Q. E. D.

a 15. def. 1.
b 5. 1.
c 16. 1.
d 19. 1.

*Coroll.*

Hinc, Recta circulum tangens, ita ut eum non ſecet, in unico puncto tangit.

P R O P. III.




*Si in circulo* EABC *recta quædam linea* BD *per centrum extenſa quandam* A C *non per centrum extenſam bifariam ſecet, (in* F) *& ad angulos rectos ipſam ſecabit; & ſi ad angulos rectos eam ſecet, bifariam quoque eam ſecabit.*

Ex centro E ducantur E A, E C.

a hyp. b 15. def. 1. c 8. 1. d 10 def. 1. e hyp. & 12. ax. f 5. 1. g 26. 1.

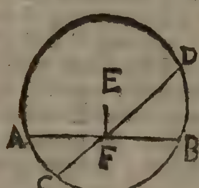
1. *Hyp*. Quoniam AF *a* = FC, & EA*b* = EC, latuſque EF commune eſt, *c* erunt anguli EFA, EFC pares, & *d* conſequenter recti. Q. E. D.

2. *Hyp*. Quoniam ang. EFA *e* = EFC, & ang. EAF *f* = ECF, latuſque EF commune, *g* erit AF = FC. Biſecta eſt igitur A C. Q. E. D.

*Coroll.*

Hinc, in triangulo quovis æquilatero & Iſoſcele linea ab angulo verticis biſecans baſim, perpendicularis eſt baſi. & contra perpendicularis ab angulo verticis biſecat baſim.

P R O P. IV.



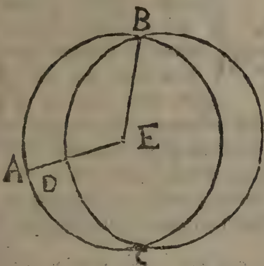

*Si in circulo* A C D *duæ rectæ lineæ* A B, C D *ſeſe mutuo ſecent non per centrum* E *extenſæ, ſeſe mutuo bifariam non ſecabunt.*

Nam ſi una per centrum

trum tranſeat, patet hanc non biſecari ab altera, quæ ex hyp. per centrum non tranſit.

Si neutra per centrum tranſit, ex E centro duc E F. Si jam ambæ A B, C D forent biſectæ in F, anguli EFB, EFD *a* ambo eſſent recti, & proinde æquales. *b* Q. E. A. a 3. 3. b 9. ax.

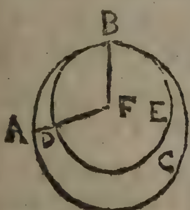
PROP. V.




*Si duo circuli* B A C, B D C *ſeſe mutuo ſecent, non erit illorum idem centrum* E.

Alias enim ductis ex communi centro E rectis E B, E D A, eſſent E D *a* = E B *a* = E A. *b* Q. E. A. a 15. def. 1. b 9. ax.

PROP. VI.




*Si duo circuli* B A C, BDE, *ſeſe mutuo interius tangant* (in B) *eorum non erit idem centrum* F.

Alias ductis ex centro F rectis F B, F D A, eſſent F D *a* = F B *a* = F A. *b* Q. F. N. a 15. defin. b 9 ax.

## PROP. VII.




*Si in* A B *diametro circuli quodpiam ſumatur punctum* G , *quod circuli centrum non ſit , ab eoque puncto in circulum quædam rectæ lineæ* GC , GD , GE *cadunt ; maxima quidem erit ea* (G A) *in qua centrum* F , *minima vero reliqua* GB. *aliarum vero illi, quæ per centrum ducitur , propinquior* G C *remotiore* G D *ſemper major eſt. Duæ autem ſolum rectæ lineæ* GE GH *æquales ab eodem puncto in circulum cadunt , ad utraſque partes minimæ* G B , *vel maximæ* G A.

a 23. 1. Ex centro F duc rectas F C, F D, F E; & *a* fac ang. BFH = BFE.

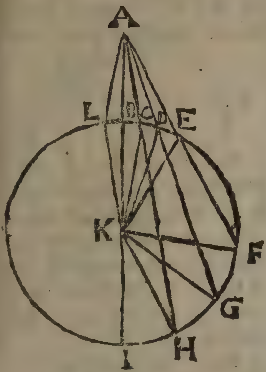
a 20, 1. 1. G F + F C (hoc eſt G A) *a* ⊏ G C. Q. E. D.

b 15. def. 1. 2. Latus F G commune eſt, & FC *b* = F D,
c 9. ax. atque ang. G F C *c* ⊏ G F D *d* ergo baſ. G C
d 24. 1. ⊏ G D. Q. E. D.

e 20. 1. 3. F B (F E) *e* ⊐ G E + G F. ergo abla-
f 5. ax. to communi F G *f* remanet B G ⊐ E G. Q. E. D.

4. Latus F G commune eſt , & F E = F H ;
g conſtr. atque ang. BFH *g* = BFE. *h* ergo GE = GH.
h 4. 1. Quod vero nulla alia G D ex puncto G æquetur ipſi G E , vel G H , jamjam oſtenſum eſt. Q. E. D.

# P R O P. VIII.




*Si extra circulum ſumatur punctum quodpiam* A, *ab eoque puncto ad circulum deducantur quædam lineæ* AI, AH, AG, AF, *quarum una quidem* AI *per centrum* K *protendatur, reliquæ vero ut libet; in cavam peripheriam cadentium rectarum linearum maxima quidem eſt illa* A I, *quæ per centrum ducetur, aliarum autem ei quæ per centrum tranſit propinquior* AH *remotiore* AG *ſemper major eſt. In convexam vero peripheriam cadentium rectarum linearum minima quidem eſt illa* A B, *quæ inter punctum* A, *& diametrum* B I *interponitur; aliarum autem ea, quæ eſt minimæ propinquior* A C *remotiore* A D *ſemper minor eſt. Duæ autem tantum rectæ lineæ* A C, A L *æquales ab eo puncto in ipſum circulum cadunt, ad utraſque partes minimæ* A B, *vel maximæ* A I.

Ex centro K duc rectas K H, K G, K F; K C, K D, K E. & fac ang. A K L = A K C.

1. A I (A K + K H) *a* ⊏ A H. Q. E. D. *a* 20. 1.

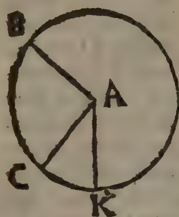
2. Latus A K commune eſt; & K H = K G; atque ang. AKH ⊏ AKG. *b* ergo baſ. A H ⊏ AG. Q. E. D. *b* 24. 1.

3. KA *c* ⊐ KC + CA. aufer hinc inde æquales KC, KB, *d* erit AB ⊐ AC. *c* 20. 1. *d* 5. ax.

4. A C + C K *e* ⊐ A D + D K. aufer hinc inde æquales C K, D K, *f* erit A C ⊐ A D. Q. E. D. *e* 21. 1. *f* 5. ax.

5. Latus K A eſt commune & K L = K C
g conſtr. atque ang. A K L g = A K C, h ergo L A =
h 4. 1. C A. hiſce vero nulla alia æquatur, ex mox
oſtenſis. ergo, &c.

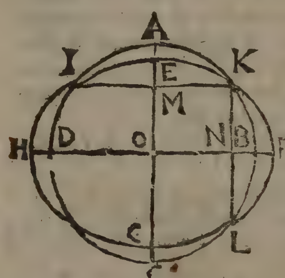
## Prop. IX.




*Si in circulo* B C K *acceptum fuerit punctum aliquod* A, *& ab eo puncto ad circulum cadant plures, quam duæ rectæ lineæ æquales* A B, A C, A K, *acceptum punctum* A *centrum eſt ipſius circuli.*

a 7. 3. Nam a à nullo puncto extra centrum plures quam duæ rectæ lineæ æquales duci poſſunt ad circumferentiam. Ergo A eſt centrum. Q. E. D.

## Prop. X.

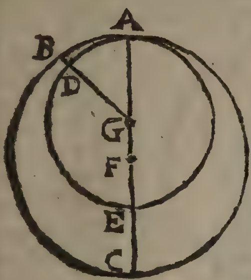



*Circulus* IAKBL *circulum* IEKFL *in pluribus quam duobus punctis non ſecat.*

Secet, ſi fieri poteſt, in tribus punctis I K L. Iunctæ I K K L. biſecentur in M & N.
a cor. 1. 3. a Ambo circuli centrum habent in ſingulis perpendicularibus M C, N H, & proinde in earum interſectione O. ergo ſecantes circuli idem centrum habent.
b 5. 3. b Q. F. N.

PROP.

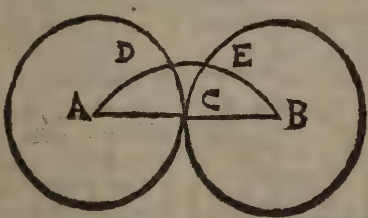
PROP. XI.




*Si duo circuli* GADE, FABC *ſeſe intus contingant, atque accepta fuerint eorum centra* G, F; *ad eorum centra adjuncta recta linea* F G, *& producta, in* A *contactum circulorum cadet.*

Si fieri poteſt, recta F G protracta ſecet circulos extra contactum A, ſic ut non F G A, ſed FGDB ſit recta linea. ducatur G A. Et quia G D *a* = GA, & G B *b* $\sqsupset$ GA, (cum recta FGB tranſeat per F centrum majoris circuli) erit G B $\sqsupset$ GD. *c* Q. E. A. a 15.*def.* 1. b 7. 3. c 9.*ax.*

PROP. XII.

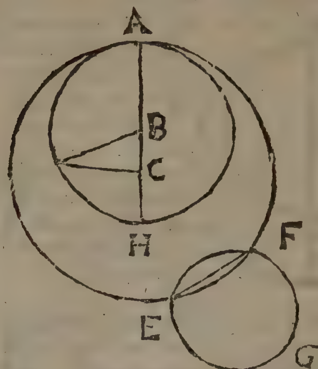



*Si duo circuli* ACD, BCE *ſeſe exterius contingant, linea recta* A B *quæ ad eorum centra* A, B *adjungitur, per contactum* C *tranſibit.*

Si fieri poteſt, ſit recta ADEB ſecans circulos extra contactum C in punctis D, E. Duc A C, CB. erit A D + E B (A C + C B) *a* $\sqsubset$ A D-EB. *b* Q. E. A. a 20. 1. b 9.*ax.*

PROP.

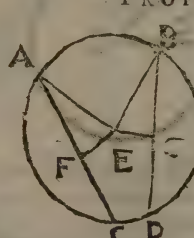
## PROP. XIII.




*Circulus* C A F *circulum* BAH *non tangit in pluribus punctis, quam uno* A, *ſive intus, ſive extra tangat.*

a 11. 3. 1. Tangat, ſi fieri poteſt, intus in punctis A, H. *a* ergo recta C B centra connectens, ſi producatur cadet tam in A, quam
b 15. def. 1. in H. Quoniam igitur CH *b* $=$ CA, & BH $\sqsubset$
c 15. def. 1. CH. erit BA (*c* BH) $\sqsubset$ CA. Q. E. A.
d 9. ax. 2. Sin dicatur exterius contingere in punctis
e 2. 3. E & F, *e* ducta recta E F in utroque circulo erit. Circuli igitur ſe mutuo ſecant, quod non ponitur.

## PROP. XIV.




*In circulo* E A B C *æquales rectæ lineæ* A C B D, *æqualiter diſtant à centro* E, *&* *quæ* A C, B D *æqualiter diſtant à centro, æquales ſunt inter ſe.*

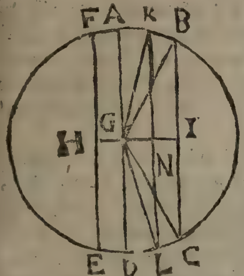
Ex centro E duc perpendiculares E F,
a 3. 3. E G : *a* quæ biſecabunt A C, D B. connecte E A E B.

b 7. ax. 1. *Hyp.* A C $=$ B D. ergo A F *b* $=$ B G. ſed & E A

EA = EB. ergo FEq *c* = EAq − AFq = EBq − BGq *c* = EGq. *d* ergo FE = EG. Q.E.D. c 47.1. & 3.ax. d Schol.48.1.

2. *Hyp*. EF = EG. ergo AFq *c* = EAq − EFq = EBq − EGq *c* = GBq. ergo AF *d* = GB. *e* proinde AD = BC. Q.E.D. e 6 ax.

PROP. XV.

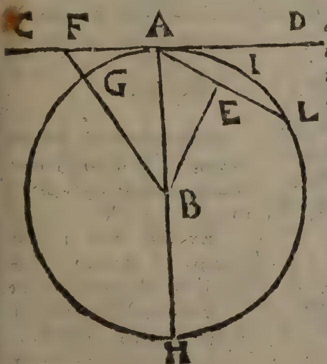



*In circulo* GABC *maxima quidem linea eſt diameter* AD; *aliarum autem centra* G *propinquior* FE *remotiore* BC *ſemper major eſt.*

1. Duc GB, GC. Diameter AD (*a* GB + GC) *b* ⊏ BC Q.E.D. a 15.def.1. b 10.1.

2. Sit diſtantia GI ⊏ GH. accipe GN = GH. per N duc KL perpend. GI. junge GK, GL. & quia GK = GB, & GL = GC; eſtque ang. KGL ⊏ BGC, *c* erit KL (FE) ⊏ BC. Q.E.D. c 24.1.

PROP. XVI.




*Quæ* CD *ab extremitate diametri* HA *cujuſque circuli* BALH *ad angulos rectos ducitur, extra ipſum circulum cadet, & in locum inter ipſam rectam lineam, & peripheriam comprehen-*

*prehenſum altera recta linea* A L *non cadet*, *&* *ſemicirculi quidem angulus* BAI *quovis angulo acuto rectilineo* B A L *major eſt* ; *reliquus autem* D A I *minor.*

1. Ex centro B ad quodvis punctum F in recta A C duc rectam B F. Latus B F ſubtendens angulum rectum B A F a majus eſt latere B A, quod opponitur acuto BFA. ergo cum B A (BG) pertingat ad circumferentiam, B F ulterius porrigetur, adeoque punctum F, & eadem ratione quodvis aliud rectæ A C, extra circulum ſitum erit. Q. E. D.

a 19. 1.

2. Duc BE perpendic. AL. Latus BA oppoſitum recto angulo B E A b majus eſt latere B E, quod acutum BAE ſubtendit: ergo punctum E, adeoque tota E A cadit intra circulum. Q.E D.

b 19. 1.

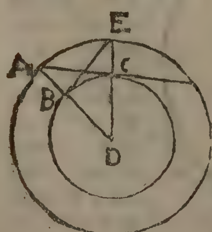
3. Hinc ſequitur angulum quemvis acutum, nempe EAD angulo contactus DAI majorem eſſe. Item angulum quemvis acutum B A L angulo ſemicirculi BAI minorem eſſe. Q.E.D.

*Coroll.*

Hinc, recta à diametri circuli extremitate ad angulos rectos ducta ipſum circulum tangit.

Ex hac propoſitione paradoxa conſequuntur,& mirabilia bene multa,quæ vide apud interpretes.

## PROP. XVII.




*A dato puncto* A *rectam lineam* A C *ducere*, *quæ datum circulum* D B C *tangat.*

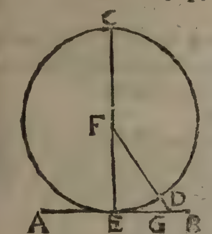
Ex D dati circuli centro ad datum punctum A ducatur recta D A ſecans peripheriam in B. Centro D deſcribe per A alium circulum A E;

A E; & ex B duc perpendicularem ad A D, quæ occurrat circulo A E in E. duc E D occurrentem circulo B C in C. ex A ad C ducta recta tanget circulum D B C.

Nam D B *a* = D C, & D E *a* = D A, & ang. D communis eſt: *b* ergo ang. ACD = EBD, rect. *c* ergo AC tangit circulum C. Q. E. F.

a 15. *def.* 1.
b 4. 1.
c *cor.* 16. 3.

PROP. XVIII.

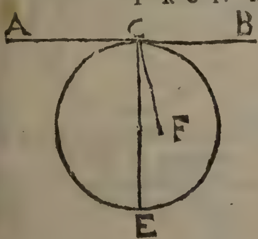



*Si circulum* FEDC *tangat recta quæpiam linea* A B, *à centro autem ad contactum* E *adjungatur recta quædam linea* F E; *quæ adjuncta fuerit* F E *ad ipſam contingentem* A B *perpendicularis erit.*

Si negas, ſit ex F centro alia quædam F G perpendicularis ad contingentem, *a* ſecabit ea circulum in D. Quum igitur ang. F G E rectus dicatur *b* erit ang. F E G acutus. *c* ergo F E (F D) = F G. *d* Q. E. A.

a 2. *def.* 3.
b *cor.* 17. 1.
c 19. 1.
d 9 *ax.*

PROP. XIX.



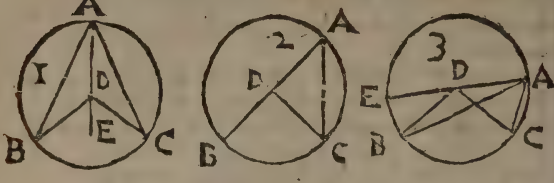

*Si circulum tetigerit recta quæpiam linea* A B, *à contactu autem* C *recta linea* C E *ad angulos rectos ipſi tangenti excitetur, in excitata* C E *erit centrum circuli.*

Si negas, ſit centrum extra C E in F, & ab F ad contactum ducatur F C. Igitur ang. F C B rectus eſt; & *a* proinde par angulo E C B recto per hypoth. *b* Q. E. A.

a 11. *ax.*
b 9. *ax.*

PROP.

## Prop. XX.

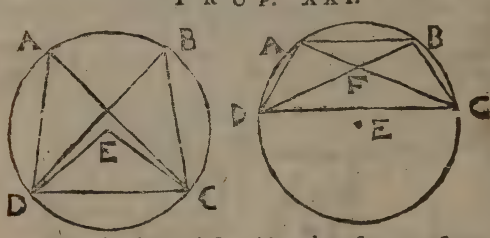



*In circulo* DABC, *angulus* BDC *ad centrum duplex eſt anguli* BAC *ad peripheriam, cum fuerit eadem peripheria* BC *baſis angulorum.*

Duc diametrum ADE.

a 32. 1. Externus angulus BDE a = DAB + DBA b =
b 5. 1. 2 DAB. Similiter ang. EDC = 2 DAC. ergo in primo caſu totus BDC = 2 BAC; ſed in ter-
c 20. ax. tio caſu c reliquus angulus BDC = 2 BAC. Q. E. D.

## Prop. XXI.



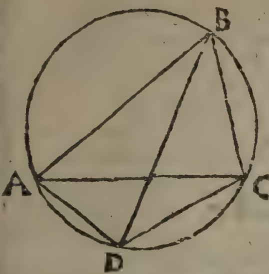

*In circulo* EDAC *qui in eodem ſegmento ſunt anguli,* DAC *&* DBC *ſunt inter ſe æquales.*

1. *Caſ.* Si ſegmentum DABC ſemicirculo ſit majus, ex centro E, duc ED, EC. Eritque 2 ang.
a 20. 3. A a = E a = 2 B. Q. E. D.

2. *Caſ.* Sin ſegmentum ſemicirculo majus non fuerit, ſumma angulorum trianguli ADF æquatur ſummæ angulorum in triangulo BCF. De-
b 15. 1. mantur hinc inde AFD b = BFC, & ADB c =
c per 1. caſ. ACB, remanent DAC = DBC. Q. E. D.

PROP.

PROP. XXII.




*Quadrilaterorum* ABCD *in circulo deſcriptorum anguli* ADC, ABC, *qui ex adverſo, duobus rectis ſunt æquales.*

Duc AC, BD. Ang. ABC + BCA + BAC *a* = 2 Rect. Sed BDA *b* = BCA, & BDC *b* = BAC. *c*ergo ABC + ADC = 2 Rect. Q. E. D.

a 32. 1.
b 21. 3.
c 1. ax

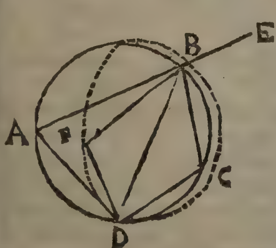
*Coroll.*

1. Hinc, ſi * AB unum latus quadrilateri in circulo deſcripti producatur, erit angulus externus EBC æqualis angulo interno ADC, qui opponitur ei ABC, qui eſt deinceps externo EBC. ut patet ex 13. 1. & 3. ax.

* *vide ſeq. diagram.*

2. Item circa Rhombum circulus deſcribi nequit; quia adverſi ejus anguli vel cedunt duobus rectis, vel eos excedunt.

*SCHOL.*

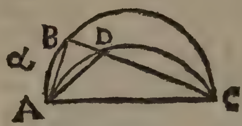



*Si in quadrilatero* ABCD *anguli* A, & C *qui ex adverſo duobus rectis æquantur, circa quadrilaterum circulus deſcribi poteſt.*

Nam circulus per quoſlibet

bet tres angulos B, C, D tranſibit (ut patebit ex 5.4.) dico eundem per A tranſire. Nam ſi neges, tranſeat per F. ergo ductis rectis BF, FD, BD ;
a 22. 3. b hyp. c 3 ax. d 21. 1. ang. C+F *a*= 2 Rect. *b*= C+ A *c* quare A=F. *d* Q. E. A.

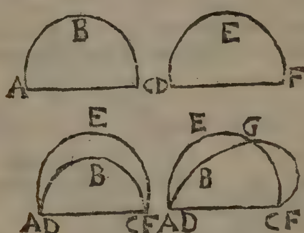
PROP. XXIII.




*Super eadem recta linea AC duo circulorum ſegmenta ABC, ADC ſimilia & inæqualia non conſtituentur ad eaſdem partes.*

Nam ſi dicantur ſimilia, duc CB ſecantem circumferentias in D, & B, & junge AD, ac
a 10 def. 3. b 16. 1. AB. Quia ſegmenta ponuntur ſimilia, *a* erit ang. ADC = ABC *b* Q. E. A.

PROP. XXIV.

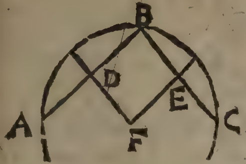



*Super æqualibus rectis lineis AC, DF ſimilia circulorum ſegmenta ABC, DEF ſunt inter ſe æqualia.*

Baſis AC ſuperpoſita baſi DF ei congruet, quia AC = DF. ergo ſegmentum ABC congruet ſegmento DEF (alias enim aut intra cadet, aut extra, *a* atque ita ſegmen-
a 23 3. ta non erunt ſimilia, contra Hyp. aut ſaltem partim intra, partim extra, adeoque ipſum in tribus punctis ſecabit. *b* Q. E. A.) *c* proinde ſe-
b 10 3. c 8. ax. gmentum. ABC = DEF. Q. E. D.

PROP.

PROP. XXV.

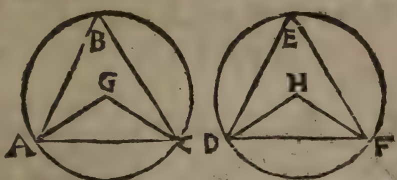



*Circuli ſegmento ABC dato, deſcribere circulum, cujus eſt ſegmentum.*

Subtendantur utcunque duæ rectæ AB, BC, quas biſeca in D, & E. Ex D, & E duc perpendiculares DF, EF occurrentes in puncto F. Hoc erit centrum circuli.

Nam centrum *a* tam in DF, quam in EF exſiſtit. ergo in communi puncto F. Q. E. F. a Cor. 1. 3.

PROP. XXVI.

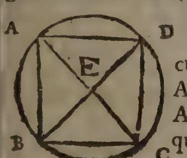



*In æqualibus circulis* GABC, HDEF *æquales anguli æqualibus peripheriis* AC, DF *inſiſtunt, ſive ad centra* G, H, *ſive ad peripher.* B, E *conſtituti inſiſtant.*

Ob circulorum æqualitatem, eſt GA = HD, & GC = HF item per hyp. ang. G = H. *a* ergo AC = DF. Sed & ang. B *b* = $\frac{1}{2}$ G = *c* $\frac{1}{2}$ H *b* = E. *d* ergo ſegmenta ABC, DEF ſimilia, *e* & proinde paria ſunt. *f* ergo etiam reliqua ſegmenta AC, DF æquantur. Q. E. D.

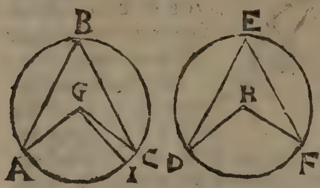
a 4. 1.
b 10. 3.
c hyp.
d 10. def. 3.
e 24. 3.
f 3. ax.

*Scholium.*




In circulo ABCD, ſit arcus AB par arcui DC; erit AD parall. BC. Nam ducta AC, *a* erit ang. ACB = CAD. a 26. 3. quare per 27. 1.

## PROP. XXVII.

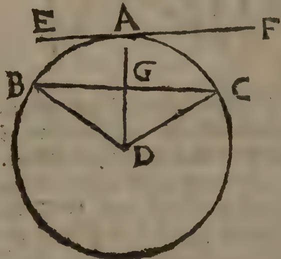



*In æqualibus circulis, G A B C, HDEF, anguli qui æqualibus peripheriis* AC, D F. *inſiſtunt, ſunt inter ſe æquales, ſive ad centra* G, H, *ſive ad peripherias* B, E *conſtituti inſiſtant.*

Nam ſi fieri poteſt, ſit alter eorum AGC ⊏ D H F. fiatque A G I = D H F. ergo arcus AI *a* = DF *b* = AC. *c* Q. E. A.

a 26. 3.
b *hyp.*
c 9 *ax.*

### SCHOL.




*Linea recta* E F, *quæ ducta ex* A *medio puncto peripheriæ alicujus* B C, *circulum tangit, parallela eſt rectæ lineæ* B C, *quæ peripheriam illam ſubtendit.*

Duc è centro D ad contactum A rectam DA, & connecte DB, DC.

Latus DG commune eſt; & DB = DC, atque ang. BDA *a* = CDA (ob arcus B A, C A *b* æquales) *c* ergo anguli ad baſim DGB, DGC æquales, & *d* proinde recti ſunt. Sed interni anguli GAE, GAF *e* etiam recti ſunt. *f* ergo B C, EF ſunt parallelæ. Q. E. D.

a 17. 3.
b *hyp.*
c 4. 1.
d 10 *def* 1.
e *hyp.*
f 28. 1.

PROP.

PROP. XXVIII.

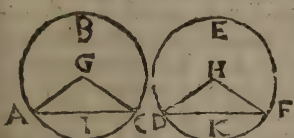



*In æqualibus circulis* GABC, HDEF, *æquales rectæ lineæ* AC, DF *æquales peripherias auferunt; majorem quidem* ABC *majori* DEF, *minorem autem* AIC *minori* DKF.

E centris G, H, duc GA, GC; & HD, HF. Quoniam GA = HD, & GC = HF, atque AC *a* = DF; *b* erit ang. G = H. *c* ergo arcus AIC = DKF. *d* proinde reliquus ABC = DEF. Q. E. D. a *hyp.* b 8. 1. c 16. 3. d 3. *ax.*

Quod ſi ſubtenſa AC ſit ⊏ vel ⊐ DF, erit ſimili modo arcus AC ⊏ vel ⊐ DF.

PROP. XXIX.

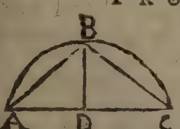



*In æqualibus circulis* GABC, HDEF, *æquales peripherias* ABC, DEF *æquales rectæ lineæ* AC, DF *ſubtendunt*.

Duc GA, GC; & HD, HF. Quia GA = HD; & GC = HF; & (ob arcus AC, DF *a* pares) etiam ang. G *b* = H; *c* erit baſ. AC = DF. Q. E. D. a *hyp.* b 27. 3. c 4. 1.

Hæc & tres proxime præcedentes intelligantur etiam de eodem circulo.

PROP. XXX.



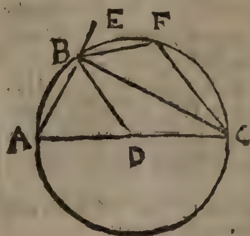

*Datam peripheriam* ABC *biſariam ſecare.*

Duc AC; quam biſeca in D. ex D duc perpendicularem DB occurrentem arcui in B. Dico factum.

Iungantur enim A B, C B. Latus D B commune eſt; & A D *a* = DC; & ang. ADB *b* = CDB. *c* ergo A B = B C. *d* quare arcus A B = B C. Q. E. F.

a conſt.
b 12. ax.
c 4. 1.
d 28. 3.

## PROP. XXXI.




*In circulo angulus* A B C, *qui in ſemicirculo, rectus eſt; qui autem in majore ſegmento* BAC, *minor recto; qui vero in minore ſegmento* B F C, *major eſt recto. Et inſuper angulus majoris ſegmenti recto quidem major eſt, minoris autem ſegmenti angulus, minor eſt recto.*

a 5. 1.
b 2. ax.
c 32. 1.
d 10. def. 1.
e cor. 17. 1.
f 22. 3.
g 9. ax.

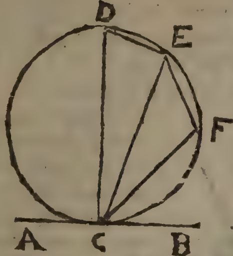
Ex centro D duc DB. Quia DB = DA, erit ang. A *a* = DBA. pariter ang. DCB *a* = DBC. *b* ergo ang. A B C = A + A C B *c* = E B C, *d* proinde A B C, & E B C recti ſunt. Q. E. D. *e* ergo B A C acutus eſt. Q. E. D. ergo cum B A C + B F C *f* = 2 Rect. erit B F C obtuſus. denique angulus ſub recta C B, & arcu B A C major eſt recto A B C. factus vero ſub C B, & B F C peripheria minoris ſegmenti, recto E B C *g* minor eſt. Q. E. D.

## SCHOLIUM.

*In triangulo rectangulo* A B C, *ſi hypotenuſa* A C *biſecetur in* D, *circulus centro* D, *per* A *deſcriptus tranſibit per* B. ut facile ipſe demonſtrabis ex hac, & 21. 1.

PROP.

## PROP. XXXII.



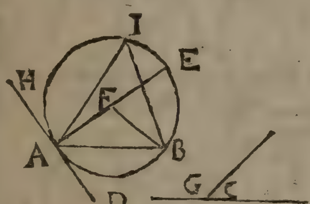

*Si circulum tetigerit aliqua recta linea* AB, *à contactu autem producatur quædam recta linea* CE *circulum ſecans: anguli* ECB, ECA, *quos ad contingentem facit, æquales ſunt iis, qui in alternis circuli ſegmentis conſiſtunt, angulis* EDC, EFC.

Sit CD latus anguli EDC perpendiculare ad AB (*a* perinde enim eſt) *b* ergo CD eſt diameter. *c* ergo ang. CED in ſemicirculo rectus eſt. *d* ergo ang. D + DCE = Rect. *e* = ECB + DCE. *f* ergo ang. D = ECB. Q. E. D.

Cum igitur ang. ECB + ECA *g* = 2 Rect. *h* = D + F; aufer hinc inde æquales ECB, & D, *k* remanent ECA = F. Q. E. D.

a 26. 3.
b 19. 3.
c 31. 3.
d 32. 1.
e conſtr.
f 3. ax.
g 13. 1.
h 22. 3.
k 3. ax.

## PROP. XXXIII.




*Super data recta linea* AB *deſcribere circuli ſegmentum* AIEB, *quod capiat angulum* AIB *æqualem dato angulo rectilineo* C.

*a* Fac ang. BAD = C. per A duc AE perpendicularem ad HD. ad alterum terminum datæ AB fac ang. ABF = BAF. cujus alterum latus ſecet AE in F. centro F per A deſcribe circulum, quod tranſibit per B. (quia ang. FBA

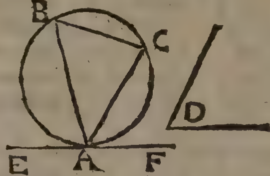
a 23. 1.

b conſtr. c 6. 1. — $b$ = F A B, $c$ ideoque F B = F A) ; ſegmentum AIB eſt id quod quæritur.

d cor. 16. 3. e 32. 3. f conſtr. — Nam quia H D diametro A E perpendicularis eſt, $d$ tangit HD circulum, quem ſecat AB. ergo ang. AIB $e$ = BAD $f$ = C. Q. E. F.

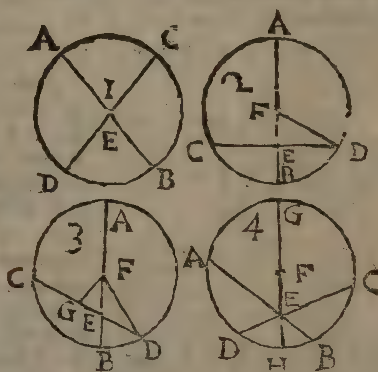
## PROP. XXXIV.

*A dato circulo* ABC *ſegmentum* A B C *abſcindere capiens angulum* B *æqualem dato angulo rectilineo* D.




a 17. 3. b 23. 1. — $a$ Duc rectam E F, quæ tangat datum circulum in A. $b$ ducatur item AC faciens ang. FAC = D. Hæc auferet ſegmentum ABC capiens angulum B $c$ = CAF $d$ = D. Q. E. F. (c 32. 3. d conſtr.)

## PROP. XXXV.




*Si in circulo* FBCA *duæ rectæ lineæ* A B , D C *ſeſe mutuo ſecuerint , rectangulum comprehenſum ſub*

*ſub ſegmentis* A E, E B *unius, æquale eſt ei quod ſub ſegmentis* C E, E D *alterius comprehenditur, rectangulo.*

*Caſ.* 1. Si rectæ ſeſe in centro ſecent, res clara eſt.

2. Si una A B tranſeat per centrum F, & reliquam C D biſecet, duc F D. Eſtque Rectang. AEB + FEq *a* = FBq *b* = FDq *c* = EDq + FEq *d* = C E D + FEq. *e* ergo Rectang. AEB = CED. Q. E. D. a 5. 2. b *ſch.* 48. 1. c 47. 1. d *hyp.* e 3. *ax.*

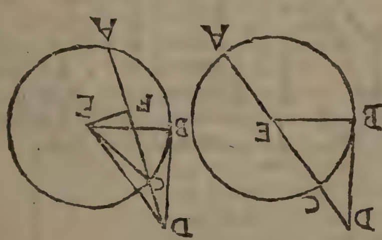
3. Si una A B diameter ſit, alteramque C D ſecet inæqualiter, biſeca C D per F G perpendicularem ex centro.

Æquantur iſta:
- Rectang. AEB + FEq.
- *f* FBq (FDq) f 5. 2.
- *g* FGq + GDq. g 47. 1.
- FGq + *h* GEq + Rectang. CED. h 5. 2.
- *k* FEq + CED. k 47. 1.

*l* Ergo Rectang. AEB = CED. l 3. *ax.*

4. Si neutra rectarum A B, C D per centrum tranſeat, per interſectionis punctum E duc diametrum G H. Per modo demonſtrata Rectang. AEB = GEH = CED. Q. E. D.

## PROP. XXXVI.



*Si extra circulum* E B C *ſumatur punctum aliquod* D, *ab eoque puncto in circulum cadant duæ rectæ lineæ* DA, DB; *quarum altera* DA *circulum*

 *ſecet,*

*ſecet, altera vero* D B *tanget; Quod ſub tota ſecante* D A, *&* *exterius inter punctum* D, *&* *convexam peripheriam aſſumpta* D C *comprehenditur rectangulum, æquale erit ei, quod à tangente* D B *deſcribitur, quadrato.*

1. *Caſ.* Si ſecans A D tranſeat per centrum E, junge E B; *a* faciet hæc cum D B rectum angulum; quare D Bq + E B Q (E Cq) *b* = E Dq *c* = A D x D C + E Cq *d* ergo A D x D C = D Bq. Q. E. D.

a 18. 3.
b 47. 1.
c 6. 2.
d 3. ax.

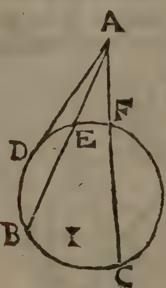
2. *Caſ.* Sin A D per centrum non tranſeat, duc EC, EB, ED; atque EF perpend. A D, quare *a* biſecta eſt A C in F.

a 3. 3.

Quoniam igitur BDQ + EBq *b* = DEq *b* = EFq + FDq *c* = EFq + ADC + FCQ *d* = ADC + C Eq (E Bq); *e* erit B Dq = A D C. Q. E. D.

b 47. 1.
c 6. 2.
d 47. 1.
e 3. ax.

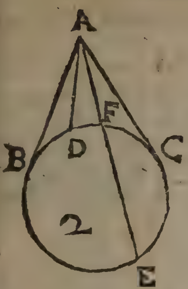
*Coroll.*




1. Hinc, ſi à puncto quovis A extra circulum aſſumpto, plurimæ lineæ rectæ A B, A C circulum ſecantes ducantur, rectangula comprehenſa ſub totis lineis A B, A C, & partibus externis A E, A F inter ſe ſunt æqualia. Nam ſi ducatur tangens AD; erit CAF = ADq *a* = BAE.

a 36. 3.

2. Con-




2. Conſtat etiam duas rectas A B, A C ab eodem puncto A ductas, quæ circulum tangant, inter ſe æquales eſſe.

Nam ſi ducatur A E ſecans circulum; erit A Bq $^{a}=$ EAF $^{b}=$ ACq. a 36. 3. b 36. 3.

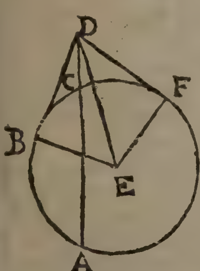
3. Perſpicuum quoque eſt ab eodem puncto A extra circulum aſſumpto, duci tantum poſſe duas lineas, A B, A C quæ circulum tangant.

Nam ſi tertia A D tangere dicatur, erit A D $^{c}=$ A B $^{c}=$ A C. $^{d}$Q. F. N. c 2. cor. d 8 3.

4. E contra conſtat, ſi duæ rectæ æquales A B, A C ex puncto quopiam A in convexam peripheriam incidant, & earum una A B circulum tangat, alteram quoque circulum tangere.

Nam ſi fieri poteſt, non A C, ſed altera A D circulum tangat. ergo A D $^{e}=$ A C $^{f}=$ A B. $^{g}$Q. E. A. e 2. cor. f hyp. g 8 3.

## PROP. XXXVII.




*Si extra circulum* E B F *ſumatur punctum* D, *ab eoque in circulum cadant duæ rectæ lineæ* D A, D B; *quarum altera* D A *circulum ſecet, altera* D B *in eum incidat; ſit autem quod ſub tota ſecante* D A, *& exterius inter punctum, & convexam peripheriam aſſumpta* D C, *comprehenditur rectangulum, æquale ei, quod ab incidente* D B

*D B deſcribitur quadrato, incidens ipſa D B circulum tanget.*

a 17. 3. b hyp. c 36. 3. d 1. ax & ſch. 48, 1. e 8. 1. f 12. ax. g cor. 16. 3.

Ex D *a* ducatur tangens DF; atque ex E centro duc ED, EB, EF. Quia DBq *b* = ADC *c* = D Fq, *d* erit D B = D F. Sed E B = E F, & latus E D commune eſt; *e* ergo ang. E B D = E F D. Sed E F D rectus eſt, *f* ergo E B D etiam rectus eſt. *g* ergo D B tangit circulum. Q. E. D.

*Coroll.*

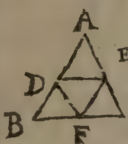
h 8. 1. Hinc, *h* ang. EDB = EDF.

LIB.

# LIB. IV.

## *Definitiones.*

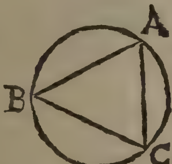
I Figura rectilinea in figura rectilinea inſcribi dicitur, cum ſinguli ejus figuræ, quæ inſcribitur, anguli ſingula latera ejus in qua inſcribitur, tangunt.

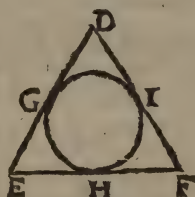



*Sic triangulum* DEF *eſt inſcriptum in triangulo* ABC.

II. Similiter & figura circa figuram deſcribi dicitur, cum ſingula ejus, quæ circumſcribitur, latera ſingulos ejus figuræ angulos tetigerint, circa quam illa deſcribitur.

*Ita triangulum* ABC *eſt deſcriptum circa triangulum* DEF.



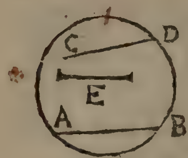

III. Figura rectilinea in circulo inſcribi dicitur, cum ſinguli ejus figuræ, quæ inſcribitur, anguli tetigerint circuli peripheriam.

IV. Figura vero rectilinea circa circulum deſcribi dicitur, cum ſingula latera ejus, quæ circumſcribitur, circuli peripheriam tangunt.

V. Similiter & circulus in figura rectilinea inſcribi dicitur, cum circuli peripheria ſingula latera tangit ejus figuræ, cui inſcribitur.

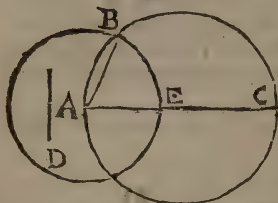
VI. Circulus autem circa figuram deſcribi di-

dicitur, cum circuli peripheria ſingulos tangit ejus figuræ, quam circumſcribit, angulos.




VII. Recta linea in circulo accommodari, ſeu coaptari dicitur, cum ejus extrema in circuli peripheria fuerint; ut recta linea A B.

PROP. I. *Probl.* 1.

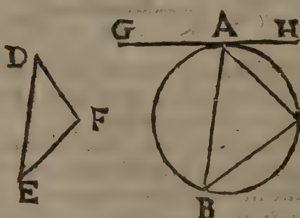



*In dato circulo* A B C *rectam lineam* A B *accommodare æqualem datæ rectæ lineæ* D, *quæ circuli diametro* A C *non ſit major.*

a 3. *poſt.* & 3. 1. b 15. *def* 1. c *conſtr.*

Centro A, ſpatio AE = D *a* deſcribe circulum dato circulo occurrentem in B. Erit ducta AB *b* = AE *c* = D. Q. E. F.

PROP. II. *Probl.* 2.




*In dato circulo* A B C *triangulum* ABC *deſcribere dato triangulo* DEF *æquiangulum.*

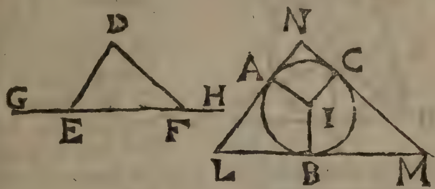
a 17. 3. b 23. 1.

Recta G H circulum datum *a* tangat in A. *b* Fac ang. HAC = E; *b* & ang. GAB = F, & junge BC. Dico factum.

Nam

Nam ang. B*c* = HAC*d* = E; & ang. c 32. 3.
C*c* = GAB*d* = F; *e* quare etiam ang. BAC = D. d *conſtr.*
ergo triang. BAC circulo inſcriptum triangulo e 32. 1.
DEF æquiangulum eſt. Q. E. F.

## PROP. III *Probl.* 3.



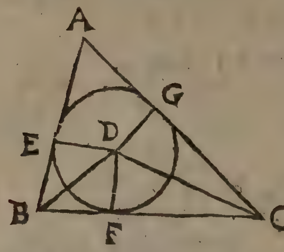

*Circa datum circulum* IABC *triangulum* LNM *deſcribere, dato triangulo* DEF *æquiangulum.*

Produc latus EF utrinque. *a* Fac ad centrum a 23. 1.
I ang. AIB = DEG. & ang. BIC = DFH.
deinde in punctis A, B, C circulum *b* tangant b 17. 3.
tres rectæ LN, LM, MN. Dico factum.

Nam quod coibunt rectæ LN, LM, MN,
atque ita triangulum conſtituent, patet; *c* quia c 13. *ax.*
anguli LAI, LBI *d* recti ſunt, adeoque ducta d 18. 3.
AB angulos faciet LAB, LBA duobus rectis mi-
nores. Quoniam igitur ang. AIB + L *e* = 2 e *Schol.* 32. 1.
Rect. *f* = DEG + DEF; & AIB *g* = DEG *h* erit f 13. 1. g *conſtr.*
ang. L = DEF. Simili argumento ang. M = DFE. h 3. *ax.*
*k* ergo etiam ang. N = D. ergo triang. LNM k 32. 1.
circulo circumſcriptum dato EDF eſt æquian-
gulum. Q. E. F.

PROP.

PROP. IV. *Probl.* 4.



*In dato triangulo* A B C. *circulum* E F G *inſcribere.*

a 9. 1. Duos angulos B, & C *a* biſeca rectis B D, C D coeuntibus in D. Ex

b 12. 1. D *b* duc perpendiculares D E, DF, DG. circulus centro D per E deſcriptus tranſibit per G, & F, tangetque tria latera trianguli.

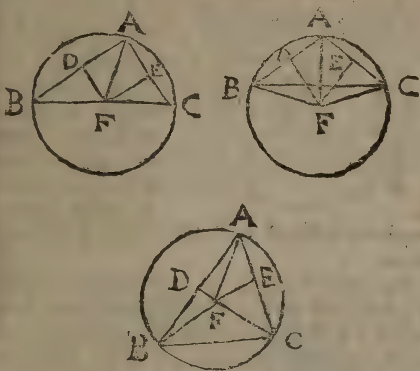
c conſtr. d 12. ax. e 26. 1. Nam ang. DBE *c* = DBF; & ang. DEB *d* = DFB; & latus DB commune eſt: *e* ergo DE = DF. Simili argumento DG = DF. Circulus igitur centro D deſcriptus tranſit per E, F, G; & cum anguli ad E, F, G ſint recti, tangit omnia trianguli latera. Q. E. F.

*Scholium.*

Petr. Herig. Hinc, *cognitis lateribus trianguli, invenientur eorum ſegmenta, quæ fiunt à contactibus circuli inſcripti.* Sic,

Sit AB 12, AC 18, BC 16. Erit AB + BC = 28. ex quo ſubduc 18 = AC = AE + FC, remanet 10 = BE + BF. ergo BE, vel BF = 5. proinde FC, vel CG = 11. quare GA, vel AE = 7.

## PROP. V. *Probl.* 5.




*Circa datum triangulum* ABC *circulum* **FABC** *deſcribere.*

Latera quævis duo B A , A C *a* biſeca perpendicularibus D F , E F concurrentibus in **F.** Hoc erit centrum circuli, a 10,& 11.1.

Nam ducantur rectæ FA, FB, FC. Quoniam AD*b*=DB ; & latus D F commune eſt ; & ang. FDA *c* = FDB, *d* erit FB=FA. eodem modo b *conſtr.* F C = F A. ergo circulus centro F per dati trianguli angulos B, A, C tranſibit. Q. E. F. c *conſtr.* & 12.*ax.* d 4.1.

*Coroll.*

* Hinc , ſi triangulum fuerit acutangulum , * 31.3. centrum cadet intra triangulum; ſi rectangulum, in latus recto angulo oppoſitum ; ſi denique obtuſangulum, extra triangulum.

*Schol.*

Eadem methodo deſcribetur circulus , qui tranſeat per data tria puncta , non in una recta linea exiſtentia:

PROP.

PROP. VI. *Probl. 6.*



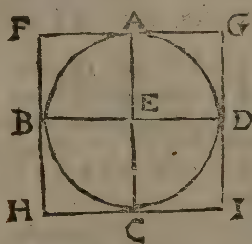

*In dato circulo* EABCD *quadratum* ABCD *inſcribere.*

a 11, 1. *a* Duc diametros AC, BD ſe mutuo ſecantes ad angulos rectos in centro E. junge harum terminos rectis AB, BC, CD, DA. Dico factum.

b 26. 3. c 29. 3. Nam quia 4 anguli ad E recti ſunt, *b* arcus, & *c* ſubtenſæ AB, BC, CD, DA pares ſunt. ergo ABCD æquilaterum eſt; ejuſque omnes

d 31. 3. e 29. def. 1. anguli in ſemicirculis, adeoque *d* recti ſunt. *e* ergo ABCD eſt quadratum, dato circulo inſcriptum. Q. E. F.

PROP. VII. *Probl. 7.*




*Circa datum circulum* EABCD *quadratum* FHIG *deſcribere.*

Duc diametros AC, BD ſe mutuo ſecantes perpendiculariter. per

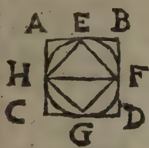
a] 17. 3. harū extrema *a* duc tangentes concurrentes in F, H, I, G. Dico factum. Nam ob

b 18 3. c 28. 1. angulos ad A, & C *b* rectos, *c* erit FG parall. HI. eodem modo FH parall. GI. ergo FHIG eſt parallelogrammum; & quidem rectangulum. ſed & æquilaterum, quia FG *d*= HI *d*= BD *e*=

d 34. 1. e 15. def. 1. f 29. def. 1. CA *d*= FH *d*= GI. quare FHIG eſt *f* quadratum, dato circulo circumſcriptum. Q. E. F.

*SCHOL.*

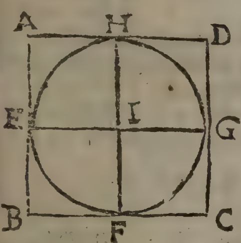
SCHOL.




Quadratum ABCD circulo circumſcriptum, duplum eſt quadrati EFGH circulo inſcripti.

Nam rectang. HB = 2 HEF. & HD = 2 HGF. per 41. 1.

PROP. VIII. *Probl.* 8.



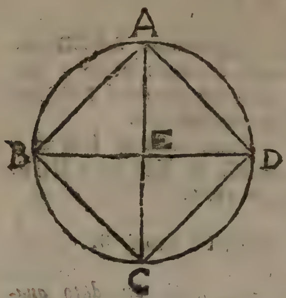

*In dato quadrato* ABCD *circulum* IEFGH *inſcribere.*

Latera quadrati biſeca in punctis H, E, F, G; junge HF, EG ſeſe ſecantes in I. circulus centro I per H deſcriptus quadrato inſcribetur.

Nam quia AH, BF *a* pares ac *b* parallelæ ſunt, *c* erit AB parall. HF parall. DC. eodem modo AD parall. EG parall. BC. ergo IA, ID, IB, IC ſunt parallelogramma. Ergo AH *d* = AE *e* = HI = EI = IF = IG. Circulus igitur centro I per H deſcriptus tranſibit per H, E, F, G, tangetque quadrati latera, cum anguli ad H, E, F, G ſint recti. Q. E. F.

a 7. ax.
b 34. 1.
c 33. 1.
d 7. ax.
e 34. 1.

PROP. IX. *Probl.* 9.

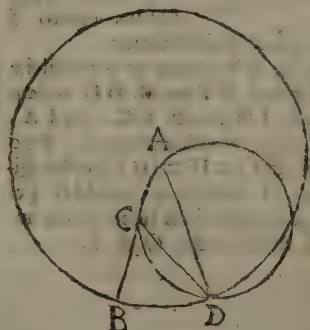



*Circa datum quadratum* A B C D *circulum* E A-B C D *deſcribere.*

Duc diametros A C, B D ſecantes in E. centro E per A deſcribe circulum. Is dato quadrato circumſcriptus eſt.

a 4 cor. 32. 1. b 6. 1. Nam anguli ABD, & BAC *a* ſemirecti ſunt; *b* ergo EA = EB. eodem modo EA = ED = EC. Circulus igitur centro E deſcriptus per A,B,C,D dati quadrati angulos tranſit. Q.E.F.

PROP. X. *Probl.* 10.




*Iſoſceles triangulum* A B D *conſtituere, quod habeat utrumque eorum quæ ad baſim ſunt angulorum* B & A D B *duplum reliqui* A.

Accipe quamvis rectam AB, quam *a* ſeca in C, ita ut AB x BC = ACq. Centro A per B deſcribe circulum A B D;

a 11. 2. in

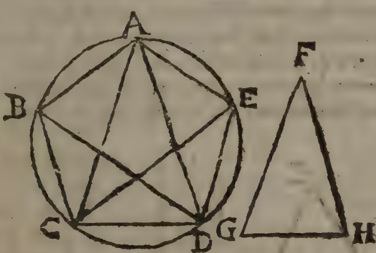
in hoc *b* accommoda B D = A C, & junge AD. erit triang. ABD quod quæritur. b 1. 4.

Nam duc D C; & per C D A *c* deſcribe circulum. Quoniam AB x BC = ACq. *d* liquet BD tangere circulum A C D, quem ſecat C D. *e* ergo ang. BDC = A. ergo ang. BDC + CDA *f* = A + CDA = BCD. ſed BDC + CDA = BDA *h* = CBD. *k* ergo ang. BCD = CBD. ergo DC *l* = DB *m* = AC. *n* quare ang. CDA = A = BDC. ergo ADB = 2 A = ABD. Q. E. F. c 5. 4. d 37. 3. e 32. 3. f 1. ax. g 32. 1. h 5. 1. k 1. ax. l 6. 1. m conſtr. n 5. 1.

*Coroll.*

Cum omnes anguli A, B, D *o* conficiant $\frac{5}{5}$ 2 Rect. (2 Rect.) liquet A eſſe $\frac{1}{5}$ 2 Rect. o 32. 1.

PROP. XI. *Probl.* II.




*In dato circulo* ABCDE *pentagonum æquilaterum & æquiangulum* ABCDE *inſcribere.*

*a* Deſcribe triangulum Iſoſceles FGH, habens utrumque angulorum ad baſim duplum anguli ad verticem. *b* Huic æquiangulum C A D inſcribe circulo. Angulos ad baſim ACD, & ADC *c* biſeca rectis D B, C E occurrentibus circumferentiæ in B, & E. connecte rectas CB, BA, AE, ED. Dico factum. a 10. 4. b 2. 4. c 9. 1.

Nam ex conſtr. liquet quinque angulos CAD, d 26 3. CDB, BDA, DCE, ECA pares eſſe; quare *d* e 29 3. arcus *e* & ſubtenſæ DC, CB, BA, AE, DE æquantur. Pentagonum igitur æquilaterum eſt. f 27. 3. Eſt vero etiam æquiangulum, *f* quia ejus anguli g 2. ax. BAE, AED, &c. inſiſtunt arcubus *g* æqualibus BCDE, ABCD, &c.

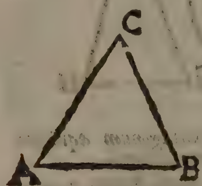
Hujus problematis praxis facilior tradetur ad 10. 13.

*Coroll.*

Hinc, angulus pentagoni æquilateri & æquianguli æquatur $\frac{3}{5}$ 2 Rect. vel $\frac{6}{5}$ Rect.

*Schol.*

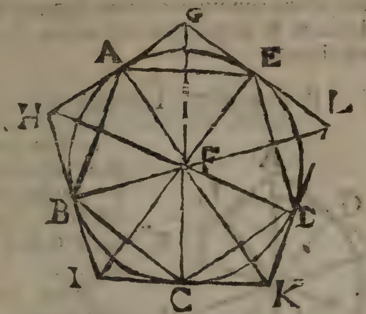
Petr. Herig. *Vniverſaliter figuræ imparium laterum inſcribuntur circulo beneficio triangulorum Iſoſcelium, quorum anguli æquales ad baſim multiplices ſunt eorum, qui ad verticem ſunt, angulorum; parium vero laterum figuræ in circulo inſcribuntur ope Iſoſcelium triangulorum, quorum anguli ad baſim multiplices ſeſquialteri ſunt eorum, qui ad verticem ſunt, angulorum.*




Ut in triangulo Iſoſcele CAB, ſi ang. A = 3 C = B; A B erit latus Heptagoni. Si A = 4 C; erit A B latus Enneagoni, &c. Sin vero A = $1\frac{1}{2}$ C, erit A B latus quadrati. Et ſi A = $2\frac{1}{2}$ C, ſubtendet AB ſextam partem circumferentiæ: pariterque ſi A = $3\frac{1}{2}$ C; erit AB latus octagoni, &c.

PROP.

PROP. XII. *Probl.* 12.




*Circa datum circulum* FABCDE *pentagonum æquilaterum & æquiangulum* HIKLG *deſcribere.*

*a* Inſcribe pentagonum ABCDE æquilaterum & æquiangulum; duc è centro rectas FA, FB, FC, FD, FE, iiſque totidem perpendiculares GAH, HBI, ICK, KDL, LEG concurrentes in punctis H, I, K, L, G. Dico factum. Nam quia GA, GE ex uno puncto G *b* tangunt circulum, *c* erit GA = GE. *d* ergo ang. GFA = GFE. ergo ang. AFE = 2 GFA. eodem modo ang. AFH = HFB; & proinde ang. AFB = 2 AFH. Sed ang. AFE *e* = AFB. *f* ergo ang. GFA = AFH. ſed & ang. FAH *g* = FAG; & latus FA eſt commune, *h* ergo HA = AG = GE = EL, &c. *k* ergo HG, GL, LK, KI, IH latera pentagoni æquantur: ſed & anguli etiam, utpote *l* æqualium AGF, AHF, &c. dupli; ergo, &c.

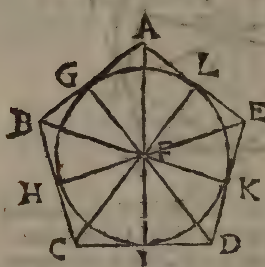
a 11. 4.
b cor. 16. 3.
c 1. cor. 36. 3.
d 8. 1.
e 27. 3.
f 7. ax.
g 12. ax.
h 26. 1.
k 2. ax.
l 32. 1.

*Coroll.*

Eodem pacto, ſi in circulo quæcunque figura æquilatera & æquiangula deſcribatur, & ad extrema ſemidiametrorum ex centro ad angulos

ductarum, excitentur lineæ perpendiculares, hæ perpendiculares conſtituent aliam figuram totidem laterum & angulorum æqualium circulo circumſcriptam.

P R O P. XIII. *Probl.* 13.




*In dato pentagono æquilatero & æquiangulo* ABCDE *circulum* F G H K *inſcribere.*

Duos pentagoni angulos A, & B *a* biſeca rectis AF, BF concurrentibus in F. Ex F duc perpendiculares F G, F H, F I, F K, F L. Circulus centro F per G deſcriptus tanget omnia pentagoni latera.

*a* 9. 1.

*b hyp.* *c conſtr.* *d* 4. 1. *e hyp.*

Duc F C, F D, F E. Quoniam B A *b* = B C; & latus B F commune eſt; & ang. F B A *c* = FBC, *d* erit AF = FC; & ang. FAB = FCB. Sed ang. FAB *e* = $\frac{1}{2}$ BAE *e* = $\frac{1}{2}$ BCD. ergo ang. FCB = $\frac{1}{2}$ BCD. eodem modo anguli totales C, D, E omnes biſecti ſunt. Quum igitur ang. FGB *f* = FHB, & ang. FBH = FBG, & latus FB ſit commune, *g* erit FG = FH. ſimiliter omnes FH, FI, FK, FL, FG æquantur. Ergo circulus centro F per G deſcriptus tranſit per H, I, K, L; *h* tangitque pentagoni latera, cum anguli ad ea puncta ſint recti. Q. E. F.

*f* 12. ax. 1 *g* 26. 1. *h* cor. 16. 3.

*Coroll.*

Hinc, ſi duo anguli proximi figuræ æquilateræ & æquiangulæ biſecentur, & à puncto, in quo coeunt lineæ angulos biſecantes, ducantur rectæ

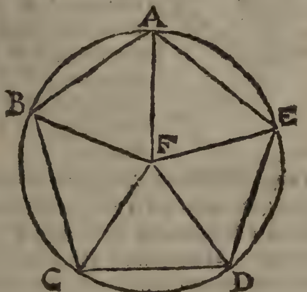
lineæ

lineæ ad reliquos figuræ angulos, omnes anguli figuræ erunt biſecti.

*Schol.*

Eadem methodo in qualibet figura æquilatera & æquiangula circulus deſcribetur.

PROP. XIV. *Probl.* 14.




*Circa datum Pentagonum æquilaterum & æquiangulum* ABCDE *circulum* FABCD *deſcribere.*

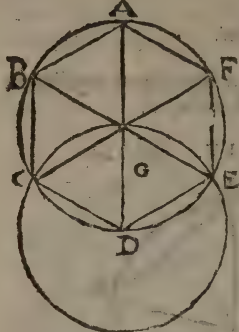
Duos pentagoni angulos biſeca rectis AF, BF concurrentibus in F. Circulus centro F per A deſcriptus pentagono circumſcribitur.

Ducantur enim FC, FD, FE. *a* Biſecti itaque ſunt anguli C, D, E. *b* ergo FA, FB, FC, FD FE æquantur. ergo circulus centro F deſcriptus, per A, B, C, D, E, pentagoni angulos tranſibit. Q. E. F. a *cor.* 13. 4. b 6. 1.

*Schol.*

Eadem arte circa quamlibet figuram æquilateram & æquiangulam circulus deſcribetur.

### PROP. XV. *Probl.* 15.




*In dato circulo G-ABCDEF hexagonum & æquilaterum & æquiangulum* ABCDEF *inſcribere.*

Duc diametrum AD; centro D per centrum G deſcribe circulum, qui datum ſecet in C, & E. duc diametros CF, EB. junge AB, BC, CD, DE, EF, FA. Dico factum.

a 32. 1. b 15. 1. c cor. 13. 1. d 26. 3. e 29. 3. f 27. 3.

Nam ang. CGD *a* = $\frac{1}{3}$ 2 Rect. *a* = DGE *b* = AGF *b* = AGB. *c* ergo BGC = $\frac{1}{3}$ 2 Rect. = FGE. *d* ergo arcus *e* & ſubtenſæ AB, BC, CD, DE, EF æquantur. Hexagonum igitur æquilaterum eſt: ſed & æquiangulum, *f* quia ſinguli ejus anguli arcubus inſiſtunt æqualibus. Q. E. F.

*Coroll.*

1. Hinc latus Hexagoni circulo inſcripti ſemidiametro æquale eſt.

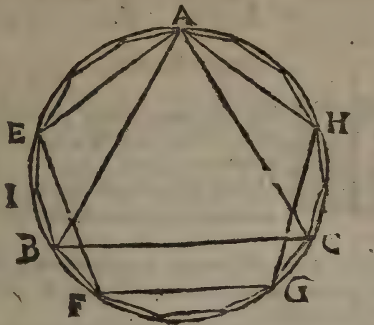
2. Hinc facile triangulum æquilaterum ACE in circulo deſcribetur.

*Schol. Probl.*

Andr. Tacq. a 1. 1.

*Hexagonum ordinatum ſuper data recta* CD *ita conſtrues.* *a* Fac triangulum CGD æquilaterum ſuper data CD. centro G per C, & D deſcribe circulum. Is capiet Hexagonum ſuper data CD.

## PROP. XVI. *Probl. 16.*




*In dato circulo* AEBC *quindecagonum æquilaterum & æquiangulum inſcribere.*

Dato circulo [a] inſcribe pentagonum æquilaterum AEFGH; *b* itemque triangulum æquilaterum ABC. erit BF latus quindecagoni quæſiti. a 11. 4. b 3 4.

Nam arcus A B *c* eſt $\frac{1}{3}$, vel $\frac{5}{15}$ peripheriæ, cujus AF eſt $\frac{2}{5}$ vel $\frac{6}{15}$. ergo reliquus B F $= \frac{1}{15}$ periph. ergo quindecagonum, cujus latus B F, æquilaterum eſt; ſed & æquiangulum, *d* cum ſinguli ejus anguli arcubus inſiſtant æqualibus, quorum unuſquiſque eſt $\frac{13}{15}$ totius circumferentiæ. ergo, &c. c conſtr. d 27. 3.

*Schol.*

Circulus dividitur Geometrice in partes
- 4, 8, 16, &c. per 6, 4, & 9, 1.
- 3, 6, 12, &c. per 15, 4, & 9, 1.
- 5, 10, 20, &c. per 11, 4, & 9, 1.
- 15, 30, 60, &c. per 16, 4 & 9, 1.

Cæterum diviſio circumferentiæ in partes datas etiamnum deſideratur; quare pro figurarum quarumcunq; ordinatarum conſtructionibus ſæpe ad mechanica artificia recurrendum eſt, propter quæ Geometræ practici conſulendi ſunt.

# LIB. V.

## *Definitiones.*

I. Pars eſt magnitudo magnitudinis, minor majoris, cum minor metitur majorem.

II. Multiplex autem eſt major minoris, cum minor metitur majorem.

III. Ratio eſt duarum magnitudinum ejuſdem generis mutua quædam ſecundum quantitatem habitudo.

*In omni ratione ea quantitas, quæ ad aliam refertur, dicitur antecedens rationis; ea vero, ad quam alia refertur, conſequens rationis dici ſolet. ut in ratione 6 ad 4; antecedens eſt 6, & conſequens 4.*

Nota.

*Cujuſque rationis quantitas innoteſcit dividendo antecedentem per conſequentem. ut ratio 12 ad 5 effertur per* $\frac{12}{5}$. *item quantitas rationis* A *ad* B *eſt* $\frac{A}{B}$. *Quare non raro brevitatis cauſa, quantitates rationum ſic deſignamus,* $\frac{A}{B} \sqsubset$, *vel* $=$, *vel* $\sqsupset \frac{C}{D}$; *hoc eſt, ratio* A *ad* B *major eſt ratione* C *ad* D, *vel ei æqualis, vel minor. Quod probe animadvertat, quiſquis hæc legere volet.*

*Rationis, ſive proportionis ſpecies, ac diviſiones vide apud interpretes.*

IV. Proportio vero eſt rationum ſimilitudo.

*Rectius quæ hic vertitur proportio, proportionalitas, ſive analogia dicitur; nam proportio idem denotat quod ratio, ut pleriſque placet.*

V. Rationem habere inter ſe magnitudines dicuntur, quæ poſſunt multiplicatæ ſe mutuo ſuperare.

VI. In

| | | |
|---|---|---|
| E, 12. F, 30. | A, 4. B, 6. C, 10. D, 15. | G, 24. H, 60. |

VI. In eadẽ ratione magnitudines dicuntur eſſe, prima A ad ſecundam B, & tertia C ad quartam D, cum primæ A, & tertiæ C æquemultiplicia E, & F à ſecundæ B, & quartæ D æquemultiplicibus G, & H, qualiſcunque ſit hæc multiplicatio, utrumque E, F ab utroque G, H, vel una deficiunt, vel una æqualia ſunt, vel una excedunt, ſi ea ſumantur E, G; & F, H quæ inter ſe reſpondent.

*Hujus nota eſt* ::. ut A. B :: C. D. *hoc eſt* A *ad* B, *&* C *ad* D *in eadem ſunt ratione. aliquando ſic ſcribimus* $\frac{A}{B}=\frac{C}{D}$ *id eſt*, A. B :: C. D.

VII. Eandem autem habentes rationem (A.B :: C.D) proportionales vocentur.

| | | |
|---|---|---|
| E, 30. F, 60. | A, 6. B, 4. C, 12. D, 9. | G, 28. H, 63. |

VIII. Cum vero æquemultiplicium, E multiplex primæ magnitudinis A exceſſerit G multiplicem ſecundæ B; at F multiplex tertiæ C non exceſſerit H multiplicem quartæ D; tunc prima A ad ſecundam B majorem rationem habere dicetur, quam tertia C ad quartam D.

Si $\frac{A}{B} \sqsubset \frac{C}{D}$, *neceſſarium non eſt ex hac definitione, ut* E *ſemper excedat* G; *quum* F *minor eſt quam* H; *ſed conceditur hoc fieri poſſe.*

IX. Proportio autem in tribus terminis pauciſſimis conſiſtit. *Quorum ſecunda eſt inſtar duorum.*

X. Cum autem tres magnitudines A, B, C proportionales fuerint, prima A ad tertiam C duplicatam rationem habere dicetur ejus, quam habet ad ſecundam B: at quum quatuor magnitudines A, B, C, D, proportionales fuerint, prima A ad quartam D triplicatam rationem habere

dicetur

dicetur ejus, quam habet ad ſecundam B; & ſemper deinceps uno amplius, quamdiu proportio extiterit.

*Duplicata ratio exprimitur ſic* $\frac{A}{C} = \frac{A}{B}$ *bis. Hoc eſt, ratio* A *ad* C *duplicata eſt rationis* A *ad* B. *Triplicata autem ſic* $\frac{A}{D} = \frac{A}{B}$ *ter.* id eſt, *ratio* A *ad* D *triplicata eſt rationis* A *ad* B.

∺ *denotat continue proportionales.* ut A, B, C, D; *item* 2, 6, 18, 64 ſunt ∺

XI. Homologæ, ſeu ſimiles ratione, magnitudines dicuntur, antecedentes quidem antecedentibus, conſequentes vero conſequentibus.

Ut ſi A. B :: C. D; *tam* A *&* C, *quam* B *&* D *homologæ magnitudines dicuntur.*

XII. Alterna ratio, eſt ſumptio antecedentis ad antecedentem, & conſequentis ad conſequentem.

*Vt ſit* A. B :: C. D. *ergo alterne, vel permutando, vel viciſsim,* A. C :: B. D. *per* 16. 5.

*In hac definitione, & 5. ſequentibus imponuntur nomina ſex modis argumentandi, quibus mathematici frequenter utuntur; quarum illationum vis innititur propoſitionibus hujus libri, quæ in explicationibus citantur.*

XIII. Inverſa ratio, eſt ſumptio conſequentis ceu antecedentis, ad antecedentem velut ad conſequentem.

*Vt* A. B :: C. D. *ergo inverſe,* B. A :: D. C. *per* cor. 4. 5.

XIV. Compoſitio rationis, eſt ſumptio antecedentis cum conſequente, ceu unius, ad ipſam conſequentem.

*Vt* A. B :: C. D. *ergo componendo,* A+B. B :: C+D. D. *per* 18. 5.

XV. Diviſio rationis, eſt ſumptio exceſſus, quo conſequentem ſuperat antecedens, ad ipſam conſequentem.

*Vt* A. B :: C. D. *ergo dividendo*, A- B. B :: C- D. D. *per* 17.5.

X V I. Converſio rationis, eſt ſumptio antecedentis ad exceſſum, quo ſuperat antecedens ipſam conſequentem.

*Vt* A. B :: C. D. *ergo per converſam rationem*, A. A- B :: C. C- D. *per* cor. 19.5.

X V I I. Ex æqualitate ratio eſt; ſi plures duabus ſint magnitudines, & his aliæ multitudine pares, quæ binæ ſumantur, & in eadem ratione; cum ut in primis magnitudinibus prima ad ultimam, ſic & in ſecundis magnitudinibus prima ad ultimam ſeſe habuerit. Vel aliter; ſumptio extremorum, per ſubductionem mediorum.

X V I I I. Ordinata proportio eſt, cum fuerit quemadmodum antecedens ad conſequentem, ita antecedens ad conſequentem: fuerit etiam ut conſequens ad aliud quidpiam, ita conſequens ad aliud quidpiam.

*Vt ſi* A. B :: D. E. *item* B. C :: E. F. *erit ex æquo* A. C :: D. F. *per* 22. 5.

X I X. Perturbata autem proportio eſt; cum tribus poſitis magnitudinibus, & aliis, quæ ſint his multitudine pares, ut in primis quidem magnitudinibus ſe habet antecedens ad conſequentem, ita in ſecundis magnitudinibus antecedens ad conſequentem: ut autem in primis magnitudinibus conſequens ad aliud quidpiam, ſic in ſecundis magnitudinibus aliud quidpiam ad antecedentem.

*Vt ſi* A. B :: F. G. *item* B. C :: E. F. *erit ex æquo perturbate* A. C :: E. G. *per* 23. 5.

X X. Quotlibet magnitudinibus ordine poſitis, proportio primæ ad ultimam componitur ex proportionibus primæ ad ſecundam, & ſecundæ ad tertiam, & tertiæ ad quartam, & ita deinceps, donec extiterit proportio.

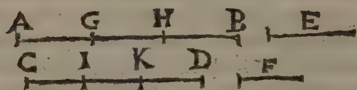
Sint

Sint quotcunque A, B, C, D; ex hac def.
$\frac{A}{D} = \frac{A}{B} + \frac{B}{C} + \frac{C}{D}$.

*Axioma.*

Æquemultiplices eidem multiplici, ſunt quoq; inter ſe æquemultiplices.

PROP. I.



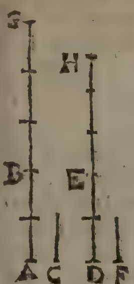

*Si ſint quotcunque magnitudines* A B, C D, *quotcunque magnitudinum* E, F *æqualium numero, ſingulæ ſingularum, æquemultiplices; quam multiplex eſt unius* E *una magnitudo* A B, *tam multiplices erunt & omnes* AB+CD *omnium* E+F.

Sint A G, G H, H B partes quantitatis A B ipſi E æquales. item CI, IK, KD partes quantitatis C D ipſi F pares. Harum numerus illarum numero æqualis ponitur. Quum igitur
a 2. ax. AG+CI *a*=E+F; *a* & GH+IK=E+F; *a* & HB+KD=E+F, liquet AB+CD æque multoties continere E+F, ac una AB unam E continet. Q. E. D.

PROP.

PROP. II.

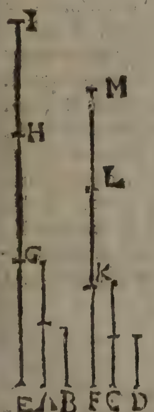
*Si prima* A B *ſecundæ* C *æque fuerit multiplex, atque tertia* D E *quartæ* F; *fuerit autem & quinta* B G *ſecundæ* C *æque multiplex, atque ſexta* E H *quartæ* F, *erit & compoſita prima cum quinta* (A G) *ſecundæ* C *æque multiplex, atque tertia cum ſexta* (DH) *quartæ* F.




Numerus partium in A B ipſi C æqualium æqualis ponitur numero partium in D E ipſi F æqualium. Item numerus partium in B G ponitur æqualis numero partium in E H. *a* ergo numerus partium in A B+B G æquatur numero partium in DE+EH. hoc eſt tota AG æquemultiplex eſt ipſius C, atque tota G H ipſius F. Q. E. D. *a 2.ax.*

PROP. III.

*Sit prima* A *ſecundæ* B *æquemultiplex, atque tertia* C *quartæ* D; *ſumantur autem* E I, F M *æquemultiplices primæ & tertiæ; erit & ex æquo, ſumptarum utraque utriuſque æquemultiplex: altera quidem* E I *ſecundæ* B, *altera autem* F M *quartæ* D.

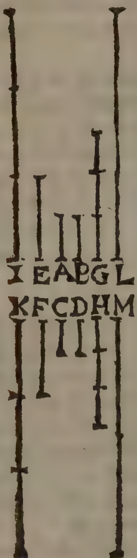



Sint E G, G H, H I partes multiplicis E I ipſi A pares; item F K, K L, L M partes multiplicis F M ipſi C æquales. *a* Harum numerus illarum numero æquatur. porro A, id eſt E G, vel G H, vel G I ipſius B ponitur æquemultiplex atque C, vel FK, &c. ipſius D. *a 1.p.*

*b* ergo

b 2. 5. *b* ergo E G + G H æquemultiplex eſt ſecundæ
c 2. 5. B, atque F K + K L quartæ D. *c* Simili argumento E I (E H + H I) tam multiplex eſt ipſius B, quam F M (F L + L N) ipſius D. Q. E. D.

## PROP. IV.




*Si prima A ad ſecundam B eandem habuerit rationem, & tertia C ad quartam D; etiam E & F æquemultiplices primæ A, & tertiæ C ad G, & H æquemultiplices ſecundæ B, & quartæ D, juxta quamvis multiplicationem, eandem habebunt rationem, ſi prout inter ſe reſpondent, ita ſumptæ fuerint.* (E. G :: F. H.)

Sume I, & K ipſarum E, & F; item L & M ipſarum G, & H æquemultiplices. *a* Erit I ipſius A æquemultiplex atque
a 3. 5. K ipſius C; *a* pariterque L tam multiplex ipſius B quam M ipſius D. Itaque cum ſit A. B *b* :: C. D; juxta 6 *def.* ſi
b hyp. I ⊏, =, ⊐ L; conſequenter pari modo K ⊏, =, ⊐ M. ergo cum I, & K ipſarum E, & F ſumptæ ſint æquemultiplices, atque L, & M ipſarum G & H; erit juxta 7. *def.* E. G :: F. H. Q. E. D.

*Coroll.*

*Hinc demonſtrari ſolet inverſa ratio.*

Nam quoniam A. B :: C. D, ſi E ⊏, =, ⊐
c 6 def. 5. G; *c* erit ſimiliter F ⊏, =, ⊐ H. ergo liquet, quod

quod ſi G ⊏, =, ⊐ E, eſſe H ⊏, =, ⊐ F.
d ergo B. A :: D. C. Q. E. D. d 6. def. 5.

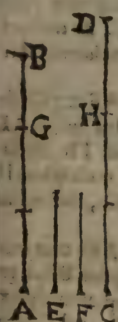
### Prop. V.




*Si magnitudo* A B *magnitudinis* C D *æque fuerit multiplex, atque ablata* AE *ablata* CF; *etiam reliqua* EB *reliquæ* FD *ita multiplex erit, ut tota* AB *totius* CD.

Accipe aliam quandam GA, quæ reliquæ FD ita ſit multiplex, atque tota AB totius CD, vel ablata AE ablatæ CF. a ergo tota GA + AE totius CF + FD æquemultiplex eſt, ac una AE a 1. 5. unius CF, hoc eſt, ac AB ipſius CD. b ergo GE = b 6. ax. AB. c proinde, ablata communi AE, manet GA c 3. ax. = EB. ergo, &c.

### Prop. VI.

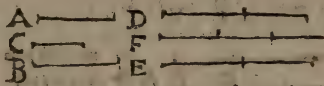



*Si duæ magnitudines* A B, C D *duarum magnitudinum* E, F *ſint æquemultiplices; & detractæ quædam ſint,* AG, *&* CH, *earundem* E, *&* F *æquemultiplices; & reliquæ* G B, HD *eiſdem* E, F *aut æquales ſunt, aut æque ipſarum multiplices.*

Nam quia numerus partium in A B ipſi E æqualium ponitur æqualis numero partium in CD ipſi E æqualium; item numerus partium in A G æqualis numero partium in C H. ſi hinc A G, inde CH detrahatur, a remanet numerus partium in a 3. ax. reliqua G B æqualis numero partium in H D. ergo ſi GB ſit E ſemel, erit HD etiam C ſemel. ſi GB ſit E aliquoties, erit H D etiam C toties accepta. Q. E. D.

 Prop.

## PROP. VII.




*Æquales A & B ad eandem C eandem habent rationem; & eadem C ad æquales A & B.*

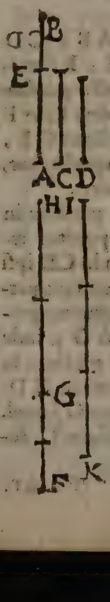
Sumantur D & E æqualium A & B æquemultiplices, & F utcunque multiplex ipſius C; *a* erit D = E. quare ſi D ⊏, =, ⊐ F, erit ſimiliter E ⊏, =, ⊐ F. *b* ergo A. C :: B. C. inverſe igitur C. A *c* :: C. B. Q. E. D.

a 8 ax.
b 6 def. 5.
c cor. 4 5.

*Schol.*

Si loco multiplicis F ſumantur duæ æquemultiplices, eodem modo oſtendetur æquales magnitudines ad alias inter ſe æquales eandem habere rationem.

## PROP. VIII.




*Inæqualium magnitudinum* A B, C, *major* A B *ad eandem* D *majorem rationem habet, quam minor* C. *Et eadem* D *ad minorem* C *majorem rationem habet, quam ad majorem* AB.

Ex majori AB aufer AE = C. ſumatur HG tam multiplex ipſius AE, vel C, quam GF reliquæ FB. Multiplicetur D, donec ejus multiplex IK major evadat quam HG, ſed minor quam HF.

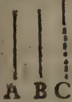
Quoniam HG ipſius AE *a* tam multiplex eſt, quam GF ipſius EB, *b* erit tota HF totius AB æquemultiplex, atque una HG unius AE, vel C. ergo cum HF ⊏ IK (quæ multiplex eſt ipſius D) ſed HG ⊐ IK, *c* erit $\frac{AB}{D} \sqsubset \frac{C}{D}$ Q. E. D.

a conſt.
b 1. 5.
c 8. def. 5.

Rurſus

Rurſus quia IK $\sqsubset$ HG, at IK $\sqsupset$ HF (ut prius dictum) d erit $\frac{D}{C} \sqsubset \frac{D}{AB}$ Q. E. D.

## PROP. IX.




*Quæ ad eandem eandem habent rationem, æquales ſunt inter ſe. Et ad quas eadem eandem habet rationem, eæ quoque ſunt inter ſe æquales.*

1. *Hyp.* Sit A. C :: B. C. dico A = B. Nam ſit A $\sqsubset$, vel $\sqsupset$ B, a erit ideo $\frac{A}{C} \sqsubset$, vel $\sqsupset \frac{B}{C}$. contra Hyp. a 8. 5.

2. *Hyp.* Sit C. B :: C. A. dico A = B. nam ſit A $\sqsubset$ B, b ergo $\frac{C}{B} \sqsubset \frac{C}{A}$. contra Hyp. b 8. 5.

## PROP. X.

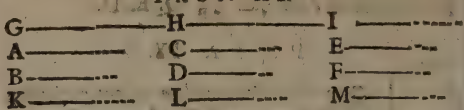



*Ad eandem magnitudinem rationem habentium, quæ majorem rationem habet, illa major eſt: ad quam vero eadem majorem rationem habet, illa minor eſt.*

1. *Hyp.* Sit $\frac{A}{C} \sqsubset \frac{B}{C}$. Dico A $\sqsubset$ B. Nam ſi dicatur A = B, a erit A. C :: B. C. contra a 7. 5.
Hyp. Sin A $\sqsupset$ B, b erit $\frac{A}{C} \sqsupset \frac{B}{C}$ etiam contra b 8. 5.
Hyp.

2. *Hyp.* Sit $\frac{C}{B} \sqsubset \frac{C}{A}$. Dico B $\sqsupset$ A. Nam dic B = A. c ergo C. B :: C. A. contra Hyp. vel c 7. 5.
dic B $\sqsubset$ A. d ergo $\frac{C}{A} \sqsubset \frac{C}{B}$ etiam contra Hyp. d 8. 5.

PROP. XI.




*Quæ eidem ſunt eædem rationes, & inter ſe ſunt eædem.*

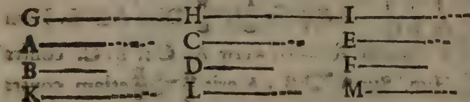
Sit A.B :: E. F. item C. D :: E.F. dico A. B :: C.D. ſume ipſarum A, C, E æquemultiplices G, H, I; atque ipſarum B, D, F æquemultiplices K, L, M. Et quoniam [a] A.B :: E.F. ſi G ⊏, =, ⊐ K, [b] erit pari modo I ⊏, =, ⊐ M, pariterque quia [a] E.F :: C. D. ſi I ⊏, =, ⊐ M, [b] erit H ſimiliter ⊏, =, ⊐ L. ergo ſi G ⊏, =, ⊐ K, erit ſimiliter H ⊏, =, ⊐ L, [c] quare A. B :: C. D. Q. E. D.

a hyp.
b 6. def. 5.
c 6. def. 5.

*Schol.*

Quæ eiſdem rationibus ſunt eædem rationes, ſunt quoque inter ſe eædem.

PROP. XII.




*Si ſint magnitudines quotcunque* A, & B; C & D; E, & F *proportionales; quemadmodum ſe habuerit una antecedentium* A *ad unam conſequentium* B, *ita ſe habebunt omnes antecedentes*, A, C, E *ad omnes conſequentes*, B, D, F.

Sume antecedentium æquemultiplices G, H, I; & conſequentium K, L, M. Quoniam quam multiplex eſt una G unius A, [a] tam multiplices ſunt omnes G, H, I omnium A, C, E; pariterque quam multiplex eſt una K unius B, [a] tam multiplices ſunt omnes K, L, M omnium B, D, F; ſi G ⊏, =, ⊐ K, erit ſimiliter

a 1. 5.

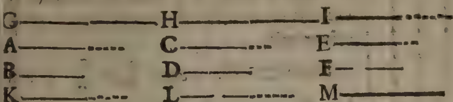
G+

G+H+I $\sqsubset$, =, $\sqsupset$ K+L+M. *b* quare A.B :: A+C+E.B+D+F. Q. E. D. b 6. *def.* 5.

*Coroll.*

Hinc, ſi ſimilia proportionalia ſimilibus proportionalibus addantur, tota erunt proportionalia.

## PROP. XIII.



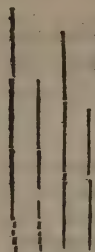
*Si prima A ad ſecundam B eandem habuerit rationem, quam tertia C ad quartam D; tertia vero C ad quartam D majorem habuerit rationem, quam quinta E ad ſextam F; prima quoque A ad ſecundam B majorem rationem habebit, quam quinta E ad ſextam F.*

Sume ipſarum A, C, E æquemultiplices G, H, I: ipſarumque B, D, F æquemultiplices K, L, M. Quia A.B :: C. D; ſi H $\sqsubset$ L, *a* erit G $\sqsubset$ K. Sed quia $\frac{C}{D} \sqsubset \frac{E}{F}$, *b* fieri poteſt ut ſit H $\sqsubset$ L, & I non $\sqsubset$ M. ergo fieri poteſt ut G $\sqsubset$ K, & I non $\sqsubset$ M. *c* ergo $\frac{A}{B} \sqsubset \frac{E}{F}$. Q. E.D. a 6. *def.* 5. b 8. *def.* 5. c 8 *def.* 5.

*SCHOL.*

Quod ſi $\frac{C}{D} \sqsupset \frac{E}{F}$, erit quoque $\frac{A}{B} \sqsupset \frac{E}{F}$. Item ſi $\frac{A}{B} = \frac{C}{D} \sqsubset \frac{E}{F}$, erit $\frac{A}{B} \sqsubset \frac{E}{F}$. & ſi $\frac{A}{B} = \frac{C}{D} = \frac{E}{F}$, erit $\frac{A}{B} \sqsupset \frac{E}{F}$.

## PROP. XIV.



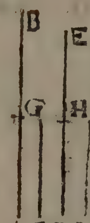
*Si prima* A *ad ſecundam* B *eandem habuerit rationem, quam tertia* C *ad quartam* D; *prima vero* A, *quam tertia* C *major fuerit; erit & ſecunda* B *major quam quarta* D. *Quod ſi prima* A *fuerit æqualis tertiæ* C, *erit & ſecunda* B *æqualis quartæ* D; *ſi vero* A *minor, &* B *minor erit.*

a 8. 5. Sit A $\sqsubset$ C. *a* ergo $\frac{A}{B} \sqsubset \frac{C}{B}$. *b* ſed
b hyp.
c 13. 5. A B C D $\frac{A}{B} = \frac{C}{D}$. *c* ergo $\frac{C}{D} \sqsubset \frac{C}{B}$. *d* ergo B $\sqsubset$ D.
d 10. 5.
e 7. 5. Simili argumento ſi A $\sqsupset$ C, *d* erit B $\sqsupset$ D. Sin
f hyp.
g 11. & 9. 5. ponatur A = C; ergo C. B *e* :: A. B *f* :: C. D. *g* ergo B = D. Quæ E. D.

### SCHOL.

A fortiori, ſi $\frac{A}{B} \sqsupset \frac{C}{D}$, atque A $\sqsubset$ C, erit B $\sqsubset$ D. Item ſi A = B, erit C = D. Et ſi A $\sqsubset$, vel $\sqsupset$ B, erit pariter C $\sqsubset$, vel $\sqsupset$ D.

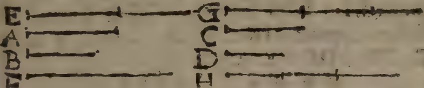
## PROP. XV.




*Partes* C & F *cum pariter multiplicibus* AB, & DE *in eadem ſunt ratione, ſi prout ſibi mutuo reſpondent, ita ſumantur.* (AB. DE :: C. F.)

Sint AG, GB partes multiplicis AB ipſi C æquales: item DH, HE partes multiplicis DE ipſi F æquales. *a* Harum numerus illarum numero æquatur. ergo quum *b* A G. C ::
a hyp.
b 7. 5. A C D F DH. F; *b* atque GB. C :: HE. F. *c* erit
c 12. 5. AG + GB (AB) DH + HE (DE) :: C. F. Q. E. D.

## PROP. XVI.




*Si quatuor magnitudines* A, B, C, D *proportionales fuerint; & viciſſim proportionales erunt.* (A.C :: B. D.)

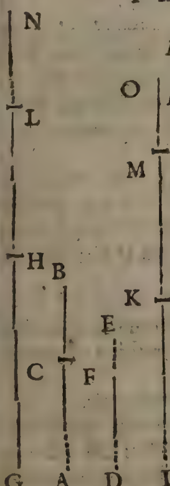
Accipe E & F æquemultiplices ipſarum A & B. ipſarumque C & D æquemultiplices G & H. Itaque E. F [a] :: A.B. [b] :: C.D [a] :: G.H Quare ſi E $\sqsubset$, $=$, $\sqsupset$ G, [c] erit ſimiliter F $\sqsubset$, $=$, $\sqsupset$ H. [d] ergo A.C :: B.D. Q. E. D.

a 15. 5.
b hyp.
c 11. 5 & 14. 5.
d 6. def. 5.

### SCHOL.

Alterna ratio locum tantum habet, quando quantitates ejuſdem ſunt generis. Nam Heterogeneæ quantitates non comparantur.

## PROP. XVII.




*Si compoſitæ magnitudines proportionales fuerint* (AB.CB :: DE. FE;) *hæ quoque diviſæ proportionales erunt.* (AC.CB :: DF. FE.)

Accipe GH, HL, IK, KM ordine æquemultiplices ipſarum AC, CB, DF, FE. item LN, MO æquemultiplices ipſarum CB, FE. Tota GL totius AB [a] tam multiplex eſt, quam una GH unius AC, [b] id eſt quam IK ipſius DF; [c] hoc eſt quam tota IM totius DE; Item HN (HL + LN) ipſius C B [d] æquemultiplex eſt, ac K O (K M + M O) ipſius F E. Quum igitur per hyp. AB, BC :: DE. EF. ſi GL $\sqsubset$, $=$, $\sqsupset$ HN, etiam ſimiliter.

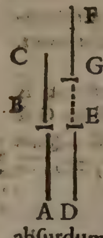
a 1. 5.
b conſtr.
c 1. 5.
d 2. 5.

e 6. def. 5. militer *e* erit IM ⊏, =, ⊐ KO. aufer hinc inde æquales HL, KM. ſi reliqua GH ⊏, =, ⊐

f 5. ax. LN, *f* erit ſimiliter IK ⊏, =, ⊐ MO. *g* unde

g 6. def. 5. AC. CB :: DF. FE. Q. E. D.

PROP. XVIII.

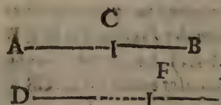


*Si diviſæ magnitudines ſint proportionales* (AB. BC :: DE. EF,) *hæ quoque compoſitæ proportionales erunt* (AC. CB :: DF. FE.)

Nam ſi fieri poteſt, ſit AB. CB :: DF. FG ⊐ FE. *a* ergo erit diviſim AB. BC :: DG. GF. *b* hoc eſt DG. GF :: DE. EF. ergo cum DG ⊏ DE, *c* erit GF ⊏ EF. Q. E. A. Simile abſurdum ſequetur, ſi dicatur AB. CB :: DE. GF ⊏ FE.

a 17. 5.
b hyp. & 11. 5.
c 14. 5.
d 9. ax.

PROP. XIX.



*Si quemadmodum totum* AB *ad totum* DE, *ita oblatum* AC *ſe habuerit ad ablatum* DF; *& reliquum* CB *ad reliquum* FE, *ut totum* AB *ad totum* DE, *ſe habebit.*

a hyp. Quoniam *a* AB. DE :: AC. DF, *b* erit per-

b 16. 5. mutando AB. AC :: DE. DF. *c* ergo diviſim

c 17. 5. AC. CB :: DF. FE. quare rurſus *b* permutando

d hyp. & 11. 5. AC. DF :: CB. FE; *d* hoc eſt AB. DE :: CB. FE. Q. E. D.

*Coroll.*

1. Hinc, ſi ſimilia proportionalia ſimilibus proportionalibus ſubducantur, reſidua erunt proportionalia.

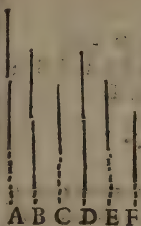
2. *Hinc demonſtrabitur converſa ratio.*

Sit AB. CB :: DE. FE. Dico AB. AC :: DE. DF. Nam *a* permutando AB. DE :: CB. FE. *b* ergo AB. DE :: AC. DF. quare iterum permutando, AB. AC :: DE. DF. Q. E. D.

a 16. 5.
b 19. 5.

PROP.

## Prop. XX.




*Si ſint tres magnitudines* A, B, C; *& aliæ* D, E, F *ipſis æquales numero, quæ binæ & in eadem ratione ſumantur* (A. B :: D. E, *atque* B. C :: E. F;) *ex æquo autem prima* A *major fuerit, quam tertia* C; *erit & quarta* D *major quam ſexta* F. *Quod ſi prima* A *tertiæ* C *fuerit æqualis; erit & quarta* D *æqualis ſextæ* F. *Sin illa minor, hæc quoque minor erit,*

*Hyp.* Si A ⊏ C. quoniam *a* E. F :: B.C. *b* erit inverſe F. E :: C.B. *c* Sed $\frac{C}{B}$ ⊐ $\frac{A}{B}$ *d* ergo $\frac{F}{E}$ ⊐ $\frac{A}{B}$ vel $\frac{D}{E}$. *e* ergo D ⊏ F. Q.E.D.

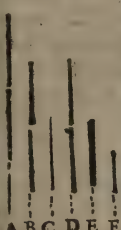
a *hyp.*
b *cor.* 4. 5.
c *hyp.* & 8. 5.
d *ſchol.* 13. 5.
e 10. 5.

2. *Hyp.* Simili argumento, ſi A ⊐ C, oſtendetur D ⊐ F.

3. *Hyp.* Si A = C. quoniam F. E :: C. B :: *f* A. B :: D. E. *g* erit D = F. Q.E.D.

f 7. 5.
g 11. 5. & 9. 5.

## Prop. XXI.




*Si ſint tres magnitudines* A, B, C; *& aliæ* D, E, F *ipſis æquales numero, quæ binæ & in eadem ratione ſumantur, fueritque perturbata eorum proportio,* (A. B :: E. F. *atque* B. C :: D. E;) *ex æquo autem prima* A *quam tertia* C *major fuerit; erit & quarta* D *quam ſexta* F *major. Quod ſi prima fuerit tertiæ æqualis, erit & quarta æqualis ſextæ: ſin illa minor, hac quoque minor erit.*

1. *Hyp.* A ⊏ C. Quoniam *a* D. E :: B. C, invertendo erit E.D :: C.B. atqui $\frac{C}{B}$ *b* ⊐ $\frac{A}{B}$.

a *hyp.*
b 8. 5.

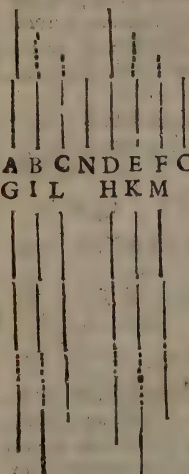
c ergo

e *ſchol.* 13. 5. d 10 5. *e* ergo $\frac{E}{D} \sqsupset \frac{A}{B}$, hoc eſt $\frac{E}{F}$. *d* ergo D $\sqsubset$ F. Q. E. D.

2. *Hyp.* Similiter, ſi A $\sqsupset$ C, erit D $\sqsupset$ F.

e 7. 5. f *hyp.* g 9 5. 3. *Hyp.* Si A = C. quoniam E. D *e* :: C.B :: *e* A.B :: *f* E. F. *g* erit D = F. Q. E. D.

## PROP. XXII.




*Si ſint quotcunque magnitudines* A, B, C ; *& aliæ ipſis æquales numero* D, E, F, *quæ binæ & in eadem ratione ſumantur* (A.B :: D.E. & B. C :: E. F;) *& ex æqualitate in eadem ratione erunt* (A.C :: D.F.)

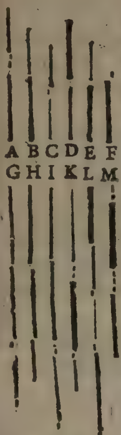
Accipe G, H ipſarum A, D ; & I, K ipſarum B, E ; item L, M ipſarum E, F æquemultiplices.

a *hyp.* b 4 5. Quoniam *a* A. B :: D.E. *b* erit G. I :: H. K. eodem modo, erit I. L :: K.M. ergo ſi G $\sqsubset$, =, $\sqsupset$ L, *c* erit H. $\sqsubset$, =, $\sqsupset$ M; *d* ergo A.C :: D.F. Eodem pacto ſi ulterius C. N :: F.O, erit ex æquali A.N :: D.O. Q. E. D.

c 20. 5. d 6. *def* 5.

PROP.

PROP. XXIII.




*Si ſint tres magnitudines* A, B, C, *aliæque* D, E, F *ipſis æquales numero; quæ binæ in eadem ratione ſumantur; fuerit autem perturbata earum proportio.* ( A. B :: E. F. & B.C :: D. E. ) *etiam ex æqualitate in eadem ratione erunt.*

Sume G, H, I, ipſarum A, B, D; item K, L, M ipſarum C, E, F æquemultiplices. erit G. H [a] :: A. B [b] :: E. F [a] :: L. M. porro quia [b] B. C :: D. E. erit [c] H. K :: I. L. ergo G, H, K; & I, L, M habent ſe juxta 21. 5. quare ſi G ⊏, =, ⊐ K, erit ſimiliter I ⊏, =, ⊐ M. [d] proinde A. C :: D. F. Q. E. D.

a 15. 5.
b hyp.
c 4. 5.
d 6. def. 5.

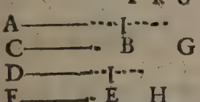
Eodem modo ſi plures fuerint magnitudinibus tribus, &c.

*Coroll.*

Ex *his ſequitur, rationes ex iiſdem rationibus compoſitas eſſe inter ſe eaſdem. item, earundem rationum eaſdem partes inter ſe eaſdem eſſe.

* 21 & 23. 5. & 10. def. 5.

PROP. XXIV.



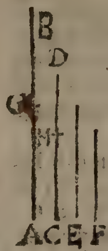

*Si prima* AB *ad ſecundam* C *eandem habuerit rationem quam tertia* DE *ad quartam* F; *habuerit autem & quinta* BG *ad ſecundam* C *eandem rationem, quam ſexta* EH *ad quartam* F; *etiam compoſita prima cum quinta* (AG) *ad ſecundam* C *eandem habebit rationem, quam tertia cum ſexta* (DH) *ad quartam* F.

Nam quia [a] AB. C :: DE. F. atque ex hyp. & inverſe C. BG :: F. EH, erit [b] ex æquali AB. BG :: DE. EH. ergo componendo AG. BG :: DH. EH. [c] item BG. C :: EH. F. [b] ergo rurſus ex æquo, AG. C :: DH. F. Q. E. D.

a hyp.
b 22. 5.
c hyp.

PROP.

## PROP. XXV.




*Si quatuor magnitudines proportionales fuerint, (AB. CD :: E. F.) maxima AB & minima F, reliquis CD & E majores erunt.*

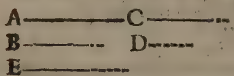
Fiant AG = E; & CH = F.

a hyp.
b 7.5.
c 19.5.
d hyp.
e ſchol. 14.5.

Quoniam AB. CD *a* :: E. F *b* :: AG. CH; *c* erit AB. CD :: GB. HD. *d* ſed AB $\sqsubset$ CD. *e* ergo GB $\sqsubset$ HD. atqui AG + F = E + CH. ergo AG + F + GB $\sqsubset$ E + CH + HD, hoc eſt AB + F $\sqsubset$ E + CD. Q. E. D.

Quæ ſequuntur propoſitiones non ſunt Euclidis; ſed ex aliis deſumptæ, ob frequentem earum uſum Euclidæis ſubjungi ſolent.

## PROP. XXVI.



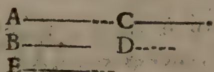

*Si prima ad ſecundam habuerit majorem proportionem, quam tertia ad quartam; habebit convertendo, ſecunda ad primam minorem proportionem, quam quarta ad tertiam.*

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{B}{A} \sqsupset \frac{D}{C}$. Nam concipe

a 13.5.
b 10.5.
c 8.5
d cor 4.5.

$\frac{C}{D} = \frac{E}{B}$. *a* ergo $\frac{A}{B} \sqsubset \frac{E}{B}$. *b* quare A $\sqsubset$ E. *c* ergo $\frac{B}{A} \sqsupset \frac{B}{E}$, *d* vel $\frac{D}{C}$. Q. E. D.

## PROP. XXVII.



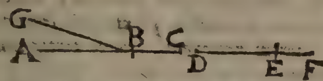

*Si prima ad ſecundam habuerit majorem proportionem, quam tertia ad quartam; habebit quoque viciſſim prima ad tertiam majorem proportionem, quam ſecunda ad quartam.*

Sit

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{A}{C} \sqsubset \frac{B}{D}$. Nam puta $\frac{E}{B} = \frac{C}{D}$.

*a* ergo A $\sqsubset$ E. *b* ergo $\frac{A}{C} \sqsubset \frac{E}{C}$, *c* vel $\frac{B}{D}$. Q. E. D. a 10. 5. b 8. 5. c 16. 5.

## PROP. XXVIII.



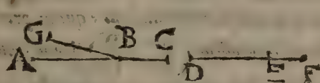

*Si prima ad ſecundam habuerit majorem proportionem, quam tertia ad quartam; habebit quoque compoſitá prima cum ſecunda ad ſecundam majorem proportionem, quam compoſitá tertia cum quarta ad qu artam.*

Sit $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Dico $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Nam cogita $\frac{GB}{BC} = \frac{DE}{EF}$. *a* ergo AB $\sqsubset$ GB. adde utrinque BC, a 10. 5.

*b* erit AC $\sqsubset$ GC. *c* ergo $\frac{AC}{BC} \sqsubset \frac{GC}{BC}$. *d* hoc eſt $\frac{DF}{FE}$. b 4. ax. c 8. 5. d 18. 5.

Q. E. D.

## PROP. XXIX.




*Si compoſita prima cum ſecunda ad ſecundam majorem habuerit proportionem, quam compoſita tertia cum quarta ad quartam; habebit quoque dividendo prima ad ſecundam majorem proportionem quam tertia ad quartam.*

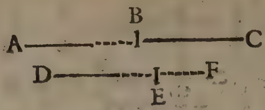
Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Intellige $\frac{GC}{BC} = \frac{DE}{EF}$. *a* ergo AC $\sqsubset$ GC. aufer commune a 10. 5.

BC, *b* erit AB $\sqsubset$ GB. *c* ergo $\frac{AB}{BC} \sqsubset \frac{CB}{BC}$ *d* vel $\frac{DE}{EF}$. b 5. ax. c 8. 5. d 17. 5.

Q.E.D.

PROP.

PROP. XXX.




*Si compoſita prima cum ſecunda ad ſecundam habuerit majorem proportionem, quam compoſita tertia cum quarta ad quartam; habebit, per converſionem rationis, prima cum ſecunda ad primam minorem rationem, quam tertia cum quarta ad tertiam.*

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Nam quia

a hyp. $\frac{AC}{BC}$ *a* $= \frac{DF}{FE}$, *b* erit dividendo $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. *c* conver-

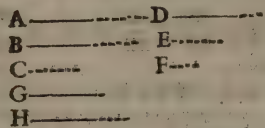
b 19. 5.

c 16. 5.

d 28. 5. tendo igitur $\frac{BC}{AB} \sqsupset \frac{EF}{DE}$. *d* ergo componendo

$\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Q. E. D.

PROP. XXXI.




*Si ſint tres magnitudines A, B, C, & aliæ ipſis æquales numero D, E, F; ſitque major proportio primæ priorum ad ſecundam, quam primæ poſteriorum ad ſecundam* $\left(\frac{A}{B} \sqsubset \frac{D}{F};\right)$ *item ſecundæ priorum ad tertiam major, quam ſecundæ poſteriorum ad tertiam* $\left(\frac{B}{C} \sqsubset \frac{E}{F};\right)$ *erit quoque ex æqualitate major proportio primæ priorum ad tertiam, quam primæ poſteriorum ad tertiam* $\left(\frac{A}{C} \sqsubset \frac{D}{F}.\right)$

a 10. 5. Concipe $\frac{G}{C} = \frac{E}{F}$. *a* ergo E $\sqsubset$ G. *b* ergo $\frac{A}{C} \sqsubset \frac{A}{B}$.

b 8. 5.

c 13. 5.

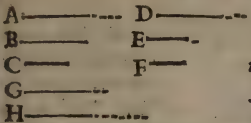
d 10. 5.

e 8. 5.

f 12. 5. Rurſus puta $\frac{H}{G} = \frac{D}{E}$. *c* ergo $\frac{H}{G} \sqsupset \frac{A}{B}$; *d* ergo fortius $\frac{H}{G} \sqsupset \frac{A}{G}$. *d* quare A $\sqsubset$ H. *e* proinde $\frac{A}{C} \sqsubset \frac{H}{C}$, vel $\frac{D}{F}$.

Q. E. D.

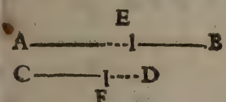
## PROP. XXXII.




*Si ſint tres magnitudines* A,B.C *; & aliæ ipſis æquales* D, E, F *; ſitque major proportio primæ priorum ad ſecundam, quam ſecundæ poſteriorum ad tertiam* $\left(\frac{A}{B} \sqsubset \frac{E}{F};\right)$ *item ſecundæ priorum ad tertiam major quam primæ poſteriorum ad ſecundam* $\left(\frac{B}{C} \sqsubset \frac{D}{E};\right)$ *erit quoque ex æqualitate major proportio primæ priorum ad tertiã, quam primæ poſteriorum ad tertiam* $\left(\frac{A}{C} \sqsubset \frac{D}{F}.\right)$

Hujuſce demonſtratio plane ſimilis eſt demonſtrationi præcedentis.

## PROP. XXXIII.

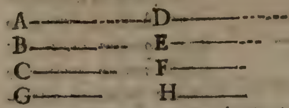



*Si fuerit major proportio totius* AB *ad totum* CD, *quam ablati* AE *ad ablatum* CF *; erit & reliqui* EB *ad reliquum* FD *major proportio, quam totius* AB *ad totum* CD.

Quoniam $\frac{AB}{CD}$ [a] $\sqsubset \frac{AE}{CF}$, *b* erit permutando $\frac{AB}{AE} \sqsubset \frac{CD}{CF}$. *c* ergo per converſionem rationis $\frac{AB}{EB} \sqsupset \frac{CD}{FD}$. permutando igitur $\frac{AB}{CD} \sqsupset \frac{EB}{FD}$. Q. E. D.

a *hyp.*
b *17. 5.*
c *30. 5.*

## PROP. XXXIV.

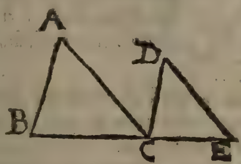



*Si ſint quotcunque magnitudines, & aliæ ipſis æquales numero, ſitque major proportio primæ priorum ad primam poſteriorum, quam ſecundæ ad ſecundam; & hæc major quam tertiæ ad tertiam; & ſic deinceps: habebunt omnes priores ſimul ad omnes poſteriores ſimul, majorem proportionem, quam omnes priores, relicta prima, ad omnes poſteriores, relicta quoque prima; minorem autem, quam prima priorum ad primam poſteriorum; majorem denique etiam, quam ultima priorum ad ultimam poſteriorum.*

*Horum demonſtratio eſt penes interpretes; quos adeat, qui eam deſiderat: nos omiſimus, brevitatis ſtudio; & quia illorum nullus uſus in his elementis.*

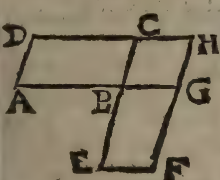
# LIB. VI.

## *Definitiones.*




I. Similes figuræ rectilineæ ſunt (A B C, D C E,) quæ & angulos ſingulos ſingulis æquales habent; atque etiam latera, quæ circum angulos æquales, proportionalia.

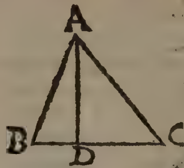
*Ang.* B = D C E; *&* A B. B C :: D C. C E. *item ang.* A = D; *atque* BA. AC :: CD. DE. *denique ang.* A C B = E. *atque* B C. C A :: C E. E D.




II. Reciprocæ autem ſunt (B D, B F,) cum in utraque figura antecedentes, & conſequentes rationum termini fuerint. (hoc eſt, AB. BG :: EB. BC.)




III. Secundùm extremam & mediam rationem recta linea A B ſecta eſſe dicitur, cum ut tota A B ad majus ſegmentum AC, ita majus ſegmentum A C ad minus C B ſe habuerit. (A B. A C :: A C. C B.)




IV. Altitudo cujuſque figuræ ABC eſt linea perpendicularis A D, à vertice A ad baſim B C deducta.

V. Ratio ex rationibus componi dicitur, cum rationum quantitates inter ſe multiplicatæ, aliquam effecerint rationem.

*Vt ratio* A *ad* C, *componitur ex rationibus* A
a 20.def.5. b 15.5. *ad* B, *&* B *ad* C. *nam* $\frac{A}{B} + \frac{B}{C} \overset{a}{=} \frac{Ab}{C} \overset{b}{=} \frac{AB}{BC}$.

## PROP. I.

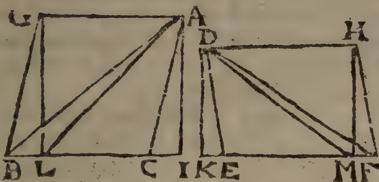



*Triangula* ABC, ACD, *& parallelogramma* BCAE, CDFA, *quorum eadem fuerit altitudo, ita ſe habent inter ſe, ut baſes* BC, CD.

a 3. 1. *a* Accipe quotvis BG, HG, ipſi BC æquales; item DI = CD. & connecte AG, AH, AI.

b 38. 1. *b* Triangula ACB, ABG, AGH æquantur; *b* item triang. ACD = ADI. ergo triangulum ACH tam multiplex eſt trianguli ACB, quam baſis HC baſis BC. & æquemultiplex eſt triang. ACI trianguli ACD, ac baſis CI baſis CD.
c ſch.38 1. d 6.def 5. e 41. 1.& 15. 5. cum igitur ſi HC $\sqsubset, =, \sqsupset$ CI, *c* erit ſimiliter triang. AHC $\sqsubset, =, \sqsupset$ ACI. *d* ideoque BC, CD :: triang. ABC. ACD :: *e* pgr. CE. CF. Q.E.D.

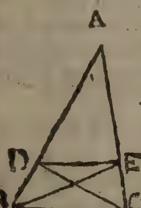
*Schol.*




Hinc, *triangula* A B C, D E F, *& parallelo-grammа* A G B C, D E F H, *quorum æquales ſunt baſes* BC, EF, *ita ſe habent ut altitudines* AI, DK.

a Sume IL = CB; & KM = EF; ac junge LA, LG, MD, MH. liquet eſſe triang. ABC. DEF :: b ALI. DKM :: c AI. DK :: d pgr. AGBC. DEFH. Q. E. D. a 3. 1. b 7. 5. c 1. 6. d 41. 1. & 15. 5.

## PROP. II.




*Si ad unum trianguli* A B C *latus* B C, *parallela ducta fuerit recta quædam linea* D E, *hæc proportionaliter ſecabit ipſius trianguli latera* (A D. B D :: A E. E C.) *Et ſi trianguli latera proportionaliter ſecta fuerint* (A D. B D :: A E. E C) *quæ ad ſectiones* D, E *adjuncta fuerit recta linea* DE, *erit ad reliquum ipſius trianguli latus* BC *parallela.* Ducantur CD, BE.

1. *Hyp.* Quia triang. DEB a = DEC; b erit triang. ADE. DBE :: ADE. ECD. atqui triang. ADE. DBE c :: AD. DB. & triang. ADE. DEC c :: AE. EC. d ergo AD. DB :: AE. EC. a 37. 1. b 7. 5. c 1. 6. d 11. 5.

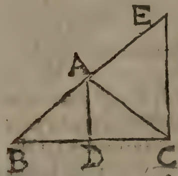
2. *Hyp.* Quia AD. DB :: AE. EC. e hoc eſt triang. ADE. DBE :: ADE. ECD; f erit triang. DBE = ECD. g ergo DE, BC ſunt parallelæ. Q. E. D. e 1. 6. f 9. 5. g 39. 1.

 *Schol.*

*Schol.*

Imo, ſi plures ad unum trianguli latus parallelæ ductæ fuerint, erunt omnia laterum ſegmenta proportionalia, ut facile deducitur ex hac.

PROP. III.



*Si trianguli* BAC *angulus* BAC *bifariam ſectus ſit, ſecans autem angulum recta linea* AD *ſecuerit & baſim, baſis ſegmenta eandem habebunt rationem quam reliqua ipſius trianguli latera* (BD. DC :: AB. AC.) *Et ſi baſis ſegmenta eandem habeant rationem quam reliqua ipſius trianguli latera* (BD. DC :: AB. AC.) *recta linea* AD *quæ à vertice* A *ad ſectionem* D *ducitur, bifariam ſecat trianguli ipſius angulum* BAC.

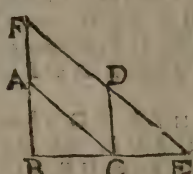
Produc BA; & fac AE = AC. & junge CE.

1. *Hyp.* Quoniam AE = AC, erit ang. ACE *a* = E *b* = $\frac{1}{2}$ BAC *c* = DAC. *d* ergo DA, CE parallelæ ſunt. *e* quare BA. AE (AC) :: BD. DC. Q. E. D.

a 5. 1.
b 32. 1.
c hyp.
d 27. 1.
e 2. 6.
f 2. 6.
g 29. 1.
h 5. 1.
k 1. ax.

2. *Hyp.* Quoniam BA. AC. (AE) :: BD. DC. *f* erunt DA, CE parallelæ: *g* ergo ang. BAD = E; & ang. DAC *g* = ACE *h* = E. *k* ergo ang. BAD = DAC. biſectus igitur eſt ang. BAC. Q. E. D.

PROP. IV.



*Æquiangulorum triangulorum* ABC, DCE *proportionalia ſunt latera, quæ circum æquales angulos* B, DCE (AB. BC :: DC. CE, &c.) *& homologa ſunt latera* AB, DC, &c. *quæ æqualibus angulis* ACB, E, &c. *ſubtenduntur.*

Statue

Statue latus B C in directum lateri C E, & produc B A, ac E D donec *a* occurrant. a 32. 1. & 13. ax.

Quoniam ang. B *b* = ECD, *c* ſunt B F, C D parallelæ. Item quia ang BCA *b* = CED, *c* ſunt C A, E F parallelæ. Figura igitur C A F D eſt parallelogramma. *d* ergo A F = C D; *d* & A C = F D. Liquet igitur A B. A F (C D) *e* :: B C. CE. *f* permutando igitur AB. BC :: CD. CE. *e* item B C. C E :: F D. (A C) D E. *f* ergo permutando BC. AC :: CE. DE. quare etiam *g* ex æquo AB. AC :: CD. DE. ergo, &c. b hyp. c 28. 1. d 34. 1. e 2. 6. f 16. 5. g 22. 5.

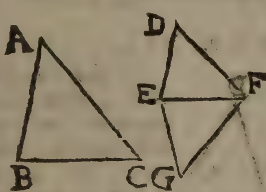
*Coroll.*

Hinc AB. DC :: BC. CE :: AC. DE.

*Schol.*

Hinc ſi in triangulo FBE ducatur uni lateri FE parallela AC; erit triangulum ABC ſimile toti FBE.

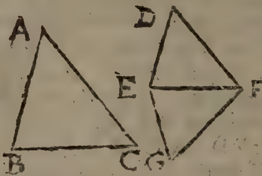
PROP. V.




*Si duo triangula* A B C, D E F *latera proportionalia habeant* (A B. BC :: DE. E F. *&* A C. B C :: D F. E F. item A B. A C :: D E D F) *æquiangula erunt triangula, & æquales habebunt eos angulos, ſub quibus homologa latera ſubtenduntur.*

Ad latus E F *a* fac ang. F E G = B; *a* & ang. EFG = C, *b* quare etiam ang. G = A. ergo G E. E F *c* :: A B. B C :: *d* D E. E F. *e* ergo G E = D E. Item G F F E *c* :: A C. C B *d* :: DF. FE. *e* ergo GF = DF. Triangula igitur DEF, GEF ſibi mutuo æquilatera ſunt. *f* ergo ang. D = G = A. *f* & ang. FED = FEG = B. *g* proinde & ang. DFE = C. ergo, &c. a 23. 1. b 32. 1. c 4. 6. d hyp. e 11. 5. & 9. 5. f 8. 1. g 32. 1.

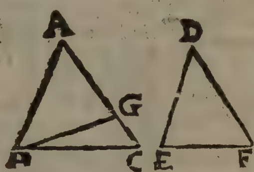
## Prop. VI.




*Si duo triangula* A B C, D E F *unum angulum* B *uni angulo* D E F *æqualem, & circum æquales angulos* B, D E F *latera proportionalia habuerint* (A B. B C :: D E. E F;) *æquiangula erunt triangula* A B C, D E F; *æqualeſque habebunt angulos, ſub quibus homologa latera ſubtenduntur.*

a 32. 1. Ad latus E F fac ang. FEG = B, & ang. EFG
b 4. 6. = C. *a* unde & ang. G = A. ergo GE. EF *b* ::
c hyp. AB. BC *c* :: DE. EF. *d* ergo DE = GE. atqui
d 9. 5. ang. D E F *e* = B *f* = G E F. *g* ergo ang. D
e hyp. = G = A. *h* proinde etiam ang. E F D = C.
f conſtr. Q. E. D.
g 4. 1.
h 32. 1.

## Prop. VII.

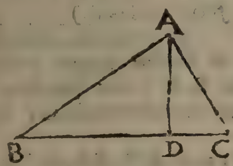



*Si duo triangula* ABC, DEF *unum angulum* A *uni angulo* D *æqualem, circa autem alios angulos* A B C, E *latera proportionalia habeant* (A B. B C :: D E. E F;) *reliquorum autem ſimul utrunque* C, F *aut minorem aut non minorem recto, æquiangula erunt triangula* ABC, DEF, *& æquales habebunt eos angulos circum quos proportionalia ſunt latera.*

a hyp. Nam ſi fieri poteſt, ſit ang. ABC ⊏ E. fac
b 32. 1. igitur ang. ABG = E; ergo cum ang. A *a* = D,
c 4. 6. *b* erit etiam ang. AGB = F. ergo AB. BG *c* ::
d hyp. DE. EF :: AB. BC. *e* ergo BG = BC. *f* ergo
e 9. 5. ang. BGC = BCG. *g* ergo ang. BGC. vel C
f 5. 1. minor
g cor. 17. 1.

minor eſt recto; *g* proinde ang. AGB, vel F re- *g cor.13.1.*
cto major eſt. ergo anguli C & F non ſunt e-
juſdem ſpeciei, contra Hyp.

## PROP. VIII.



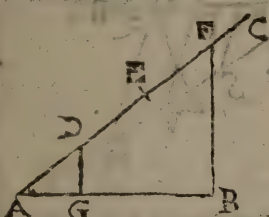

*Si in triangulo rectangulo* ABC, *ab angulo recto* B A C *in baſin* B C *perpendicularis* A D *ducta eſt; quæ ad perpendicularem triangula* A D B, ADC, *tum toti triangulo* ABC, *tum ipſa inter ſe, ſimilia ſunt.*

Nam ang. B A C *a* = B D A *a* = C D A. & *a 12.ax.*
ang. B A D *b* = C. & C A D *b* = B. ergo per *b 32. 1.*
4. 6. & 1 def.6.

*Coroll.*

Hinc 1. BD. DA *c* :: DA. DC. *c 1 def.6.*
2. BC. AC :: AC. DC. & CB.
BA :: BA. BD.

## PROP. IX.

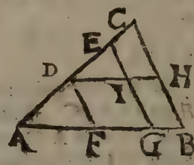



*A data recta linea* A B *imperatam partem* $\frac{1}{3}$ (A G) *auferre.*

Ex A duc infinitam A C utcunq; in qua *a* ſume tres A D, D E, E F æquales utcunque. junge F B, cui ex D *b* duc parallelam DG. Dico factum. *a 3. 1.* *b 31. 1.*

Nam GB. AG *c* :: FD. AD. ergo *d* componendo AB. AG :: AF. AD. ergo cum AD = $\frac{1}{3}$ AF, erit AG = $\frac{1}{3}$ AB. Q. E. F. *c 2. 6.* *d 18. 5*

PROP. X.

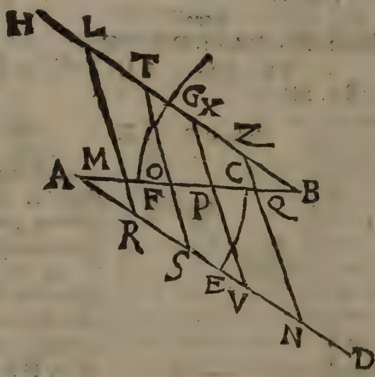



*Datam rectam lineam* A B *inſectam ſimiliter ſecare* (in F, G,) *ut data altera* A C, *ſecta fuerit* (in D, E.)

Extremitates ſectæ & inſectæ jungat recta
a 31. 1. B C. Huic ex punctis E, D a duc parallelas E G, D F rectæ ſecandæ occurrentes in G, & F. Dico factum.

a Ducatur enim D H parall. A B. Eſtque AD.
b 2. 6. c 34. 1. & 7. 5. DE b :: A F. F G, & D E. E C b :: D I. I H c :: FG. GB. Q. E. F.

*Scholium.*



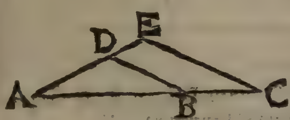

Hinc *diſcimus rectam datam* A B *in quotvis æquales partes* (puta 5.) *ſecare*. id quod facilius præſtabitur ſic;

Duc infinitam AD, eique parallelam BH etiam infinitam. Ex his cape partes æquales AR, RS, SV, VN; & BZ, ZX, XT, TL; in ſingulis una pau-

pauciores, quam deſiderentur in AB; tum rectæ ducantur L R, T S, X V, Z N. hæ quinquiſecabunt datam A B.

Nam R L, S T, V X, N Z *a* parallelæ ſunt. a 33. 1.
ergo quum A R, R S, S V, V N *b* æquales ſint, b conſtr.
*c* erunt AM, MO, OP, PQ æquales. Similiter c 2. 6.
quia BZ = ZX, erit BQ = QP. ergo A B quinquiſecta eſt. Q. E. F.

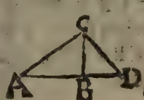
PROP. XI.




*Datis duabus rectis lineis* A B, A D, *tertiam proportionalem* D E *invenire.*

Junge B D, & ex A B protracta ſume B C = A D. per C duc C E parall. B D. cui occurrat A D producta in E. Erit D E expetita.

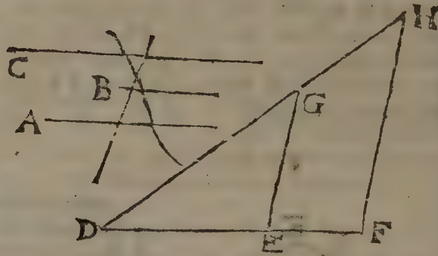
Nam AB. *a* BC. (AD) :: AD. DE. Q. E. F. 22. 6.




Vel ſic, fac ang. A B C rectum, & ang. A C D etiam rectum. b cor. 9.
*b* erit A B. B C :: B C. B D.

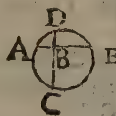
PROP.

## PROP. XII.



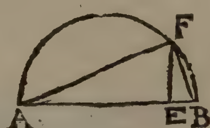
*Tribus datis rectis lineis* D E, E F, D G, *quartam proportionalem* GH *invenire.*

Connectatur E G. per F duc F H parall. E G, cui occurrat D G producta ad H. liquet eſſe
a 26 DE. EF *a* :: DG. GH. Q. E. F.



Vel ita. CD = CB + BD adapta circulo. Circino ſume A B.
a 35. 3. *a* Erit AB x BE = CB x BD. *b* qua-
b 6. 6. re AB. CB :: BD. BE.

## PROP. XIII.



*Duabus datis rectis lineis* A E, E B, *mediam proportionalem* E F *adinvenire.*

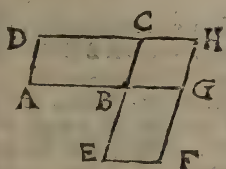
Super tota A B diametro deſcribe ſemicirculum A F B. Ex E erige perpendicularem E F occurrentem peripheriæ in F. Dico A E. E F :: E F. E B. Du-
a 31. 3. cantur enim A F, & F B. Ex trianguli *a* rectanguli

guli A F B recto angulo deducta eſt F E baſi perpendicularis; *b* ergo A E. F E :: F E. E B. Q. E. F. b *cor.* 8. 6.

*Coroll.*

Hinc, linea recta, quæ in circulo à quovis puncto diametri, ipſi diametro perpendicularis ducitur ad circumferentiam uſque, media eſt proportionalis inter duo diametri ſegmenta.

PROP. XIV.




*Æqualium, & unum* A B C *uni* E B G *æqualem habentium angulum, parallelogrammorum* B D, B F, *reciproca ſunt latera quæ circum æquales angulos.* (A B. B G :: E B. B C:) *Et quorum parallelogrammorum* B D, B F, *unum angulum* A B C *uni angulo* E B G *æqualem habentium, reciproca ſunt latera quæ circum æquales angulos, illa ſunt æqualia.*

Nam latera A B, B G circa æquales angulos faciant unam rectam: *a* quare E B, B C etiam in directum jacebunt. Producantur FG, DC; donec occurrant. a *ſch.* 15. 1.

1. *Hyp.* AB. BG *b* :: BD. BH *c* :: BF. BH *d* :: BE. BC. *e* ergo; &c. b 1. 6. c 7. 5. d 1. 6.

2. *Hyp.* BD. BH *f* :: AB. BG *g* :: BE. BC *h* :: BF. BH, *k* ergo Pgr. BD = BF. Q. E. D. e 11. 5. f 1. 6. g *hyp.* h 1. 6. k 11. & 9. 5.

PROP.

PROP. XV.




*Æqualium, & unum* ABC, *uni* DBE *æqualem habentium angulum triangulorum* ABC, DBE, *reciproca ſunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC): *Et quorum triangulorum* ABC, DBE, *unum angulum* ABC *uni* DBE *æqualem habentium reciproca ſunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC.) *illa ſunt æqualia.*

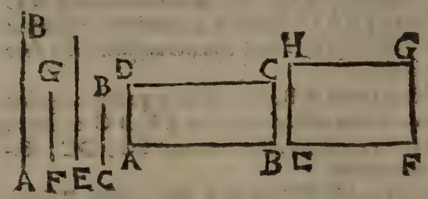
Latera CB, BD circa æquales angulos, ſtatuantur ſibi in directum; *a* ergo ABE eſt recta linea, ducatur CE.

*a ſch. 15. 1.* *b* 1. 6. *c* 7. 5. *d* 1. 6. *e* 11. 5. *f* 1. 6. *g hyp.* *h* 1. 6 *k* 11, & 9. 5.

1. *Hyp.* AB. BE *b* :: triang. ABC. CBE *c* :: triang. DBE. CBE. *d* :: DB. BC. *e* ergo, &c.

2. *Hyp.* Triang. ABC. CBE *f* :: AB. BE *g* :: DB. BC *h* :: triang. DBE. CBE. *k* ergo triang. ABC = DBE. Q. E. D.

PROP. XVI.




*Si quatuor rectæ lineæ proportionales fuerint* (AB. FG :: EF. CB,) *quod ſub extremis* AB, CB *comprehenditur rectangulum* AC, *æquale eſt ei, quod ſub mediis* EF, FG *comprehenditur, rectangulo* EG. *Et ſi ſub extremis comprehenſum rectangulum* AC *æquale fuerit ei, quod ſub mediis comprehenditur, rectangulo* EG, *illæ quatuor rectæ lineæ proportionales erunt* (AB. FG :: EF. CB.)

1. *Hyp.*

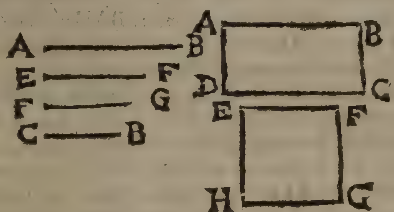
1. *Hyp*. Anguli B & F recti, ac *a* proinde pares ſunt; atque ex hyp. AB. FG :: EF. CB. *b* ergo rectang. AC = EG. Q. E. D. a 12. *ax.* b 14 6.

2. *Hyp*. *c* Rectang. AC = EG; atque ang. B = F; *d* ergo AB. FG :: EF. CB. Q. E. D. c *hyp.* d 14 6.

*Coroll.*

Hinc ad datam rectam lineam AB facile eſt datum rectangulum EG applicare, *e* faciendo AB. EF :: FG. BC. e 12. 6.

PROP. XVII.




*Si tres rectæ lineæ ſint proportionales* (AB. EF :: EF. CB,) *quod ſub extremis* AB, CB *comprehenditur rectangulum* AC, *æquale eſt ei, quod à media* EF *deſcribitur, quadrato* EG. *Et ſi ſub extremis* AB, CB *comprehenſum rectangulum* AC, *æquale ſit ei, quod à media* EF *deſcribitur, quadrato* EG, *illæ tres rectæ lineæ proportionales erunt* (AB. EF :: EF. CB.)

Accipe FG = EF.

1. *Hyp*. AB. EF *a* :: EF (FG.) CB. ergo Rectang. AC *b* = EG *c* = EFq. Q. E. D. a *hyp.* b 16 6 c 19. *def.* 1.

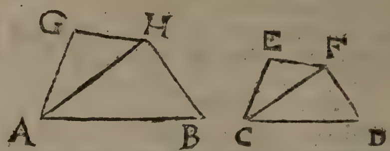
2. *Hyp*. Rectang. AC *d* = quadr. EG = EFq. *e* ergo AB. EF :: FG (EF.) BC. d *hyp.* e 16 6.

*Coroll.*

Sit A in B = Cq. ergo A. C :: C. B.

PROP.

PROP. XVIII.




*A data recta linea* AB *dato rectilineo* CEFD *ſimile ſimiliterque poſitum rectilineum* AGHB *deſcribere.*

Datum rectilineum reſolve in triangula. a fac ang. ABH = D; a & ang. BAH = DCF; a & ang. AHG = CFE; a & ang. HAG = FCE. Rectilineum AGHB eſt quæſitum.

a 23. 1.

b conſtr. Nam ang. B b = D. & ang. BAH b = DCF.

c 32. 1. c quare ang. AHB = CFD; b item ang. HAG = FCE, b & ang. AHG = CFE. c quare ang.

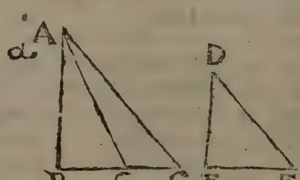
d 2. ax. G = E; & totus ang. GAB d = ECD; & totus GHB d = EFD. Polygona igitur ſibi mutuo æquiangula ſunt. Porro ob trigona æquiangula,

e 4. 6. AB. BH e :: CD. DF. & AG. GH. e :: CE. EF. item AG. AH. e :: CE. CF. & AH. AB e ::

f 22. 5. CF. CD. f unde ex æquo AG. AB :: CE. CD.

g 6. def. 6. eodem modo GH HB :: EF. FD. g ergo polygona ABHG, CDFE ſimilia ſimiliterque poſita exiſtunt. Q. E. F.

PROP. XIX.




*Similia triangula* ABC, DEF *ſunt in duplicata ratione laterum homologorum* BC EF.

a 11. 6. a Fiat BC. EF :: EF. BG. & ducatur AG.

Quia

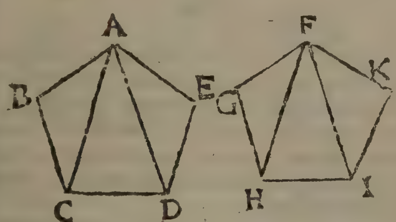
Quia AB. DE *b* :: BC. EF *c* :: EF. BG. & ang. B = E; *d* erit triang. ABG = DEF. verum triang. ABC. ABG *e* :: BC. BG; & *f* $\frac{BC}{BG} = \frac{BC}{EF}$ bis; ergo triang. $\frac{ABC}{ABG}$ hoc eſt $\frac{ABC}{DEF}$ *g* = $\frac{BC}{EF}$ bis. Q. E. D.

b cor 4 6. c *conſtr.* d 15. 6. e 1 6 f 10. *def.* 5. g 11. 5.

*Coroll.*

Hinc, ſi tres lineæ BC, EF, BG proportionales fuerint; ut eſt prima ad tertiam, ita eſt triangulum ſuper primam BC deſcriptum ad triangulum ſuper ſecundam EF ſimile ſimiliterque deſcriptum. vel ita eſt triangulum ſuper ſecundam EF deſcriptum ad triangulum ſuper tertiam ſimile ſimiliterque deſcriptum.

PROP. XX.




*Similia polygona* ABCDE, FGHIK *in ſimilia triangula* ABC, FGH; *&* ACD, FHI, *&* ADE, FIK *dividuntur*, *& numero æqualia*, *& homologa totis.* (ABC. FGH :: ABCDE. FGHIK :: ACD. FHI :: ADE. FIK.) *Et polygona* ABCDE, FGHIK *duplicatam habent eam inter ſe rationem, quam latus homologum* BC *ad homologum latus* GH.

1. Nam

a *hyp.* 1. Nam ang. B *a* = G; & AB. BC *a* :: FG.
b 6.6. GH. *b* ergo triangula ABC, FGH æquiangula
ſunt. eodem modo, triangula AED, FKI aſſi-
milantur. cum igitur ang. BCA *b* = GHF; &
c *hyp.* ang. ADE *b* = FIK; totique anguli BCD,
d 3.*ax.* GHI; atque toti CDE, HIK *c* pares ſint, *d* re-
manent ang. ACD = FHI; & ang. ADC =
e 32. 1. FIH; *e* unde etiam ang. CAD = HFI. ergo
triangula ACD, FHI ſimilia ſunt. ergo, &c.

2. Quoniam igitur triangula BCA, GHF
f 19 6. ſimilia ſunt, *f* erit $\frac{BCA}{GHF} = \frac{BC}{GH}$ bis. ob eandem
cauſam $\frac{CAD}{HFI} = \frac{CD}{HI}$ bis. denique triang. $\frac{DEA}{IKF} =$
g *hyp* & 16. 5. $\frac{DE}{IK}$ bis. quare cum BC. GH *g* :: CD. HI *g* ::
h *ſch.*23 5. DE. IK, *h* erit triang. BCA. GHF :: CAD.
k 12. 5. HFI :: DEA. IKF :: *k* polyg. ABCDE.
FGHIK :: $\frac{BC}{GH}$ bis.

### *Coroll.*

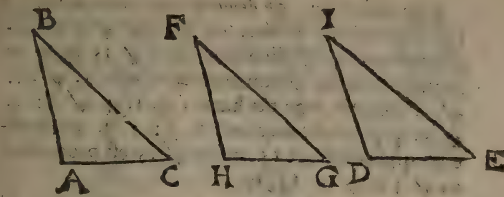
I. Hinc, ſi fuerint tres lineæ rectæ proportionales; ut eſt prima ad tertiam, ita erit polygonum ſuper primam deſcriptum ad polygonum ſuper ſecundam ſimile ſimiliterque deſcriptum. vel ita erit polygonum ſuper ſecundam deſcriptum ad polygonum ſuper tertiam ſimile ſimiliterque deſcriptum.

Unde elicitur methodus *figuram quamvis rectilineam augendi vel minuendi in ratione data. Vt ſi velis pentagoni, cujus latus* CD, *aliud facere quintuplum. inter* AB, *&* 5 AB *inveni mediam proportionalem. Super hac* * *conſtrue pentagonum ſimile* (* 18. 6.)
*dato. hoc erit quintuplum dati.*

II. Hinc etiam, ſi figurarum ſimilium homologa latera nota fuerint, etiam proportio figurarum innoteſcet; nempe inveniendo tertiam proportionalem.

PROP.

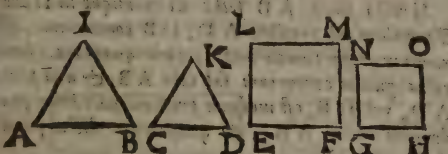
PROP. XXI.




*Quæ* (A B C, D I E) *eidem rectilineo* H F G *ſunt ſimilia, & inter ſe ſunt ſimilia.*

Nam ang. A $^a$ = H $^a$ = D. & ang. C $^a$ = G $^a$ = E; & ang. B $^a$ = F $^a$ = I. $^a$ item AB. AC :: H F. H G :: D I. D E. $^a$ & A C. C B :: H G. G F :: D E. E I. & A B. B C :: H F. F G :: DI. I E. $^a$ ergo A B C, D I E ſimilia ſunt. Q. E. D. a 1. def. 6.

PROP. XXII.




*Si quatuor rectæ lineæ proportionales fuerint* (A B. C D :: E F. G H.) *& ab eis rectilinea ſimilia ſimiliterque deſcripta proportionalia erunt.* (A B I. C D K :: E M. G O.) *Et ſi à rectis lineis ſimilia ſimiliterque deſcripta rectilinea proportionalia fuerint* (ABI. CDK :: EM. GO) *ipſæ etiam rectæ lineæ proportionales erunt.* (AB. CD :: EF. GH.)

1. *Hyp.* $\frac{ABI}{CDK} \, {}^a\!= \frac{AB}{CD}$ bis $= {}^a \frac{EF}{GH}$ bis ${}^a\!= \frac{FM}{GO}$. a 19. 6.

$^d$ ergo ABI. CDK :: EM. GO. Q. E. D.

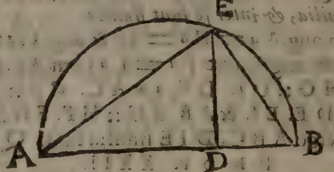
2. *Hyp.* $\frac{AB}{CD}$ bis $= {}^a \frac{ABI}{CDK} \, {}^b\!= \frac{FM}{GO} \, {}^c\!= \frac{EF}{GH}$. b hyp. c 20. 6.

bis. ergo AB. CD :: EF. GH. Q. E. D. d cor. 23 5.

*Schol.*

*Hinc deducitur, & demonſtratur ratio multiplicandi quantitates ſurdas.* ex gr. Sit $\sqrt{5}$ multiplicandus in $\sqrt{3}$. dico provenire $\sqrt{15}$. Nam ex multiplicationis definitione debet eſſe, 1. $\sqrt{3}$ :: $\sqrt{5}$. product. ergo per hanc, q. 1. q. $\sqrt{3}$ :: q. $\sqrt{5}$. q. product. hoc eſt. 1. 3 :: 5. q. product. ergo q. product. eſt 15. quare $\sqrt{15}$ eſt productus ex $\sqrt{3}$ in $\sqrt{5}$. Q. E. D.

THEOR.




Petr. Herig. *Si recta linea* AB *ſecta ſit utcunque in* D, *rectangulum ſub partibus* AD, DB *contentum, eſt medium proportionale inter earum quadrata. Item rectangulum contentum ſub tota* AB, *& una parte* AD, vel DB, *eſt medium proportionale inter quadratum totius* AB, *& quadratum dictæ partis* AD, vel DB.

Super diametrum AB deſcribe ſemicirculum. ex D erige normalem DE occurrentem peripheriæ in E. junge AE, BE.

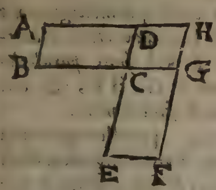
Liquet eſſe AD. DE a :: DE. DB. b ergo
a cor. 8 6. ADq. DEq :: DEq. DBq. c hoc eſt, ADq.
b 22. 6 ADB :: ADB. DBq. Q. E. D.
c 17 6.

Porro, BA. AE d :: AE. AD. e ergo BAq.
d cor. 8 6. AEq :: AEq. ADq. f hoc eſt BAq. BAD ::
e 22. 6. BAD. ADq. Eodem modo ABq. ABD ::
f 17. 6. ABD. BDq. Q. E. D.

a 1. 6. Vel ſic; ſit Z = A + E. liquet eſſe Aq. AE :: a A. E :: a AE. Eq. item Zq. ZA :: a Z. A. :: a ZA. Aq. & Zq. ZE :: a Z. E :: ZE. Eq.

PROP

## PROP. XXIII.



*Æquiangula parallelogramma* AC, CF *inter ſe rationem habent eam quæ ex lateribus componitur.* $\left(\frac{AC}{CF} = \frac{BC}{CG} + \frac{DC}{CE}.\right)$

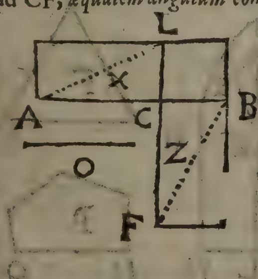
Latera circa æquales angulos C *a* ſibi in directum ſtatuantur; & compleatur parallelogrammum CH. *a ſch. 15.*

Ratio. $\frac{AC}{CF}$ *b* $= \frac{AC}{CH} + \frac{CH}{CF}$ *c* $= \frac{BC}{CG} + \frac{DC}{CE}$. *b 20 def. 5. c 1. 6.*

Q. E. D.

*Coroll.*

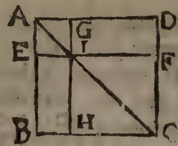
Hinc & ex 34. 1. patet primo, *Triangula, quæ unum angulum* (ad C) *æqualem habent, rationem habere ex rationibus rectarum,* AC ad CB, & LC ad CF, *æqualem angulum continentium.* *Andr. Tacq. 15. 5.*



Patet ſecundo, *Rectangula ac * proinde & parallelogramma quæcunque rationem inter ſe habere compoſitam ex rationibus baſis ad baſim, & altitudinis ad altitudinem.* Neque aliter de triangulis ratiocinaberis. ** 35. 1.*

Patet tertio, *Quomodo triangulorum ac parallelogrammorum proportio exhiberi poſſit.* Sunto parallelogramma X & Z; quorum baſes AC, CB; altitudines vero CL, CF. Fiat CL. CF :: CB. O. * erit X. Z :: AC. O. ** 14. 6. & 1. 6.*

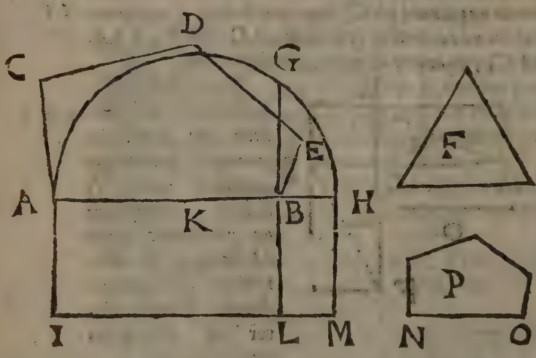
PROP. XXIV.




*In omni parallelogrammo* ABCD, *quæ circa diametrum* AC *ſunt parallelogramma* EG, HF, *& toti & inter ſe ſunt ſimilia.*

Nam parallelogramma EG, HF habent ſingula unum angulum cum toto communem. *a* ergo toti & ſibi mutuo æquiangula ſunt. *a* Item tam triangula ABC, AEI,
a 29. 1. IHC, quam triangula ADC, AGI, IFC ſunt inter ſe æquiangula. *b* ergo AE. EI :: AB. BC,
b 4 6. *b* atque AE. AI :: AB. AC ; *b* & AI. AG :: AC.
c 22 5. AD. *c* ex æquali igitur, AE. AG :: AB. AD.
d 1 *def.* 6 *d* ergo Pgra. EG, BD ſimilia ſunt. eodem modo HF, BD ſimilia ſunt. ergo, &c.

PROP. XXV.




*Dato rectilineo* ABEDC *ſimile ſimiliterque poſitum* P, *idemque alteri dato* F *æquale, conſtituere.*

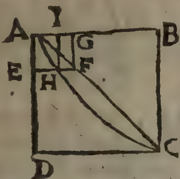
a 45. 1. *a* Fac rectang. AL = ABEDC. *b* item ſuper
b 44. 1. BL fac triang. BM = F. Inter AB, BH *c* in-
c 13. 6. veni mediam proportionalem NO. ſuper NO

*d* fac

*d* fac polygonum P ſimile dato ABEDC. Erit hoc æquale dato F, d 18 6. e cor. 20.6.

Nam A B E D C (A L.) P :: *e* A B. B H *f* :: AL. BM. ergo P *g* = BM *h* = F. Q. E. F. f 1.6. g 14. 5. h conſtr.

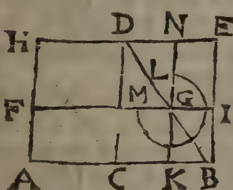
## P R O P. XXVI.




*Si à parallelogrammo* A B C D *parallelogrammum* A G F E *ablatum ſit, & ſimile toti, & ſimiliter poſitum, communem cum eo habens angulum* E A G*, hoc circa eandem cum toto diametrum* AC *conſiſtet.*

Si negas A C eſſe communem diametrum, eſto diameter AHC ſecans E F in H. & ducatur HI parall. A E. Parallelogramma E I, D B *a* ſimilia ſunt. *b* ergo AE. EH :: AD. DC *c* :: AE. EF. *d* proinde EH = EF. *f* Q. E. A. a 24 6. b 1. def.6. c hyp. d 9. 5. f 9. ax.

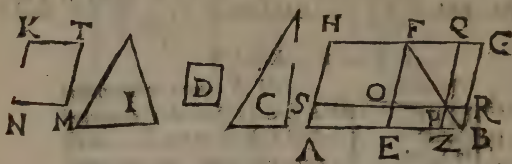
## P R O P. XXVII.




*Omnium parallelogrammorum* A D, A G *ſecundum eandem rectam lineam* A B *applicatorum, deficientiumque figuris parallelogrammis* C E, K I *ſimilibus, ſimiliterque poſitis, ei* A D, *quod à dimidia deſcribitur, maximum eſt* AD, *quod ad dimidium eſt applicatum, ſimile exſiſtens defectui* KI.

Nam quia GE *a* = G C, addito communi K I, *b* erit K E = C I *c* = A M. adde commune C G, *d* erit A G = Gnom. M B L. ſed Gnom. M B L *e* ⊐ C E (A D.) ergo A G ⊐ A D. Q. E. D. a 43. 1. b 2. ax. c 36. 1. d 2. ax. e 9. ax.

PROP. XXVIII.

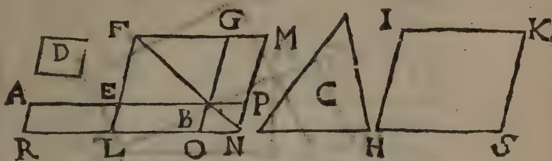



*Ad datam rectam lineam* AB, *dato rectilineo* C *æquale parallelogrammum* AP *applicare deficiens figura parallelogramma* ZR, *quæ ſimilis ſit alteri parallelogrammo dato* D. * *Oportet autem datum*
27.6. *rectilineum* C, *cui æquale* AP *applicandum eſt, non majus eſſe eo* AF, *quod ad dimidiam applicatur, ſimilibus exſiſtentibus defectibus, & ejus* AF *quod ad dimidiam applicatur, & ejus* D, *cui ſimile deeſſe debet.*

a 18. 6. Biſeca AB in E. Super EB *a* fac Pgr. EG
b ſch. 45. 1. ſimile dato D. *b* ſitque EG = C + I. *c* fac pgr.
c 25. 6. NT = I, & ſimile dato D, vel EG. duc diametrum FB. fac FO = KN; & FQ = KT. Per O, & Q duc parallelas SR, QZ. parallelogrammum AP eſt id quod quæritur.

Nam parallelogramma D, EG, OQ, NT,
d conſtr. & 24. 6 ZR *d* ſunt ſimilia inter ſe. Et Pgr. EG *e* = NT
e conſtr. + C *e* = OQ + C; *f* quare C = Gnom.
f 3. ax. OBQ *g* = AO + PG *h* = AO + EP = AP.
g 2. ax.
h 43. 1. Q. E. F.

PROP. XXIX.

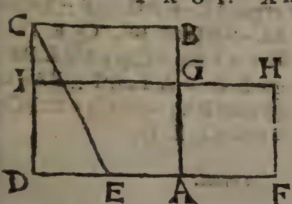



*Ad datam rectam lineam* AB, *dato rectilineo* C *æquale parallelogrammum* AN *applicare, excedens figura parallelogramma* OP, *quæ ſimilis ſit parallelogrammo alteri dato* D.

Biſeca AB in E. ſuper EB *a* fac Pgr. EG ſimile dato D. *b* ſitque pgr. HK = EG + C, & ſimile dato D vel EG. fac FEL *c* = IH; *c* & FGM = IK. per LM duc parallelas RN, MN. & AR parall. NM. Produc ABP, GBO. Duc diametrum FBN. Pgr. AN eſt quæſitum. a 18. 6. b 25. 6. c 3. 1.

Nam parallelogramma D, HK, LM, EG *d* ſimilia ſunt. *e* ergo pgr. OP ſimile eſt pgro LM, vel D. item LM *f* = HK *f* = EG + C. *g* ergo C = Gnom. ENG. atqui AL *h* = IB *k* = BM. *l* ergo C = AN. Q. E. F. d *conſtr.* e 24. 6. f *conſtr.* g 3 *ax.* h 36. 1. k 43. 1. l 2. & 1. *ax*

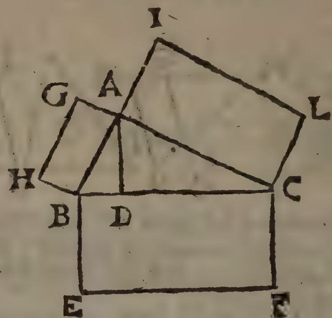
PROP. XXX.




*Propoſitam rectam lineam terminatam* AB, *extrema ac media ratione ſecare.* (AB. AG ∺ AG. GB.)

*a* Seca AB in G, ita ut AB x BG = AGq. *b* ergo BA. AG :: AG. GB. Q. E. F. a 11. 2. b 17. 6.

PROP. XXXI.




*In rectangulis triangulis* B A C, *figura quævis* B F *à latere* B C *rectam angulum* B A C *ſubtendente, deſcripta, æqualis eſt figuris* BG, AL, *quæ priori illi* B F *ſimiles, & ſimiliter poſitæ à lateribus* B A, A C *rectum angulum continentibus deſcribuntur.*

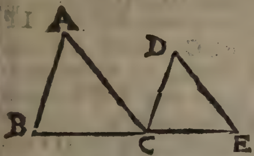
Ab angulo recto B A C demitte perpendicu-
a cor. 8. 6 larem A D. Quoniam C B. C A *a* :: C A. D C.
b cor. 20. 6. *b* erit B F. A L :: C B. D C; inverſeque A L. BF :: DC. CB. Item quia BC, BA *a* :: BA. DB. *b* erit BF. BG :: BC, DB; ac invertendo, BG.
c 24. 5. BF :: DB. BC. *c* ergo AL + BG. BF :: DC +
d ſchol. 14. 5. DB. BC. *d* ergo AL + BG = BF. Q. E D.
e 22. 6. Vel ſic. BG. BF *e* :: BAq. BCq. *e* & AL. BF ::
f 24. 5. ACq. BCq. *f* ergo B G + A L. B F :: BAq +
g ſch. 14. 5. ACq. BCq. *g* ergo cum BAq + ACq *h* = BCq,
h 47. 1. *h* erit BG + AL = BF. Q. E. D.

*Coroll.*

Ex hac propoſitione, addi poſſunt, & ſubtrahi figuræ quævis ſimiles, eadem methodo, qua quadrata adduntur & ſubtrahuntur, in ſchol. 47. 1.

PROP.

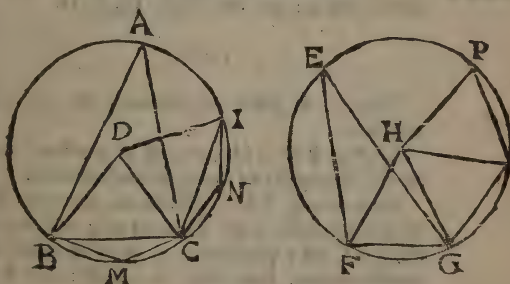
## PROP. XXXII.




*Si duo triangula* ABC, DCE, *quæ duo latera duobus lateribus proportionalia habeant* (AB. AC :: DC. DE,) *ſecundum unum angulum* ACD *compoſita fuerint, ita ut homologa eorum latera ſint etiam parallela* (AB ad DC, & AC ad DE) *tum reliqua illorum triangulorum latera* BC, CE *in rectam lineam collocata reperientur.*

Nam ang. A *a* = ACD *a* = D; & AB. a 29. 1.
AC *b* :: DC. DE. *c* ergo ang. B = DCE. ergo b *hyp.* c 6. 6.
ang, B + A *d* = ACE. ſed ang. B + A + ACB *e* = 2 d 2. *ax.*
Rect. *f* ergo ang. ACE + ACB = 2 Rect. *g* ergo e 32. 1. f 1. *ax.*
BCE eſt recta linea. Q. E. D. g 14. 1.

## PROP. XXXIII.




*In æqualibus circulis* DBCA, HFGP, *anguli* BDC, FHG *eandem habent rationem cum peripheriis* BC, FG, *quibus inſiſtunt; ſive ad centra* (ut BDC, FHG,) *ſive ad peripherias* A, E *conſtituti inſiſtant: inſuper vero & ſectores* BDC, FHG, *quippe qui ad centra conſiſtant.*

Duc

Duc rectas BC, FG. Accommoda CI=CB; & GL=FG=LP; & junge DI, HL, HP.

a 28. 3. Arcus BC *a* =C I, *a* item arcus FG, GL, LP
b 27. 3. æquantur. *b* ergo ang. BDC=CDI *b* & ang. FGH=GHL=LHP. Ergo arcus BI tam multiplex eſt arcûs BC, quam ang. BDI anguli BDC. pariterque æquemultiplex eſt arcus FP arcûs FG, atque ang. FHP anguli FHG. Ve-
c 27. 3. rum ſi arcus BI ⊏, =, ⊐ FP, *c* erit ſimiliter
d 6. def 5. ang. BDI ⊏, =, ⊐ FHP. ergo arc. BC. FG *d* ::
e 15. 5. ang. BDC. FHG *e* :: BDC. FHG *f* :: A. E.
f 20. 3.

Q. E. D.

Rurſus ang. BMC *g* = CNI; *h* atque idcirco
g 27. 3. ſegm. BCM = CIN. *k* item triang. BDC =
h 24. 3. CDI. *l* ergo ſector BDCM = CDIN. Simili
k 4. 1. ratione ſectores FHG, GHL, LHP æquantur.
l 2 ax. Quum igitur prout arcus BI ⊏, =, ⊐ FGP, ita
m 6. def. 5. ſimiliter ſector BDI ⊏, =, ⊐ FHP. *m* erit ſect. BDC. FHG :: arc. BC. FG. Q. E. D.

*Coroll.*

11. 5. Hinc 1. *Vt ſector ad ſectorem, ſic angulus ad angulum.*

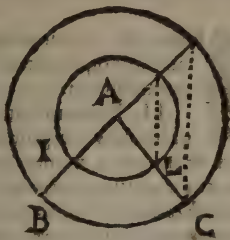
2. *Ang.* BDC *in centro eſt ad* 4 *rectos, ut arcus* BC *cui inſiſtit ad totam circumferentiam.*

Nam ut ang. BDC ad rectum, ſic arcus BC ad quadrantem. ergo BDC eſt ad 4 rectos, ut arcus BC ad 4 quadrantes, id eſt ad totam circumferentiam. item ang. A. 2 Rect :: arc. BC. periph.

Hinc 3. *Inæqualium circulorum arcus* IL, BC, *qui æquales ſubtendunt angulos, ſive ad centra, ut* IAL & BAC, *ſive ad peripheriam, ſunt ſimiles.*

Nam IL. periph. :: ang. IAL, (BAC.) 4 Rect. item arc. BC. periph :: ang. BAC. 4. Rect.

4. Rect. ergo I L. periph :: B C. periph. proinde arcus IL, & BC ſunt ſimiles. Unde




4. *Duæ ſemidiametri* A B, A C *à concentricis peripheriis arcus auferunt ſimiles* IL, BC.

LIB.

# LIB. VII.

## *Definitiones.*

I. UNitas eſt, ſecundum quam unumquodque eorum quæ ſunt, unum dicitur.

II. Numerus autem eſt, ex unitatibus compoſita multitudo.

III. Pars eſt numerus numeri, minor majoris, quum minor metitur majorem.

*Omnis pars ab eo numero nomen ſibi ſumit, per quem ipſa numerum, cujus eſt pars, metitur; ut 4 dicitur tertia pars numeri 12, quia metitur 12 per 3.*

IV. Partes autem, cum non metitur.

*Partes quæcunque nomen accipiunt à duobus illis numeris, per quos maxima communis duorum numerorum menſura utrumque eorum metitur. ut 10 dicitur $\frac{2}{3}$ numeri 15, eo quod maxima communis menſura, nempe 5, metitur 10 per 2, & 15 per 3.*

V. Multiplex vero major minoris, cum majorem metitur minor.

VI. Par numerus eſt, qui bifariam dividitur.

VII. Impar vero numerus, qui bifariam non dividitur; vel, qui unitate differt à pari.

VIII. Pariter par numerus eſt, quem par numerus metitur per numerum parem.

IX. Pariter autem impar eſt, quem par numerus metitur per numerum imparem.

X. Impariter vero impar numerus eſt, quem impar numerus metitur per numerum imparem.

XI. Primus numerus eſt, quem ſola unitas metitur.

XII. Primi inter ſe numeri ſunt, quos ſola unitas, communis menſura, metitur.

XIII.

XIII. Compoſitus numerus eſt, quem numerus quiſpiam metitur.

XIV. Compoſiti autem inter ſe numeri ſunt, quos numerus aliquis communis menſura metitur.

*In hac definitione & præcedenti unitas non eſt numerus.*

XV. Numerus numerum multiplicare dicitur, cum toties compoſitus fuerit is qui multiplicatur, quot ſunt in ipſo multiplicante unitates, & procreatus fuerit aliquis.

*Hinc, in omni multiplicatione unitas eſt ad multiplicatorem ut multiplicatus ad productum.*

*Nota, quod ſæpe cum multiplicandi ſunt quivis numeri, puta* A *in* B, *literarum conjunctio productum denotat. Sic* AB = A *in* B. *item* CDE = C *in* D *in* E.

XVI. Cum autem duo numeri ſeſe multiplicantes aliquem fecerint, qui factus erit, planus appellabitur; Qui vero numeri ſeſe mutuo multiplicarint, latera illius dicentur. *Sic* 2 (C) in 3 (D) = 6 = C D *eſt numerus planus.*

XVII. Cum vero tres numeri mutuo ſeſe multiplicantes fecerint aliquem, qui procreatus erit, ſolidus appellabitur; Qui autem numeri mutuo ſeſe multiplicarint, latera illius dicentur. *Sic*, 2 (C) *in* 3 (D) *in* 5 (E) = 30 = CDE *eſt numerus ſolidus.*

XVIII. Quadratus numerus eſt, qui æqualiter æqualis, vel qui ſub duobus æqualibus numeris continetur. *Sit* A *latus quadrati; quadratus ſic notatur*, AA, *vel* Aq.

XIX. Cubus vero, qui æqualiter æqualis æqualiter, vel qui ſub tribus æqualibus numeris continetur. *Sit* A *latus cubi; cubus notatur ſic*, AAA, *vel* Ac.

*In hac definitione, & tribus præcedentibus, unitas eſt numerus.*

XX. Nu-

XX. Numeri proportionales ſunt, cum primus ſecundi, & tertius quarti æquemultiplex eſt, vel eadem pars; vel deniq; cum pars primi ſecundum, & eadem pars tertii æque metitur quartum, vel vice verſa. A. B :: C. D. hoc eſt, 3. 9 :: 5. 15.

XXI. Similes plani, & ſolidi numeri ſunt, qui proportionalia habent latera.

*Latera nempe non quælibet, ſed quædam.*

XXII. Perfectus numerus eſt, qui ſuis ipſius partibus eſt æqualis.

Ut 6. & 28. *Numerus vero qui ſuis ipſius partibus minor eſt, abundans appellatur: qui vero major, diminutus. ut* 12 *eſt abundans,* 15 *eſt diminutus.*

XXIII. Numerus numerum metiri dicitur per illum numerum, quem multiplicans, vel à quo multiplicatus, illum producit.

*In diviſione, unitas eſt ad quotientem, ut dividens ad diviſum. Nota, quod numerus alteri lineola interjecta ſubſcriptus diviſionem denotat. Sic* $\frac{A}{B}$ = A diviſ. per B. *item* $\frac{CA}{B}$ = C *in* A diviſ. per B.

Termini ſive radices proportionis dicuntur duo numeri, quibus in eadem proportione minores ſumi nequeunt.

## *Poſtulata.*

1. POſtuletur, cuilibet numero quotlibet ſumi poſſe æquales, vel multiplices.

2. Quolibet numero ſumi poſſe majorem.

3 Additio, ſubtractio, multiplicatio, diviſio, extractioneſque radicum, ſeu laterum, numerorum quadratorum, & cuborum concedantur etiam, tanquam poſſibilia.

*Axio-*

## *Axiomata.*

1. QUicquid convenit uni æqualium numerorum, convenit & reliquis æqualibus numeris.

2. Partes eidem parti, vel iiſdem partibus, eædem, ſunt quoque inter ſe eædem.

3. Qui numeri æqualium numerorum, vel ejuſdem, eædem partes fuerint, æquales inter ſe ſunt.

4. Quorum idem numerus, vel æquales, eædem partes fuerint, æquales inter ſe ſunt.

5. Unitas omnem numerum per unitates, quæ in ipſo ſunt, hoc eſt, per ipſummet numerum metitur.

6. Omnis numerus ſeipſum metitur per unitatem.

7. Si numerus numerum multiplicans, aliquem produxerit, metietur multiplicans productum per multiplicatum, multiplicatus autem eundem per multiplicantem.

*Hinc nullus numerus primus planus eſt aut ſolidus, quadratus, vel cubus.*

8. Si numerus numerum metiatur, & ille per quem metitur, eundem metietur per eas, quæ in metiente ſunt, unitates, hoc eſt, per ipſum numerum metientem.

9. Si numerus numerum metiens, multiplicet eum, per quem metitur, vel ab eo multiplicetur, illum quem metitur, producit.

10. Numerus quotcunque numeros metiens, compoſitum quoque ex ipſis metitur.

11. Numerus quemcunque numerum metiens, metitur quoque omnem numerum quem ille metitur.

12. Numerus metiens totum & ablatum, metitur & reliquum.

PROP.

PROP. I.

A.....E..G.B 8 5 3
C...F..D $\frac{5}{3}$ $\frac{3}{2}$ $\frac{2}{1}$
H---

*Si duobus numeris inæqualibus propoſitis* (AB, CD) *detrahatur ſemper minor* CD *de majore* AB (& reliquus EB de CD &c.) *alterna quadam detractione, neque reliquus unquam præcedentem metiatur, quoad aſſumpta ſit unitas* GB; *qui principio propoſiti ſunt numeri* AB, CD *primi inter ſe erunt.*

Si negas, habeant AB, CD communem menſuram, numerum H. Ergo H metiens CD;
a 11 ax. 7. *a* etiam AE metitur; proinde & reliquum FB;
b 12. ax. 7. *a* ergo & CF, atque *b* idcirco reliquum FD; *a* quare & ipſum EG. ſed totum EB metiebatur; *b* ergo & reliquum GB metitur, numerus unitatem.
c 9 ax. 1. *c* Q. E. A.

PROP. II.

9 6
A.........E......B 15 9 6
6 3
C......F...D $\frac{9}{6}$ $\frac{6}{3}$ $\frac{3}{0}$
G---

*Duobus numeris datis* AB, CD *non primis inter ſe, maximam eorum communem menſuram* FD *reperire.*

Detrahe minorem numerum CD ex majori
a 6. ax 7. AB, quoties potes. Si nihil relinquitur, *a* patet ipſum CD eſſe maximam communem menſuram. Si relinquitur aliquid EB, deme hunc ex CD; & reliquum FD ex EB, & ſic deinceps, donec aliquis FD præcedentem EB metiatur.
b 1. 7. (nam *b* hoc fiet antequam ad unitatem perveniatur.) Erit FD maxima communis menſura.

c conſtr. Nam FD *c* metitur EB, *d* ideoque & CF;
d 11. ax 7. *e* proinde & totum CD; *d* ergo ipſum AE; atque
e 12. ax. 7. idcirco totum AB metitur. Liquet igitur FD communem eſſe menſuram. Si maximam eſſe negas,

gas, ſit major quæpiam G. ergo G metiens CD, *d* metitur A E, *e* & reliquum E B, *d* ipſumque C F. *e* proinde & reliquum F D, *g* major minorem. *h* Q. E. A. *g ſuppoſ.* *h 9 ax. 1.*

*Coroll.*

Hinc, numerus metiens duos numeros, metitur quoque maximam eorum communem menſuram.

P R O P. III.

A ............ 12
B ........ 8
D .... 4
C ...... 6
E .. 2
F - - -

*Tribus numeris datis* A, B, C *non primis inter ſe, maximam eorum communem menſuram* E *reperire.*

Inveni D maximam communem menſuram duorū A, B. Si D metitur tertium C, liquet D maximam eſſe trium communem menſuram. Si D non metitur C, erunt ſaltem D, & C compoſiti inter ſe, ex coroll. præcedentis. Sit igitur ipſorum D, & C maxima communis menſura E. erit E is quem quæris.

Nam E *a* metitur C, & D; *a* ac D ipſos A, & B metitur; *b* ergo E metitur ſingulos A, B, C; nec major aliquis (F) eos metietur; nam ſi hoc affirmas, *c* ergo F metiens A, & B, eorum maximam communem menſuram D metitur. Eodem modo, F metiens D, & C, *c* eorum maximam communem menſuram E, *d* major minorem, metitur. *e* Q. E. A. *a conſtr.* *b 11. ax. 7.* *c cor. 1. 7.* *d ſuppoſ.* *e 9 ax. 1.*

*Coroll.*

Hinc, numerus metiens tres numeros, maximam quoque eorum communem menſuram metitur.

PROP. IV.

A ...... 6
B ....... 7
B ................. 18
B ......... 9.

*Omnis numerus A, omnis numeri B, minor majoris, aut pars eſt, aut partes.*

a 4. def. 7. Si A & B primi ſint inter ſe, *a* erit A tot partes numeri B, quot ſunt in A unitates. (ut $6 = \frac{6}{7}\,7$.) Sin A metiatur B, *b* liquet A eſſe partem ipſius B. (ut $6 = \frac{1}{3}\,18$.) denique ſi A & B aliter compoſiti inter ſe fuerint, *c* maxima communis menſura determinabit, quot partes A conficiat ipſius B; ut $6 = \frac{2}{3}\,9$.

b 3. def. 7.

c 4. def. 7.

PROP. V.

A ...... 6 D .... 4
6 6 4 4
B ...... G ...... C 12. E .... H .... F 8

*Si numerus* A *numeri* B C *pars fuerit, & alter* D *alterius* E F *eadem pars; & ſimul uterque* (A + D) *utriuſque ſimul* (B C + E F) *eadem pars erit, quæ unus* A *unius* B C.

Nam ſi B C in ſuas partes B G, G C ipſi A æquales; atque E F in ſuas partes F H, H F ipſi D æquales reſolvantur; *a* erit numerus partium in B C æqualis numero partium in E F. Quum igitur A + D *b* = BG + EH = GC + HF, erit A + D toties in B C + E F, quoties A in B C. Q. E. D.

a hyp.

b conſt & 2. ax. 1.

Vel ſic brevius. Sit $a = \frac{x}{2}$ & $b = \frac{y}{2}$. *c* ergo

c 2. ax. 1.

$$a + b = \frac{x}{2} + \frac{y}{2} = \frac{x + y}{2}.$$ Q. E. D.

PROP.

PROP. VI.

| | | |
|---|---|---|
| 3 3 | 4 4 | |
| A ... G ... B 6. | D .... H .... E 8 | |
| C ......... 9 | F ............ 12 | |

*Si numerus* AB *numeri* C *partes fuerit; & alter* DE *alterius* F *eædem partes; & ſimul uterq;* (AB + DE) *utriuſq; ſimul* (C + F) *eædem partes erit, quæ unus* AB *unius* C.

Divide A B in ſuas partes A G, G B; & D E in ſuas D H, H E. Partium in utroque A B, D E æqualis eſt multitudo, ex hypoth. Quum igitur A G *a* ſit eadem pars numeri C, quæ D H numeri F, *b* erit A G + D H eadem pars compoſiti C + F, quæ unus A G unius C. *b* Eodem modo G B + H E eadem pars eſt ejuſdem C + F, quæ unus G B unius C; *c* ergo AB + DE eædem partes eſt ipſius C + F, quæ AB ipſius C. Q. E. D. a *hyp.* b 5. 7. c 2. *ax.* 7.

Vel ſic. Sit $a = \frac{2}{3}x$. & $b = \frac{2}{3}y$. *d* ergo $a + b = \frac{2}{3}x + \frac{2}{3}y = \frac{2}{3}y + \frac{2}{3}x$. Q. E. D. d 1. *ax.* 1.

PROP. VII.

| | | |
|---|---|---|
| 5 3 | | |
| A ..... E ... B 8 | | |
| 6 | 10 | 6 |
| G ...... C | .......... F | ...... D 16 |

*Si numerus* A B *numeri* C D *pars fuerit, qualis ablatus* A E *ablati* C F; *& reliquus* E B *reliqui* F D *eadem pars erit, qualis totus* AB *totius* CD.

*a* Sit E B eadem pars numeri G C, quæ A B ipſius CD, vel AE ipſius CF. *b* ergo AE + EB eadem eſt pars ipſius CF + GC, quæ AE ipſius CF, vel AB ipſius CD. *c* ergo GF = CD. aufer communem CF, *d* manet GC = FD. *e* ergo EB eadem eſt pars reliqui FD (GC) quæ totus AB totius CB. Q. E. D. a 1. *poſt.* 7. b 5. 7. c 6 *ax* 1. d 3. *ax.* 1. e 2. *ax* 7.

Vel ſic. Sit a + b = x, & c + d = y; atque tam x = 3 y, quam a = 3 c; dico b = 3 d. Nam 3 c + 3 d *f* = 3 y = x *g* = a + b. aufer utrinq; 3 c *g* = a, & *b* remanet 3 d = b. Q. E. D. f 1. 2. g *hyp.*

## PROP. VIII.

6 2 4 2 2
A...... H.. G.... E.. L.. B 16
18 6
C.................. F..... D 24

*Si numerus* AB *numeri* CD *partes fuerit, quales ablatus* AE *ablati* CF ; *& reliquus* EB *reliqui* ED *eædem partes erit, quales totus* AB *totius* CD.

Seca AB in AG, GB partes numeri CD; item AE in AH, HE partes numeri CF ; & ſume GL = AH = HE; *a* quare HG = EL. & quia *b* AG = GB, *c* etiam HG = LB. Cum igitur totus AG eadem ſit pars totius CD, quæ ablatus AH ablati CF; *d* erit reliquus HG, vel EL, eadem etiam pars reliqui FD, quæ AG ipſius CD. Eodem pacto, quia GB eadem pars eſt totius CD, quæ HE, vel GL, ipſius CF, *d* erit reliquus LB eadem pars reliqui FD, quæ GB totius CD ; ergo EL + LB (EB) eædem eſt partes reliqui FD, quæ totus AB totius CD. Q. E. D.

*a* 3.*ax*.1.
*b* *conſtr.*
*c* 3.*ax*.1.
*d* 7.7.

Vel ſic facilius. Sit $a + b = x$. & $c + d = y$. Item tam $y = \frac{2}{3} x$, quam $c = \frac{2}{3} a$ ; vel *e* quod idem eſt, $3 y = 2x$; & $3 c = 2 a$. Dico $d = \frac{2}{3} b$. Nam $3 c + 3 d$ *f* $= 3 y = 2 x$ *f* $= 2 a + 2 b$. aufer utrinque *g* ergo $3 c + 3 d = 2 a + 2 b$. aufer utrinque $3 c$ *h* $= 2 a$; & *k* manet $3 d = 2 b$. *l* ergo $d = \frac{2}{3} b$. Q. E. D.

*e* 9.*ax*.7.
*f* 1.2.
*g* 1.*ax*.1.
*h* *hyp.*
*k* 3 *ax*.1.
*l* 8.*ax*.7.

## PROP. IX.

A ....4
4 4
B.... G.... C 8
5 D ..... 5
E..... H..... F 10

*Si numerus* A *numeri* BC *pars fuerit, & alter* D *alterius* EF *eadem pars; & viciſsim quæ pars eſt, aut partes primus* A *tertii* D, *eadem pars erit, vel eædem partes, & ſecundus* BC *quarti* EF.

Poni-

Ponitur A ⊐ D. Sint igitur B G, G C, & E H, H F partes numerorum B C, E F, hæ ipſi A, illæ ipſi D pares. Utrinque multitudo partium æqualis ponitur. Liquet vero B G *a* eandem eſſe partem, aut eaſdem partes ipſius E H, quæ G C ipſius H F; *b* quare B C (B G + G C) ipſius E F (E H + H F) eadem pars eſt aut partes, quæ unus B G (A) unius E H (D.) Q. E. D. a 1. ax. 7. & 4 7. b 5, vel 6. 7.

Vel ſic; Sit $a = \frac{b}{3}$ & $c = \frac{d}{3}$ dico $\frac{c}{a} = \frac{d}{b}$. Nam $\frac{c}{a} = \frac{3d}{3b} = \frac{d}{b}$. a 1. ax. 7.

## PROP. X.

A . . G . . B 4
C . . . . . . 6
5 5
D . . . . . H . . . . . E 10
F . . . . . . . . . . . . . . 15

*Si numerus* A B *numeri* C *partes fuerit, & alter* D E *alterius* F *eædem partes; & vicisſim quæ partes eſt primus* A B *tertii* D E, *aut pars, eædem partes erit & ſecundus* C *quarti* F, *aut pars.*

Ponitur A B ⊐ D E, & C ⊐ F. Sint A G, G B, & D H, H E partes numerorum C, & F, tot nempe in A B, quot in D E. Conſtat A G ipſius C eandem eſſe partem, quæ D H ipſius F. *a* quare viciſſim A G ipſius D H, pariterque G B ipſius H E, & *b* proinde conjunctim A B ipſius D E eadem pars erit, aut partes, quæ C ipſius F. Q. E. D. a 9 7. b 5, & 9. 7.

Applicare potes ſecundam præcedentis demonſtrationem etiam huic.

## PROP. XI.

4 3
A . . . . E . . . B 7
8 6
C . . . . . . . . F . . . . . . D 14

*Si fuerit, ut totus* A B *ad totum* C D, *ita ablatus* A E *ad ablatum* C F; *& reliquus* E B *ad reliquum*

*F D erit, ut totus* A B *ad totum* C D.

a 4. 7. Sit primo A B ⊐ C D; *a* ergo A B vel pars
b 10, *def.* eſt, vel partes numeri C D; *b* eademque pars eſt,
c 7. vel 8. 7. vel partes ipſe A E ipſius C F; *c* ergo reliquus E B reliqui F D eadem pars eſt, aut partes, quæ totus A B totius C D. *b* ergo A B. C D :: E B. F D. Sin fuerit A B ⊏ C D; eodem modo erit juxta modo oſtenſa, C D. A B :: F D. E B. ergo invertendo, A B. C D :: E B. F D.

## PROP. XII.

A, 4. C, 2. E, 3.
B, 8. D, 4. F, 6.

*Si ſint quotcunque numeri proportionales* (A. B :: C. D :: E. F) *erit quemadmodum unus antecedentium* A *ad unum conſequentium* B, *ita omnes antecedentes* (A + C + E) *ad omnes conſequentes* (B + D + F.)

Sint primo, A, C, E minores quam B, D, F.
a 20. *def.* 7. ergo (propter eaſdem rationes) *a* erit A eadem
b 5, & 6. 7. pars aut partes ipſius B, quæ C ipſius D. *b* ergo conjunctim A + C eadem erit pars aut partes ipſius B + D, quæ unus A unius B. Similiter A + C + E eadem pars eſt, aut partes ipſius
c 20. *def.* 7. B + D + F, quæ A ipſius B. *c* ergo A + C + E. B + D + F :: A. B. Q. E. D. Sin A, C, E, ipſis B, D, F majores ponantur, idem oſtendetur invertendo.

## PROP. XIII.

A, 3. C, 4.
B, 5. D, 12.

*Si quatuor numeri proportionales ſint* (A. B :: C. D. *& viciſſim proportionales erunt* (A. C :: B. D.)

Sint primo A & C ipſis B & D minores,
a 20. *def.* 7. atque A ⊐ C. Ob eandem proportionem, *a* erit A eadem pars, aut partes ipſius B, quæ C ipſius
b 9. & 10. 7. D. *b* ergo viciſſim A ipſius C eadem pars eſt, aut partes, quæ B ipſius D. ergo A. C :: B. D. Sin

A ⊏

A ⊏ C; atque A & C majores ſtatuantur, quam B & D, eadem res erit, proportiones invertendo.

PROP. XIV.

A, 9. D, 6.
B, 6. E, 4.
C, 3. F, 2.

*Si ſint quotcunque numeri* A, B, C, *& alii totidem* D, E, F *illis æquales multitudine, qui bini ſumantur, & in eadem ratione* (A. B :: D. E. & B. C :: E. F) *etiam ex æqualitate in eadem ratione erunt.* (A. C :: D F.)

Nam quia A. B :: D. E, *a* erit viciſſim, A. D :: B. E :: *a* C. F. *a* ergo iterum permutando, A. C :: D. F. Q. E. D. a 13. 7.

PROP. XV.

1. D.
B ... 3. E ...... 6.

*Si unitas numerum quempiam* B *metiatur; æque autem alter numerus* D *alterum quendam numerum* E *metiatur; & viciſſim æque unitas tertium numerum* D *metietur, & ſecundus* B *quartum* E.

Nam quia 1 eſt eadem pars ipſius B, quæ D ipſius E, *a* erit viciſſim 1 eadem pars ipſius D, quæ B ipſius E. Q. E. D. a 9. 7.

PROP. XVI.

B, 4. A, 3.
A, 3. B, 4.
AB, 12. BA, 12.

*Si duo numeri* A, B *ſeſe mutuo multiplicantes fecerint aliquos* AB, BA, *geniti ex ipſis* A B, B A *æquales inter ſe erunt.*

Nam quia A B = A in B, *a* erit 1 in A toties, quoties B in AB. *b* ergo viciſſim 1 in B toties erit, quoties A in AB. atqui quoniam B A = B in A, *a* erit 1 in B toties, quoties A in B A. ergo quoties 1 in A B, toties 1 in B A; & *c* proinde AB = BA. Q. E. D. a 15. def. 7. b 15. 7. c 4. ax. 7.

## PROP. XVII.

A, 3.
B, 2. C, 4.
AB, 6. AC, 12.

*Si numerus* A *duos numeros* B, C *multiplicans fecerit aliquos* AB, AC; *geniti ex ipſis eandem rationem habebunt, quam multiplicati.* (A B. A C :: B. C.)

a 15. def. 7. Nam quia AB = A in B, *a* erit 1 toties in A, quoties B in AB. *a* item quia AC = A in C, erit 1 toties in A, quoties C in A C. ergo quoties B in A B, toties C in A C. quare B. A B :: C. A C. *c* ergo viciſſim, B. C :: A B. A C. Q. E. D.

b 20. def. 7.
c 13. 7.

## PROP. XVIII.

C, 5. C, 5.
A, 3. B, 9.
$\overline{AC}$, 15. $\overline{BC}$, 45.

*Si duo numeri* A, B, *numerum quempiam* C *multiplicantes fecerint aliquos* A C, B C; *geniti ex ipſis eandem rationem habebunt, quam multiplicantes.* (A. B :: AC. BC.)

a 16. 7. Nam AC *a* = CA; & BC *a* = CB; ſic idem C multiplicans A & B producit A C, & B C.
b 17. 7. *b* ergo A. B :: AC. BC. Q. E. D.

*Schol.*

Ex his pendet modus vulgaris reducendi fractiones ($\frac{3}{5}$, $\frac{7}{9}$) ad eandem denominationem. Nam duc 9 tam in 3, quam in 5, proveniunt $\frac{27}{45} = \frac{3}{5}$. quoniam ex his, 3. 5 :: 27. 45. item duc 5 in 7, & 9, prodeunt $\frac{35}{45} = \frac{7}{9}$. quia 7. 9 :: 35. 45.

## PROP. XIX.

A, 4. B, 6. C, 8. D, 12.
A D, 48. BC, 48.

*Si quatuor numeri proportionales fuerint,* (A B :: C. D;) *qui ex primo & quarto fit numerus* A D, *æqualis eſt ei, qui ex ſecundo & tertio fit, numero* BC.

BC. *Et ſi qui ex primo & quarto ſit numerus* AD, *æqualis ſit ei, qui ex ſecundo & tertio ſit, numero* BC, *ipſi quatuor numeri proportionales erunt.* (A. B :: C. D.) a 17. 7.
1. *Hyp.* Nam AC. AD *a* :: C. D *b* :: A. b hyp.
B *c* :: AC. BC. *d* ergo AD = BC. Q. E. D. c 18. 7. d 9. 5.
2. *Hyp.* Quoniam *e* AD = BC, erit AC. e hyp.
AD *f* :: AC. BC. Sed AC. AD *g* :: C. D, & f 7. 5. g 7. 7.
AC. BC *h* :: A. B. *k* ergo C. D. :: A. B. Q. E. D. h 18. 7. k 11. 5.

## PROP. XX.

A. B. C.
4. 6. 9.
AC, 36. BB, 36.
D, 6.

*Si tres numeri proportionales fuerint* (A. B :: B. C.) *qui ſub extremis continetur* (AC) *æqualis eſt ei, qui à medio efficitur* (BB.) *Et ſi qui ſub extremis continetur* (AC) *æqualis fuerit ei* (BB) *qui ſub medio, ipſi tres numeri proportionales erunt* ($\frac{A}{B}$ :: $\frac{B}{C}$.)

1. *Hyp.* Nam ſume D = B. *a* ergo A. B :: a 1. ax. 7.
D (B.) C. *b* quare AC = BD, *a* vel BB. b 19. 7.
Q. E. D.
2. *Hyp.* Quia AC *c* = BD, *d* erit A. B :: D c hyp.
(B.) C. Q. E. D. d 19. 7.

## PROP. XXI.

A ... G .. B 5. E .......... 10.
C .. H. D 3. F ...... 6.

*Numeri* AB, CD *minimi omnium eandem cum eis rationem habentium* (E, F) *metiuntur æque numeros* E, F *eandem cum eis rationem habentes, major quidem* AB *majorem* E, *minor vero* CD *minorem* F.

Nam AB. CD *a* :: E. F. *b* ergo viciſſim a hyp.
AB. E :: CD. F. *c* ergo AB eadem pars eſt, b 13. 7.
vel partes ipſius E, quæ CD ipſius F. Non partes; c 20. def. 7.
nam ſi ita, ſint AG, GB partes numeri E; & CH, HD partes numeri F. *c* ergo AG. E ::
CH.

d 13.7. e hyp. C H. F; & permutando, A G. C H $^d$ :: E. F $^e$ :: AB. CD. ergo AB, CD non ſunt minimi in ſua ratione, contra hypoth. ergo, &c.

## PROP. XXII.

A, 4. D, 12.
B, 3. E, 8.
C, 2. F, 6.

*Si fuerint tres numeri* A, B, C, *& alii ipſis multitudine æquales* D, E, F, *qui bini ſumantur, & in eadem ratione; fuerit autem perturbata eorum proportio*(A.B :: E.F & B.C :: D.E;) *etiam ex æqualitate in eadem ratione erunt* (A.C :: D.F.)

a hyp. Nam quia A. B $^a$ :: E. F, erit A F = B E; &
b 19.7. quia B.C :: $^a$ D. E, $^b$ erit B E = CD. $^c$ ergo
c 1.ax.1. d 19.7. AF = CD. $^d$ quare A. C :: D. F. Q. E. D.

## PROP. XXIII.

A, 9. B, 4.
C - - - - D - - -
E - -

*Primi inter ſe numeri* A, B, *minimi ſunt omnium eandem cum eis rationem habentium.*

Si fieri poteſt, ſint C & D minores quam A & B, atque in eadem ratio-
a 21.7. ne. $^a$ ergo C metitur A æque, ac D metitur B, puta per eundem numerum E: quoties igitur 1 in E, $^b$ toties erit C in A. $^c$ quare viciſſim quo-
b 13 def 7. c 15.7. ties 1 in C, toties E in A. ſimili diſcurſu quoties 1 in D, toties E in B. ergo E utrumque A & B metitur; qui proinde inter ſe primi non ſunt, contra Hypoth.

## PROP. XXIV.

A, 9. B, 4.
C - - -
D - - - E - -

*Numeri* A, B, *minimi omnium eandem cum eis rationem habentium, primi inter ſe ſunt.*

Si fieri poteſt, habeant A
a 9.ax 7. & B communem menſuram C; is metiatur A
b 17.7. per D, & B per E; $^a$ ergo CD = A, $^b$ & CE = B.

$^b$ quare

b quare A. B :: D. E. Sed D & E minores ſunt quam A & B, utpote eorum partes. Ergo A & B non ſunt minimi in ſua ratione, contra hypoth. b 17. 7.

P R O P. XXV.

A, 9. B, 4.
C, 3. D - -

*Si duo numeri A, B primi inter ſe fuerint, qui unum eorum A metitur numerus C, ad reliquum B primus erit.*

Nam ſi affirmes aliquem D numeros B & C metiri, a ergo D metiens C, metitur A. ergo A & B non ſunt primi inter ſe, contra Hypoth. a 11. ax. 7.

P R O P. XXVI.

A, 5. C, 8.
B, 3.
$\overline{A}\overline{B}$, 15. E - - - -
F - - - -

*Si duo numeri A, B ad quempiam C primi fuerint, etiam ex illis genitus AB ad eundem C primus erit.*

Si fieri poteſt, ſit ipſorum AB, & C communis menſura numerus E. ſitque $\frac{AB}{E} = F$; a ergo AB = EF; b quare E. A :: B. F. Quia vero A primus eſt ad C quem E metitur, c erunt E & A primi inter ſe; d adeoque in ſua proportione minimi, & e proinde æque metiuntur B, & F; nempe E ipſum B, & A ipſum F. Quum igitur E utrumque B, C metiatur, non erunt illi primi inter ſe, contra Hypoth. a 9. ax. 7. b 19. 7. c 25. 7. d 23. 7. e 21. 7.

P R O P. XXVII.

A, 4. B, 5.
Aq, 16.
D, 4.

*Si duo numeri, A, B, primi inter ſe fuerint, etiam ex uno eorum genitus (Aq) ad reliquum B primus erit.*

Sume D = A; ergo a ſinguli D, & A primi ſunt ad B. b quare A D, vel Aq, ad B primus eſt. Q. E. D. a 1. ax. 7. b 26. 7.

PROP.

PROP. XXVIII.

| | |
|---|---|
| A, 5. | C, 4. |
| B, 3. | D, 2. |
| AB, 15. | CD, 8. |

*Si duo numeri* A, B *ad duos numeros* C, D, *uterque ad utrumque, primi fuerint; & qui ex eis gignentur* AB, CD, *primi inter ſe erunt.*

a 16. 7. Nam quia A & B ad C primi ſunt, *a* erit AB ad C primus. Eadem ratione erit AB ad D

b 26 7. primus. *b* ergo AB ad CD primus eſt. Q.E.D.

PROP. XXIX.

| | |
|---|---|
| A, 3. | B, 2. |
| Aq, 9. | Bq, 4. |
| Ac, 27. | Bc, 8. |

*Si duo numeri* A, B *primi inter ſe fuerint, & multiplicans uterque ſeipſum fecerit aliquem* (Aq, & Bq;) *& geniti ex ipſis* (Aq, Bq) *primi inter ſe erunt; & ſi qui in principio* A, B *genitos ipſos* Aq, Bq *multiplicantes fecerint aliquos* (Ac, Bc;) *& hi primi inter ſe erunt: & ſemper circa extremos hoc eveniet.*

a 27 7. Nam quia A primus eſt ad B, *a* erit Aq ad B primus. & quia Aq primus ad B, *a* erit Aq ad Bq primus. Rurſus quia tam A ad B, & Bq;

b 28. 7. quam Aq ad eoſdem B, & Bq primi ſunt, *b* erit A x Aq, id eſt Ac, ad B x Bq, id eſt Bc, primus. Et ſic porro de reliquis.

PROP. XXX.

8 5

A ........ B ..... C 13. D ----

*Si duo numeri* AB, BC *primi inter ſe fuerint, etiam uterque ſimul* (AC) *ad quemlibet illorum* AB, BC *primus erit. Et ſi uterque ſimul* AC *ad unum aliquem illorum* AB *primus fuerit, etiam qui in principio numeri* AB, BC *primi inter ſe erunt.*

1. *Hyp.* Nam ſi AC, AB compoſitos velis,

a 12. ax 7. ſit D communis menſura. *a* Is metietur reliquum BC. ergo AB, BC non ſunt primi inter ſe, contra Hypoth.

2. *Hyp.*

2. *Hyp.* Poſitis AC, AB inter ſe primis, vis D ipſorum AB, BC communem eſſe menſuram. [b] Is igitur totum AC metitur. quare AC, AB non ſunt primi inter ſe, contra Hypoth. b 10. ax. 7.

*Coroll.*

Hinc numerus, qui ex duobus compoſitus, ad unum illorum primus eſt, ad reliquum quoque primus eſt.

### PROP. XXXI.

A 5, B, 8. *Omnis primus numerus* A *ad omnem numerum* B, *quem non metitur, primus eſt.*

Nam ſi communis aliqua menſura metiatur utrumque A, B; *a* non erit A primus numerus, contra Hypoth. a 11. def. 7.

### PROP. XXXII.

A, 4. D, 3.
B, 6. E, 8.
$\overline{AB}$, 24.

*Si duo numeri* A, B, *ſe mutuo multiplicantes fecerint aliquem* AB; *genitum autem ex ipſis* AB *metiatur aliquis primus numerus* D; *is etiam unum eorum, qui à principio,* A, vel B *metietur.*

Pone numerum D non metiri A; ſit vero $\frac{AB}{D}$ = E. *a* ergo AB = DE. *b* quare D. A :: B. E. *c* eſt vero D ad A primus. *d* ergo D, & A minimi ſunt in ſua ratione; *e* proinde D metitur B, æque ac A metitur E. liquet igitur propoſitum.

a 9. ax. 7.
b 19. 7.
c hyp. & 31. 7
d 23. 7.
e 21. 7.

### PROP. XXXIII.

A, 12.
B, 2.

*Omnem compoſitum numerum* A, *aliquis primus numerus* B *metitur.*

Unus vel plures numeri *a* metiantur A, quorum minimus ſit B. is primus erit. a 13. def. 7.

nam

a 13. def 7. b 11. ax. 7. nam ſi dicetur compoſitus, [a] eum minor aliquis metietur, [b] qui proinde ipſum A metietur; quare B non eſt minimus eorum, qui A metiuntur; contra Hypoth.

### PROP. XXXIV.

A, 9. *Omnis numerus A, aut primus eſt, aut eum aliquis primus metitur.*

a 33. 7. Nam A neceſſario vel primus eſt, vel compoſitus. Si primus, hoc eſt quod aſſerimus. Si compoſitus, [a] ergo eum aliquis primus metitur. Q. E. D.

### PROP. XXXV.

A, 6. B, 4. C, 8.
D, 2.
E, 3. F, 2. G, 4.

H - - I - - K - - - -
L - - -

*Numeris datis quotcunque* A, B, C *reperire minimos omnium* E, F, G *eandem rationem cum eis habentium.*

a 23. 7. b 3. 7. Si A, B, C primi ſint inter ſe, ipſi in ſua ratione minimi [a] erunt. Si compoſiti ſint, [b] eſto eorum maxima communis menſura D, qui ipſos metiatur per E, F, G. Hi minimi erunt in ratione A, B, C.

c 9. ax 7. d 17. 7. e 21. 7. f 9 ax. 7. g 1. ax. 1. h 19. 7. k ſuppoſ. l 20. def. 7. Nam D ductus in E, F, G [c] producit A B C. [d] ergo hi & illi in eadem ſunt ratione. Iam puta alios H, I, K minimos eſſe in eadem; [e] qui propterea æque metientur A, B, C nempe per numerum L. [f] ergo L in H, I, K ipſos A, B, C procreabit. [g] ergo ED = A = HL. [h] unde E. H :: L. D. Sed E [k] $\sqsubset$ H; [l] ergo L $\sqsubset$ D. ergo D non eſt maxima communis menſura ipſorum A, B, C; contra Hypoth.

*Coroll.*

Hinc, maxima communis menſura quotlibet nume-

numerorum metitur ipſos per numeros, qui minimi ſunt omnium eandem rationem cum ipſis habentium. Ex quo patet methodus vulgaris reducendi fractiones ad minimos terminos.

## PROP. XXXVI.

*Duobus numeris datis* A, B, *reperire, quem illi minimum metiuntur, numerum.*

A, 5. B, 4.
AB, 20.
D - - - - - -
E - - - F - - -

1. *Caſ.* Si A, & B primi ſint inter ſe, eſt AB quæſitus. Nam liquet A & B metiri AB. Si fieri poteſt, metiantur A & B aliquem D ⊐ AB; a 9 ax. 7. & 1. ax. 1.
puta per E, & F. *a* ergo AE = D = BF. *b* quare b 19. 7.
A. B :: F. E. Quia vero A, & B *c* primi ſunt c hyp.
inter ſe, *d* adeoque in ſua ratione minimi, *e* æque d 23. 7.
metientur A ipſum F, ac B ipſum E. Atqui e 21. 7.
B. E *f* :: AB. AE (D.) *g* ergo AB etiam me- f 17. 7. g 20. def. 7.
tietur D, ſeipſo minorem. Q. E. A.

A, 6. B, 4. F - - - -
C, 3. D, 2. G - - - H - - -
AD, 12.

2. *Caſ.* Sin A, & B inter ſe compoſiti fuerint, *h* reperiantur C, & D minimi in eadem ratione. *k* ergo h 35. 7. k 19. 7.
AD = BC. Erit AD, vel BC quæſitus. l 7. ax. 7.

Nam *l* liquet B, & A ipſum AD, vel BC metiri. Puta A, & B metiri F ⊐ AD, nempe
A per G, & B per H. *m* ergo AG = F = BH. m 9. ax. 7.
*n* unde A. B :: H. G *o* :: C. D. *p* proinde æque n 19. 7. o conſtr.
metitur C ipſum H, ac D ipſum G. atqui D. G p 21. 7.
*q* :: AD. AG (F.) ergo AD *r* metitur F, major q 17. 7.
minorem. Q. E. A. r 20 def. 7.

*Coroll.*

Hinc, ſi duo numeri multiplicent minimos eandem rationem habentes, major minorem, & minor majorem, producetur numerus minimus, quem illi metiuntur.

PROP.

PROP. XXXVII.

A, 2. B, 3.
E, ...... 6.
C - - - - F - - - D

*Si duo numeri A, B numerum quempiam C D metiantur; etiam minimus E, quem illi metiuntur, eundem C D metietur.*

Si negas, aufer E ex C D, quoties fieri poteſt, & relinquatur F D ⊐ E. quum igitur A & B *a* metiantur E, *b* & E ipſum C F, *c* etiam A, & B metiuntur C F; *a* metiuntur autem totum C D; *d* ergo etiam reliquum F D metiuntur. ergo E non eſt minimus, quem A, & B metiuntur, contra hyp.

*a hyp.*
*b conſtr.*
*c 11. ax. 7.*
*d 12. ax. 7.*

PROP. XXXVIII.

A, 3, B, 4, C, 6.
D, 12.

*Tribus numeris datis A, B, C, reperire minimum, quem illi metiuntur.*

*a* Reperi D minimum, quem duo A, & B metiuntur; quem ſi tertius C metiatur, patet D eſſe quæſitum. Quod ſi C non metiatur D, ſit E minimus, quem C, & D metiuntur. Erit E requiſitus.

*a 36. 7.*

A, 2. B, 3. C, 4.
D, 6. E, 12.
F - - - A ...

Nam ſingulos A, B, C metiri E conſtat ex 11. ax. 7. Quod vero nullum alium F minorem metiantur, facile oſtenditur. Nam ſi affirmas, *b* ergo D metitur F; *b* proinde E eundem F metitur, major minorem. Quod eſt abſurdum.

*b 37. 7.*

*Coroll.*

Hinc, ſi tres numeri numerum quempiam metiantur; etiam minimus, quem illi metiuntur, eundem metietur.

PROP.

## PROP. XXXIX.

A, 12.
B, 4, C, 3.

*Si numerum A quiſpiam numerus B metiatur, ille A quem B metitur, partem habebit C, à metiente B denominatam.*

Nam quia $\frac{A}{B}$ [a] = C, [b] erit A = BC. [c] ergo $\frac{A}{C}$ = B. Q.E.D. a hyp. b 9.ax.7. c 7.ax.7.

## PROP. XL.

A, 15.
B, 3. C, 5.

*Si numerus A partem habuerit quamlibet B, metietur illum numerus C, à quo ipſa pars B denominatur.*

Nam quia BC [a] = A, [b] erit $\frac{A}{C}$ = B. Q.E.D. a hyp. & 9.ax.7. b 7,ax.7.

## PROP. XLI.

$\frac{1}{2}$ G, 12.
$\frac{1}{3}$ H---
$\frac{1}{4}$

*Numerum reperire G; qui minimus cum ſit, habeat datas partes, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.*

[a] Inveniatur G minimus, quem denominatores 2, 3, 4 metiuntur. [b] Liquet G habere partes, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. Si fieri poteſt, H $\sqsupset$ G habeat eaſdem partes; [c] ergo 2, 3, 4 metiuntur H; & proinde G non eſt minimus, quem 2, 3, 4 metiuntur. contra conſtr. a 38.7. b 39.7. c 40.7.

L LIB.

## PROP. I.

A, 8. B, 12. C, 18. D, 27.
E - F - - G - - - H - - - -

*SI fuerint quotcunque numeri deinceps proportionales* A, B, C, D; *extremi vero ipſorum* A, D *primi inter ſe fuerint; ipſi* A, B, C, D *minimi ſunt omnium eandem cum eis rationem habentium.*

Nam, ſi fieri poteſt, ſint alii totidem E, F, G, H
a 14. 7. minores in illa ratione. *a* ergo ex æquali A. D ::
b 23. 7. E. H. ergo A, & D primi numeri, *b* adeoque in
c 21. 7. ſua ratione minimi, *c* æque metiuntur E, & H, ſeipſis minores. Q. E. A.

## PROP. II.

I.

A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.
Ac, 8. AqB, 12. ABq, 18. Bc, 27.

*Numeros reperire deinceps proportionales minimos, quotcunque juſſerit quiſpiam, in data ratione* A *ad* B.

Sint A, & B minimi in data ratione. Erunt Aq, AB, Bq tres minimi deinceps in ratione A ad B.

a 17. 7. Nam AA. AB *a* :: A. B *a* :: AB. BB. item
b 24. 7. quia A, & B *b* primi ſunt inter ſe, *c* erunt
c 29. 7. Aq, Bq inter ſe primi; *d* proinde Aq, AB, Bq
d 1. 8. ſunt ∺ minimi in ratione A ad B.

Dico porro, Ac, AqB, ABq, Bc in ratione A ad B quatuor eſſe minimos. Nam AqA.
e 17. 7. AqB *e* :: A. B *e* :: ABA (AqB.) ABB. *e* atque A. B :: ABq. BBq. (Bc) Quum igitur Ac, &

B

c f inter ſe primi ſint, g erunt Ac, AqB, f 19.7.
ABq, Bc quatuor ∺ minimi in ratione A ad B. g 1.8.
odem modo quotvis proportionales inveſtiga-
is. Q. E. F.

*Coroll.*

1. Hinc, ſi tres numeri minimi ſunt propor-
ionales, extremi quadrati erunt; ſi quatuor,
ubi.

2. Extremi quotcunque proportionales per
anc propoſ. inventi in data ratione minimi, in-
er ſe primi ſunt.

3. Duo numeri, minimi in data ratione, me-
iuntur omnes medios quotcunque minimorum
n eadem ratione; quia ſcilicet producuntur ex
lorum multiplicatione in alios quoſdam nu-
neros.

4. Hinc etiam liquet ex conſtructione, ſeries
umerorum 1, A, Aq, Ac; 1, B, Bq, Bc; Ac,
AqB, ABq, Bc, conſtare æquali multitudine
umerorum; ac proinde extremos numeros
uotcunque minimorum continue proportiona-
ium, eſſe ultimos totidem continue proportio-
nalium ab unitate. ut extremi Ac, Bc continue
proportionalium Ac, AqB, ABq, Bc, ſunt ultimi
otidem proportionalium ab unitate 1, A, Aq,
Ac; & 1, B, Bq, Bc.

5. 1, A, Aq, Ac; & B, BA, BAq; ac Bq, ABq
unt ∺ in ratione 1 ad A. item, B, Bq, Bc; &
A, AB, ABq; ac Aq, AqB ſunt ∺ in ratione
1 ad B.

PROP. III.

A, 8. B, 12. C, 18. D, 28. *Si ſint quot-
cunque numeri*
A, B, C, D *deinceps proportionales, minimi omni-
um eandem cum eis rationem habentium; illorum ex-
tremi* A, D *ſunt inter ſe primi.*

a 2. 8. Nam ſi *a* inveniantur totidem numeri minimi in ratione A ad B, illi non alii erunt, quam A, B, C, D; ergo juxta 2. coroll. præcedentis extremi A & D primi ſunt inter ſe. Q. E. D.

PROP. IV.

A, 6. B, 5. C, 4. D, 3.
H, 4. F, 24. E, 20. G, 15.
I - - K - - L - - -

*Rationibus datis quotcunque in minimis terminis,* (A ad B, & C ad D) *reperire numeros deinceps minimos in datis rationibus.*

a 36. 7. *a* Reperi E minimum, quem B, & C metiuntur; & B ipſum E *b* æque metiatur, ac A alterum
b 3. poſt 7. F, puta per eundem H. *b* item C ipſum E, ac D alterum G æque metiantur: erunt F, E, G minimi in datis rationibus. Nam A H *c* = F; &
c 9 ax 7. B H *c* = E. *d* ergo A. B :: A H. B H *e* :: F. E.
d 18. 7. Similiter C. D :: E. G. ſunt igitur F, E, G
e 7. 5 deinceps proportionales in datis rationibus. Imo minimi ſunt in iiſdem: nam puta alios I, K, L minimos eſſe. *f* ergo A & B ipſos I & K, *f* pariterque C & D ipſos K & L æque metiuntur.
f 21. 7. ergo B, & C eundem K metiuntur. *g* Quare etiam
g 37. 7. E eundem K metitur, ſeipſo minorem. Q. E. A.

A, 6. B, 5. C, 4. D, 3. E, 5. F, 7.
H, 24. G, 20. I, 15. K, 21.

Datis vero tribus rationibus A ad B, & C ad D, ac E ad F. reperi, ut prius, tres H, G, I minimos deinceps in rationibus A ad B, & C ad D. tunc ſi E numerum I metiatur,

h 3. poſt 7. *h* Sume alterum K, quem F æque metiatur; erunt quatuor H, G, I, K, deinceps minimi, in datis rationibus; quod non aliter probabimus, quam in priori parte.

A, 6.

A, 6. B, 5. C, 4. D, 3. E, 2. F, 7.
H, 24. G, 29. I, 15.
M, 48. L, 40. K, 30. N, 105.

Sin E non metiatur I, ſit K minimus, quem E, & I metiuntur; & quoties I ipſum K, toties G ipſum L, & H ipſum M metiatur. quoties vero E ipſum K, toties F ipſum N metiatur. Erunt M, L, K, N minimi deinceps in datis rationibus; quod demonſtrabimus, ut prius.

PROP. V.

C, 4. E, 3,
D, 6 F, 16 ED, 18.
CD, 24 EF, 48.

*Plani numeri* CD, EF *rationem habent ex lateribus compoſitam.* $\left(\frac{CD}{EF} = \frac{C}{E} + \frac{D}{F}\right)$

Nam quia CD. ED *a* :: C. E; *a* & ED. EF :: D. F. atque $\frac{CD}{EF}$ *b* $= \frac{CD}{ED} + \frac{ED}{EF}$; *c* erit ratio $\frac{CD}{EF} = \frac{C}{E} + \frac{D}{F}$. Q. E. D.

a 17. 7.
b 20. def. 5.
c 11. 5.

PROP. VI.

A, 16. B, 24. C, 36. D, 54. E, 81.
F, 4. G, 6. H, 9.

*Si ſint quotcunque numeri deinceps proportionales* A, B, C, D, E; *primus autem* A *ſecundum* B *non metiatur, neque alius quiſpiam ullum metietur.*

Quoniam A non metitur B, *a* neque quilibet proxime ſequentem metietur; quia A. B :: B. C :: C. D, &c. *b* Accipe tres F, G, H minimos in ratione A ad B. quoniam igitur A non metitur B, *a* neque F metietur G. *c* ergo F non eſt unitas. ſed F, & H inter ſe primi ſunt; ergo quum *e* ſit ex æquo A. C :: F. H, & F non metiatur H, *d* neque A ipſum C metietur; proinde nec B ipſum D, nec C ipſum E, &c. quia A. C *e* :: B. D *e* :: C. E, &c. Eodem modo

a 20. def. 8.
b 35. 7.
c 5 ax. 7.
d 3. 3.
e 14. 7.

 ſumptis

ſumptis quatuor vel quinque minimis in ratione A ad *B*, oſtendetur A ipſos D, & E; ac *B* ipſos E, & F non metiri, &c. Quare nullus alium metietur. Q. E. D.

## PROP. VII.

A, 3. B, 6. C, 12. D, 24. E, 48.

*Si ſint quotcunnque numeri deinceps proportionales* A, B, C, D, E; *primus autem* A *extremum* E *metiatur; is etiam metitur ſecundum* B.

a 6. 7. Si negas A metiri B, *a* ergo nec ipſum E metietur, contra Hypoth.

## PROP. VIII.

A, 24. C, 36. D, 54. B, 81.
G, 8. H, 12. I, 18. K, 27.
E, 32. L, 48. M, 72. F, 108.

*Si inter duos numeros* A, B *medii continua proportione ceciderint numeri* C, D; *quot inter eos medii continua proportione cadunt numeri, tot & inter alios* E, F *eandem cum illis habentes rationem, medii continua proportione cadent.* (L, M.)

a 35. 7. *a* Sume G, H, I, K minimos ∵ in ratione
b 14. 7. A ad C; *b* erit ex æquali, G. K :: A. B *c* :: E. F.
c hyp.
d 3. 8. Atqui G, & K *d* primi ſunt inter ſe; *e* quare G
e 21. 7. æque metitur E, ac K ipſum F. per eundem numerum metiatur H ipſum L, & I ipſum M.
f conſtr. *f* itaque E, L, M, F ita ſe habent ut G, H, I, K; hoc eſt ut A, B, C, D. Q. E. D.

## PROP. IX.

1.
E, 2. F, 3.
G, 4. H, 6. I, 9.
A, 8. C, 12. D, 18. B, 27.

*Si duo numeri* A, B, *ſint inter ſe primi, & inter eos medii continua proportione ceciderint numeri,* C, D; *quot inter eos medii continua*

*tinua proportione ceciderint numeri, totidem* (E, G, & F, I) *& inter utrumque eorum ac unitatem medii continua proportione cadent.*

Conſtat 1, E, G, A; & 1, F, I, B eſſe ∺; & totidem quot A, C, D, B, nimirum ex 4 coroll. 2. 8. Q. E. D.

## PROP. X.

A, 8. I, 12. K, 18. B, 27.
E, 4. DF, 6. G, 9.
D, 2. F, 3.
1.

*Si inter duos numeros A, B, & unitatem continue proportionales ceciderint numeri* (E, D, & F, G,) *quot inter utrumque ipſorum, & unitatem deinceps medii continua proportione cadunt numeri, totidem & inter ipſos medii continua proportione cadent,* I, K.

Nam E, D F, G; & A, D F (I,) D G (K,) B ſunt ∺, per 2. 8. ergo, &c.

## PROP. XI.

A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.

*Duorum quadratorum numerorum* Aq, Bq *unus medius proportionalis eſt numerus* A B. *& quadratum* Aq *ad quadratum* Bq, *duplicatam habet lateris* A *ad latus* B *rationem.*

a Liquet Aq, AB, Bq, eſſe ∺. b proinde etiam $\frac{Aq}{Bq} = \frac{A}{B}$ bis. Q. E. D. a 17 7. b 10. def. 5.

PROP. XII.

Ac, 27. AqB, 36. ABq, 48. Bc, 64.
A, 3. B, 4
Aq, 9. AB, 12. Bq, 16.

*Duorum cuborum numerorum* Ac, Bc *duo medii proportionales ſunt numeri* AqB, ABq. *Et cubus* Ac *ad cubum* Bc *triplicatam habet lateris* A *ad latus* B *rationem.*

a 2. 1. b 10 def. 5. *a* Nam Ac. AqB, ABq, Bc ſunt ∺ in ratione A ad B. *b* proinde $\frac{Ac}{Bc} = \frac{A}{B}$ ter. Q. E. D.

PROP. XIII.

A, 2. B, 4. C, 8.
Aq, 4. AB, 8. Bq, 16. BC, 32. Cq, 64.
Ac, 8, AqB, 16, ABq. 32. Bc, 64, BqC, 128, BCq, 256. Cc, 512.

*Si ſint quotlibet numeri deinceps proportionales,* A, B, C; *& multiplicans quiſque ſeipſum faciat aliquos; qui ab illis producti fuerint* Aq, Bq, Cq *proportionales erunt; & ſi numeri primum poſiti* A, B, C *multiplicantes jam factos* Aq, Bq, Cq, *fecerint aliquos* Ac, Bc, Cc; *ipſi quoque proportionales erunt. & ſemper circa extremos hoc eveniet.*

a 2. 8. b 14. 7. Nam Aq, AB, Bq, BC, Cq *a* ſunt ∺. *b* ergo ex æquo Aq. Bq :: Bq Cq. Q. E. D.

*a* Item Ac, AqB, ABq, Bc, BqC, BCq, Cc ſunt ∺, *b* ergo iterum ex æquo, Ac. Bc :: Bc. Cc. Q. E. D.

PROP. XIV.

Aq, 4. AB, 12. Bq, 36.
A, 2. B, 6.

*Si quadratus numerus* Aq *quadratum numerum* Bq *metiatur, & latus unius* (A) *metietur latus alterius* (B:) *& ſi unius quadrati latus* A *metietur latus alterius* B, *& quadratus* Aq *quadratum* Bq *metietur.*

a 2. & 11. 8. 1. *Hyp.* Nam Aq. AB *a* :: AB. Bq; cum igitur ex hyp. Aq metiatur Bq; idem Aq ſecundum

cundum A B *b* metietur. atqui Aq A B :: A. b 7.8
B. *c* ergo etiam A metitur B. Q. E. D. c 10.*def.*7.

2. *Hyp.* A metitur B. *c* ergo tam Aq ipſum A B, *c* quam A B ipſum Bq metitur; *d* & proinde Aq metitur Bq. Q. E. D. d 11 *ax.*7.

## PROP. XV.

A, 2. B, 6.
Ac, 8. AqB, 24. ABq, 72. Bc, 216.

*Si cubus numerus* Ac *cubum numerum* Bc *metiatur; & latus unius* (A) *metietur latus alterius* (B:) *Et ſi latus* A *unius cubi* Ac *latus* B *alterius* Bc *metiatur, & cubus* Ac *cubum* Bc *metietur.*

1. *Hyp.* Nam Ac, AqB, ABq, Bc *a* ſunt ∺. a 1. & 12.8.
ergo Ac, *b* metiens extremum Bc, *c* etiam ſe- b *hyp.* c 7. 8.
cundum AqB metietur. atqui Ac. AqB :: A. B.
*d* ergo etiam A metietur B. Q. E. D.

2. *Hyp.* A metitur B; *d* ergo Ac metitur AqB, d 10.*def.*7.
iſque ABq, & hic Bc; *e* ergo Ac metietur Bc. e 11 *ax.*7.
Q. E. D.

## PROP. XVI.

A, 4. B, 9.
Aq, 16. Bq, 81.

*Si quadratus numerus* Aq *quadratum numerum* Bq *non metiatur, neque* A *latus unius alterius latus* B *metietur: & ſi* A *latus unius quadrati* Aq *non metiatur* B *latus alterius* Bq, *neque quadratus* Aq *quadratum* Bq *metietur.*

1. *Hyp.* Nam ſi affirmes A metiri B, *a* etiam a 14 8.
Aq ipſum Bq metietur, contra hyp.

2. *Hyp.* Vis Aq metiri Bq; *a* ergo A ipſum B metietur, contra hyp.

## PROP. XVII.

A, 2. B, 3. *Si cubus numerus* Ac *cubum numerum* Bc *non metiatur, neque* A *latus unius latus* B *alterius metietur. Et ſi latus* A *unius cubi* Ac *latus* B *alterius* Bc *non metiatur, neque cubus* Ac *cubum* Bc *metietur.*

Ac, 8. Bc, 27.

a 15.8. 1. *Hyp.* Dic A metiri B; *a* ergo Ac metietur Bc. contra Hypoth.

2. *Hyp.* Dic Ac metiri *Bc*; *a* ergo A ipſum B metietur. contra Hyp.

## PROP. XVIII.

C, 6. D, 2. *Duorum ſimilium planorum numerorum* CD, EF, *unus medius proportionalis eſt numerus* DE: *& planus* CD *ad planum* EF *duplicatam habet lateris* C *ad latus homologum* E *rationem.*

CD, 12.

E, 9. F, 3. DE, 18.

EF, 27.

* 21.def.7. Quoniam * ex hyp. C. D :: E. F; permutando erit C. E :: D. F. atqui C. E *a* :: CD. DE; *a* & D. F :: DE. EF. *b* ergo CD. DE :: DE. EF. Q. E. D.

a 17.7. b 11.5.

c 10.def.5. *c* Ergo ratio CD ad EF duplicata eſt rationis CD ad DE; hoc eſt rationis C ad E, vel D ad F.

### *Coroll.*

Hinc perſpicuum eſt, inter duos ſimiles planos cadere unum medium proportionalem, in ratione laterum homologorum.

PROP. XIX.

CDE, 30. DEF, 60. FGE. 120. FGH, 240.
CD, 6. DF, 12. FG, 24.
C, 2. D, 3. E, 5. F, 4. G, 6. H, 10.

*Duorum ſimilium ſolidorum CDE, FGH, duo medii proportionales ſunt numeri DFE, FGE. Et ſolidus CDE ad ſolidum FGH triplicatam rationem habet lateris homologi C ad latus homologum F.*

Quoniam ex* hyp. C. D :: F. G; & D. E :: G. H, erit a permutando C. F :: D. G a :: E. H. atqui CD. DF b :: C. F; & DF. FG b :: D. G. c quare CD. DF :: DF. FG :: E. H. d ergo CDE. DFE :: DFE. FGE :: E. H :: FGE. FGH. ergo inter CDE, FGH cadunt duo medii proportionales, DFE, FGE. Q.E.D. e Liquet igitur rationem CDE ad FGH triplicatam eſſe rationis CDE ad DFE, vel C ad F. Q.E.D.

* 21. def. 7. a 13. 7. b 17. 7. c 11. 5. d 7. 7. e 10. def. 5.

*Coroll.*

Hinc, inter duos ſimiles ſolidos cadunt duo medii proportionales, in ratione laterum homologorum.

PROP. XX.

A, 12. C, 18. B, 27.
D, 2. E, 3. F, 6. G, 9.

*Si inter duos numeros A, B, unus medius proportionalis cadat numerus C, ſimiles plani erunt illi numeri, A, B.*

a Accipe D, & E minimos in ratione A ad C, vel C ad B. b ergo D æque metitur A, ac E ipſum C, puta per eundem F. b item D æque metitur C ac E ipſum B, puta per eundem G. c ergo DF = A, & EG = B. d quare A, & B plani ſunt numeri. Quia vero EF c = C c = DG; e erit D. E :: F. G, & viciſſim D. F :: E. G. f ergo plani numeri A, & B etiam ſimiles ſunt. Q.E.D.

a 35. 7. b 21. 7. c 9. ax. 7. d 16. def. 7. e 19. 7. f 21. def. 7.

PROP.

## PROP. XXI.

A, 16. C, 24. D, 36. B, 54.
E, 4. F, 6. G, 9.
H, 2. P, 2. M, 4. K, 3. L, 3. N, 6.

*Si inter duos numeros* A, B *duo medii proportionales cadant numeri* C, D; *ſimiles ſolidi erunt illi numeri*, A, B.

a 21. 8. *a* Sume E, F, G minimos ∺ in ratione A ad
b 20 8. C. *b* ergo E, & G ſunt numeri plani ſimiles.
c 22. *def* 7. hujus latera ſint H & P; illius K & L: *c* ergo H.
d ſchol. 18. 8. K :: P. L :: *d* E. F. Atqui E, F, G ipſos A, C,
e 21. 7. D *e* æque metiuntur, puta per eundem M; iidemque ipſos, C, D, B æque metiuntur, puta
f 9. *ax* 7. per eundem N. *f* ergo A = E M = H P M, *f* &
g 17 *def*. 7. B = G N = K L N; *g* quare A & B ſolidi ſunt numeri. Quoniam vero C *f* = F M; & D *f* =
h 17. 7. F N, erit M. N *h* :: F M. F N *k* :: C. D *l* :: E.
k 7. 5. F :: H. K :: P. L. *m* ergo A, & B ſunt numeri
l *conſtr.*
m 21 *def*. 7. ſolidi ſimiles. Q. E. D.

## PROP. XXII.

A, 4. B, 6. C, 9.

*Si tres numeri* A, B, C *deinceps ſint proportionales, primus autem* A *ſit quadratus, & tertius* C *quadratus erit.*

Inter A, & C cadit medius proportionalis.
a 20. 8. *a* ergo A, & C ſunt ſimiles plani; quare *b* cum A
b *hyp.* quadratus ſit, erit C etiam quadratus. Q. E. D.

## PROP. XXIII.

A, 8. B, 12. C, 18. D, 27.

*Si quatuor numeri* A, B, C, D *deinceps ſint proportionales; primus autem* A *ſit cubus, & quartus* D *cubus erit.*

a 21. 8. Nam A, & D *a* ſimiles ſolidi ſunt; ergo
b *hyp.* *b* cum A cubus ſit, erit D cubus. Q. E. D.

PROP.

PROP. XXIV.

A, 16. 24. B, 36.
C, 4. 6. D, 9.

*Si duo numeri A, B rationem habeant inter ſe, quam quadratus numerus C ad quadratum numerum D, primus autem A ſit quadratus; & ſecundus B quadratus erit.*

Inter C, & D numeros quadratos, * adeoque inter A, & B eandem rationem habentes, a cadit unus medius proportionalis. Ergo b cum A quadratus ſit, c etiam B quadratus erit. Q.E.D. * 8. 8. a 11. 8. b *hyp.* c 22. 8.

*Coroll.*

1. Hinc ſi fuerint duo numero ſimiles AB, CD (A. B :: C. D) primus autem AB ſit quadratus, etiam ſecundus CD quadratus erit. * 11. & 8. 8

* Nam AB. CD :: Aq. Cq.

2. Liquet ex his, proportionem cujuſvis numeri quadrati ad quemlibet non quadratum, exhiberi nullo modo poſſe in duobus numeris quadratis. unde non erit, Q. Q :: 1. 2. nec 1. 5. :: Q. Q. &c.

PROP. XXV.

C, 64. 96. 144. D, 216.
A, 8. 12. 18. B, 27.

*Si duo numeri A, B rationem inter ſe habeant, quam cubus numerus C ad cubum numerum D, primus autem A ſit cubus, & ſecundus B cubus erit.*

a Inter C, & D cubos, b adeoque inter A & B eandem rationem habentes, cadunt duo medii proportionales. ergo propter A c cubum, d etiam B cubus erit. Q. E. D. a 12. 8. b 8. 8. c *hyp.* d 23. 8.

*Coroll.*

1. Hinc etiam ſi fuerint duo numeri ABC, DEF (A. B :: D. E. & B. C :: E. F;) primus autem ABC cubus fuerit, etiam ſecundus DEF cubus erit. * 11. & 19. 8.

* Nam ABC. DEF :: Ac = Dc.

2. Patet etiam ex his, proportionem cujuſvis un-

numeri cubi ad quemlibet numerum non cubum non poſſe reperiri in duobus numeris cubis.

## PROP. XXVI.

A, 20. C, 30. B, 45.
D, 4. E, 6. F, 9.

*Similes plani numeri* A, B *rationem inter ſe habent, quam quadratus numerus ad quadratum numerum.*

a 18. 8. b 2. 8. Inter A, & B *a* cadit unus medius proportionalis C. *b* ſume tres D, E, F minimos ∺ in ratione A ad C. Extremi D, F *b* quadrati erunt. atqui ex æquali A. B *c* :: D. F. ergo A. B :: Q. Q. Q. E. D.

c 14. 7.

## PROP. XXVII.

A, 16. C, 24. D, 36. B, 54.
E, 8. F, 12. G, 18. H, 27.

*Similes ſolidi numeri* A, B, *rationem habent inter ſe, quam cubus numerus ad cubum numerum.*

a 19. 8. b 2. 8. *a* Inter A, & B cadunt duo medii proportionales, puta C & D : *b* ſume quatuor E, F, G, H minimos ∺ in ratione A ad C. *b* Extremi E, H cubi ſunt. At A. B *c* :: E. H :: C. C. Q. E. D.

c 14. 7.

### *Schol.*

*Vide Clavium.* 1. Ex his infertur, nullos numeros habentes proportionem ſuperparticularem, vel ſuperbipartientem, vel duplam, aut aliam quamcunque multiplam non denominatam à numero quadrato, eſſe ſimiles planos.

2. Nec duo quivis primi numeri, neque duo quicunque inter ſe primi, qui quadrati non ſint, ſimiles eſſe poſſunt.

LIB.

# LIB. IX.

## PROP. I.

A, 6. B, 54.
Aq, 36. 108. AB, 324.

*SI duo ſimiles plani numeri* A, B *multiplicantes ſe mutuo faciant quendam* A B, *productus* AB *quadratus erit.*

Nam A. B a :: Aq. AB; cum igitur inter A, & B b cadat unus medius proportionalis, c etiam inter Aq, & AB cadet unus med. proport. ergo cum primus Aq ſit quadratus, d etiam tertius AB quadratus erit. Q. E. D. a 17.7. b 18. 8. c 8. 8. d 22. 8

Vel ſic: Sint ab, cd ſimiles plani, nempe a.b :: c.d. x ergo a d = bc. quare abcd, vel adbc y = adad = Q: ad. x 19 7 y 1. ax 7.

## PROP. II.

A, 6. B, 54.
Aq, 36. AB, 324.

*Si duo numeri* A, B *ſe mutuo multiplicantes faciant* AB *quadratum, ſimiles plani erunt;* A, B.

Nam A. B a :: Aq. AB; quare cum inter Aq, AB b cadat unus medius proportionalis, c etiam unus inter A, & B medius cadet. d ergo A, & B ſunt ſimiles plani. Q. E. D. a 17. 7. b 11. 8. c 8. 8. d 20. 8.

## PROP. III.

A, 2. Ac, 8 Acc, 64.

*Si cubus numerus* Ac *ſeipſum multiplicans procreet aliquem* Acc, *productus* Acc *cubus erit.*

Nam 1. A a :: A. Aq b :: Aq. Ac. ergo inter 1, & Ac cadunt duo medii proportionales. Sed 1. Ac a :: Ac. Acc. c ergo inter Ac, & Acc cadunt etiam duo a 15 def. 7. b 17. 7. c 8, 8.

medii

medii proportionales. Proinde cum Ac ſit cubus,
d 13. 8. *d* erit Acc cubus. Q. E. D.

Vel ſic; aaa (Ac) in ſe ductus facit aaaaaa. (Acc;) hic cubus eſt, cujus latus aa.

## PROP. IV.

Ac, 8. Bc, 27.
Acc, 64. AcBc, 216.

*Si cubus numerus* Ac *cubum numerum* Bc *multiplicans, faciat aliquem* AcBc, *factus* AcBc *cubus erit.*

a 17. 7. Nam Ac. Bc *a* :: Acc. AcBc. ſed inter Ac
b 11. 8. & BC *b* cadunt duo medii proportionales; *c* ergo
c 8. 8. inter Acc, & AcBc totidem cadunt. itaque cum
d 13. 8. Acc ſit cubus, *d* erit AcBc etiam cubus. Q.E.D.

Vel ſic. AcBc = aaabbb (ababab) = C: ab.

## PROP. V.

Ac, 8. B, 27.
Acc, 64. AcB, 216.

*Si cubus numerus* Ac *numerum quendam* B *multiplicans, faciat cubum* AcB; *& multiplicatus* B *cubus erit.*

Nam Acc. AcB *a* :: Ac. B. Sed inter Acc, &
a 17. 7. AcB *b* cadunt duo medii proportionales. *c* ergo
b 12. 8. totidem cadent inter Ac, & B. quare cum Ac cu-
c 8. 8. d 13. 8. bus ſit, *d* etiam B cubus erit. Q. E. D.

## PROP. VI.

A, 8. Aq, 64. Ac, 512.

*Si numerus* A *ſeipſum multiplicans faciat* Aq *cubum; & ipſe* A *cubus erit.*

a hyp. Nam quia Aq *a* cubus, & AqA (Ac) *b* cu-
b 19 def. 7. bus, *c* erit A cubus. Q. E. D.
c 5. 9.

## PROP. VII.

A, 6. B, 11. AB, 66.
D, 2. E, 3.

*Si compoſitus numerus* A *numerum quempiam* B *multiplicans, quempiam faciat* AB, *factus* AB *ſolidus erit.*

Quoniam

Quoniam A compoſitus eſt, *a* metitur eum aliquis D, puta per E. *b* ergo A = D E; *c* quare DEB = AB ſolidus eſt. Q. E. D. a 13. *def.* b 9 *ax.* 7. c 17. *def.* 7.

P R O P. VIII.

I. a, 3. $a^2$, 9. $a^3$, 27. $a^4$, 81. $a^5$, 243. $a^6$, 729.

*Si ab unitate quotcunque numeri deinceps proportionales fuerint* (I, a, $a^2$, $a^3$, $a^4$, &c. ) *tertius quidem ab unitate* $a^2$ *quadratus eſt; & unum intermittentes, omnes* ($a^4$, $a^6$, $a^8$, &c.) : *quartus autem* $a^3$ *eſt cubus; & duos intermittentes omnes* ($a^6$, $a^9$, &c.) *ſeptimus vero* $a^6$, *cubus ſimul & quadratus; & quinque intermittentes omnes* ($a^{12}$, $a^{18}$, &c.)

Nam I. $a^2$ = Q. a. & $a^4$ = aaaa = Q. aa. & $a^6$ = aaaaaa = Q. aaa, &c.

2. $a^3$ = aaa = C. a. & $a^6$ = aaaaaa = C. aa. & aaaaaaaaa = C. aaa, &c.

3. $a^6$ = aaaaaa = C. aa = Q. aaa. ergo, &c.

Vel juxta Euclidem; quia I. a *a* :: a. $a^2$, *b* erit $a^2$ = Q: a. ergo cum $a^2$, $a^3$, $a^4$ ſint ∺ *c* erit tertius $a^4$ etiam quadratus. pariterq; $a^6$, $a^8$, &c. Item quia I. a *a* :: $a^2$. $a^3$. erit $a^3$ *b* = $a^2$ in a = C: a. *d* ergo quartus ab $a^3$, nempe $a^6$, etiam cubus erit, &c. ergo $a^6$ cubus ſimul & quadratus exiſtit, &c. a *hyp.* b 20. 7. c 22. 8. d 23. 8.

P R O P. IX.

I. a, 4. $a^2$, 16. $a^3$, 64. $a^4$, 256, &c.
I. a, 8. $a^2$, 64. $a^3$, 512. $a^4$, 4096.

*Si ab unitate quotcunque numeri deinceps proportionales fuerint* (I, a, $a^2$, $a^3$, &c.); *qui vero* (a) *poſt unitatem ſit quadratus; & reliqui omnes,* $a^2$, $a^3$, $a^4$, &c. *quadrati erunt. At ſi* a, *qui poſt unitatem, ſit cubus, & reliqui omnes* $a^2$, $a^3$, $a^4$, &c. *cubi erunt.*

I. *Hyp.* Nam $a^2$, $a^4$, $a^6$, &c. quadrati ſunt ex præc. item quia a ponitur quadratus, *a* erit tertius $a^3$ quadratus, pariterque $a^5$, $a^7$, &c. ergo omnes. a 22. 8.

 2. *Hyp.*

b 23. 8. 2. *Hyp*. a cubus ponitur, [b] ergò $a^4$, $a^7$, $a^{10}$
c 20. 7. cubi ſunt : atqui ex præced. $a^3$, $a^6$, $a^9$, &c. cubi
d 3. 9. ſunt. denique quia 1. a :: a. aa, [c] erit $a^2 = Q$:
e 23. 8, a. cubus autem in ſe [d] facit cubum ; ergo $a_2$ cubus eſt, & [e] proinde ab eo quartus $a^5$, pariterq; $a^8$, $a^{11}$, &c. cubi ſunt. ergo omnes. Q. E. D.

Clarius forſitan ſic; Sit quadrati a latus b. ergo ſeries a, $a^2$, $a^3$, $a^4$, &c. aliter exprimetur ſic, bb, $b_4$, $b^6$, b8, &c. liquet vero hos omnes quadratos eſſe ; & ſic etiam exprimi poſſe; Q : b, Q: bb, Q : bbb, Q : bbbb, &c.

Eodem modo, ſi b latus fuerit cubi a, ſeries ita nominari poteſt ; $b^3$, $b^6$, $b_9$, $b^{12}$, &c. vel C : b, C : $b^2$, C : $b^3$, C : $b^4$, &c.

## PROP. X.

1, a, $a^2$, $a^3$, $a^4$, $a^5$, $a^6$.
1, 2, 4, 8, 16, 32, 64.

*Si ab unitate quotcunque numeri deinceps proportionales fuerint* (1, a, $a^2$, $a^3$, &c.) ; *qui vero poſt unitatem* (a) *non ſit quadratus, neque alius ullus quadratus erit, præter* $a_2$ *tertium ab unitate, & unum intermittentes omnes* ($a^4$, $a^6$, $a_8$.) *At ſi* a, *qui poſt unitatem, non ſit cubus, neque ullus alius cubus erit præter* $a^3$ *quartum ab unitate, & duos intermittentes omnes*, $a^6$, $a^9$, $a^{12}$, *&c.*

1. *Hyp*. Nam ſi fieri poteſt, ſit $a^5$ quadratus numerus. quoniam igitur a. $a_2$ [a] :: $a^4$. $a_5$, atq;
a hyp. inverſe $a^5$. $a_4$ :: $a^2$. a; ſintque $a^5$, & $a_4$ [b] qua-
b ſuppoſ. & 8. 9. drati, primuſque $a_2$ quadratus, [c] erit a etiam
c 24. 8. quadratus, contra Hyp.

2. *Hyp*. Si fieri poteſt, ſit $a^4$ cubus. quoniam igitur [d] ex æquo $a^4$. $a^6$ :: a. $a^3$, atque in-
d 14. 7. verſe $a_6$. $a_4$ :: $a_3$. a ; [b] ſintque $a^6$, & $a_4$ cubi, & primus $a_3$ cubus, [e] etiam a cubus erit, con-
e 25. 8. tra Hypoth.

PROP.

## PROP. XI

$1, a, a_2, a_3, a_4, a_5, a_6.$
$1, 3, 9, 27, 81, 243, 729.$

*Si ab unitate quotcunq; numeri deinceps proportionales fuerint* (1, a, $a_2$, $a_3$, &c.) *minor majorem metitur per aliquem eorum qui in proportionalibus ſunt numeris.*

Quoniam 1. a :: a. aa, ^a erit $\frac{aa}{a} = a = \frac{aaa}{aa}$. a 5. ax. 7. & 20. def. 7.

Item quia 1. aa ^b :: a. aaa. ^a erit $\frac{aaa}{a} = aa = \frac{a_4}{aa} = \frac{a_5}{a_3}$ &c. denique quia 1. $a_3$ ^b :: a. $a_4$, ^a erit $\frac{a_4}{a} = a_3 = \frac{a_6}{a_3}$ &c. b 14. 7.

### Coroll.

Hinc, ſi numerus qui metitur aliquem ex proportionalibus, non ſit unus proportionalium, neque numerus per quem metitur, erit aliquis ex proportionalibus.

## PROP. XII.

$1, a, a_2, a_3, a_4,$
$1, 6, 36, 216, 1296.$
B, 3.

*Si ab unitate quotcunq; numeri deinceps proportionales fuerint* (1, a, $a_2$, $a_3$, $a_4$)*; quicunque primorum numerorum* B *ultimum* $a_4$ *metiuntur, iidem* (B) *& eum* (a) *qui unitati proximus eſt, metientur.*

Dic B non metiri a, ^a ergo B ad a primus eſt; ^b ergo B ad $a_2$ primus eſt; & ^c proinde ad $a_4$ quem metiri ponitur Q. E. A.

a 31. 7.
b 27. 7.
c 26. 7.

### Coroll.

1. Itaq; omnis numerus primus ultimum metiens, metitur quoq; omnes alios ultimum præcedentes.

2. Si aliquis numerus non metiens proximum unitati, metiatur ultimum, erit numerus compoſitus.

3. Si proximus unitati ſit primus numerus, nullus alius primus numerus ultimum metietur.

## PROP. XIII.

$1,\ a,\ a^2,\ a^3,\ a^4,$
1. 5, 25, 125, 625.
H - - G - - F - - E - -

*Si ab unitate quotcunque numeri deinceps proportionales fuerint* (a, $a^2$, $a^3$, &c,), *qui vero poſt unitatem* (a) *primus ſit; maximum nullus alius metietur, præter eos qui ſunt in numeris proportionalibus.*

Si fieri poteſt, alius quiſpiam E metiatur $a4$, a cor. 12. 9. nempe per F; *a* erit F alius extra $a$, $a^2$, $a^3$. b 2 cor 12. 9 Quia vero E metiens $a4$ non metitur $a$, *b* erit c 33 7 E numerus compoſitus; *c* ergo eum aliquis pri- d 11. ax. 7. mus metitur, *d* qui proinde ipſum $a4$ metitur; e 3 cor 12 9. *e* ideoque alius non eſt, quam $a$. ergo $a$ metitur E. Eodem modo oſtendetur F compoſitus numerus, metiens $a4$, adeoque $a$ ipſum F metiri. f 9 ax. 7. itaque quum EF *f* $= a4 = a$ in $a3$, *g* erit $a$. E :: F. g 19 7. h 20. def 7. $a3$. ergo cum $a$ metiatur E, *h* æque F metietur k cor. 11. 9. $a^3$, puta per eundem G. *k* Nec G erit $a$, vel $a^2$. ergo, ut prius, G eſt numerus compoſitus, & $a$ eum metitur. quum igitur FG *f* $= a3 = a^2$ in $a$, *g* erit $a$. F :: G. $a2$; & proinde, quia A metitur F, *h* æque G metietur $a^2$, ſcilicet per eundem H; *k* qui non eſt $a$. ergo quum G H $= a^2 = aa$. l 10 7. *l* erit H. $a$ :: $a$. G. ergo quia $a$ metitur G (ut m 10 def. 7. prius) *m* etiam H metietur $a$, numerum primum. Q. E. N.

PROP.

## PROP. XIV.

A, 30.
B, 2. C, 3. D, 5.
E -- F -- .

*Si minimum numerum A primi numeri B, C, D metiantur; nullus alius numerus primus E illum metietur, præter eos, qui à principio metiebantur.*

Si fieri poteſt, ſit $\frac{A}{E}$ = F. [a] Ergo A = E F. a 9 ax. 7.
[b] Ergo ſinguli primi numeri B, C, D ipſorum b 31. 7.
E, F unum metiuntur; non E, qui primus ponitur; ergo F, minorem ſcilicet ipſo A; contra Hypoth.

## PROP. XV.

A, 9. B, 12. C, 16.
D, 3. E, 4.

*Si tres numeri A, B, C deinceps proportionales, fuerint minimi omnium eandem cum ipſis rationem habentium; duo quilibet compoſiti, ad reliquum primi erunt.*

[a] Sume D, & E minimos in ratione A ad B. a 35. 7.
[b] ergo A = Dq, [b] & C = Eq; [b] & B = DE. Quia b 2. 8.
vero D ad E [c] primus eſt, [d] erit D + E primus ad c 24. 7.
ſingulos D, & E. * ergo D in D + E [e] = D q + d 30. 7. * 26. 7.
DE ([f] A + B) ad E primus eſt, ideoque ad C e 3. 2.
vel Eq. Q. E. D. Pari pacto DE + Eq (B + C) f prius. g 27. 7.
ad D primus eſt, & proinde ad A = Dq. Q. E. D.
Denique quia B ad D + E [h] primus eſt; is ad h 26. 7.
hujus quadratum [k] Dq + 2 DE + Eq (A + 2 k 4. 2.
B + C) primus erit. [l] quare idem B ad A + B + C, l 30. 7.
[l] adeoque ad A + C primus erit. Q. E. D.

## Prop. XVI.

A, 3. B, 5. C--- *Si duo numeri* A, B *primi inter ſe fuerint; non erit ut primus* A *ad ſecundum* B, *ita ſecundus* B *ad alium quempiam* C.

Dic A. B :: B. C. ergo quum A & B in ſua
a 23.7. ratione *a* minimi ſint, A *b* metietur B æque ac B
b 21.7. ipſum C; ſed A *c* ſeipſum etiam metitur; ergo
c 6.ax.7. A & B non ſunt primi inter ſe, contra Hypoth.

## Prop. XVII.

A, 8. B, 12. C, 18. D, 27. E---

*Si fuerint quotcunque numeri deinceps proportionales* A, B, C, D, *extremi autem ipſorum* A, D *primi inter ſe ſint; non erit ut primus* A *ad ſecundum* B, *ita ultimus* D *ad alium quempiam* E.

Dic A. B :: D. E. ergo viciſſim A. D :: B. E.
a 23.7. ergo quum A & D in ſua ratione *a* minimi ſint,
b 21.7. *b* metietur A ipſum B; *c* quare B ipſum C, & C
c 20.def.7. ſequentem D, *d* adeoque A eundem D metietur.
d 11.ax.7. Ergo A & D non ſunt primi inter ſe, contra Hypoth.

## Prop. XVIII.

A, 4. B, 6. C, 9. Bq, 16. *Duobus numeris datis* A, B, *conſiderare an poſsit ipſis tertius proportionalis* C *inveniri.*

a 9.ax.7. Si A metiatur Bq per aliquem C, *a* erit AC
b per 20.7. =Bq. unde *b* liquet eſſe A.B :: B.C. Q. E. F.

A, 6. B, 4. Bq, 16. Sin A non metiatur Bq, non erit aliquis tertius proportionalis.

c 7.ax.7. Nam dic A.B :: B.C. *a* ergo AC=Bq. *c* proinde $\frac{Bq}{A}$=C. Scilicet A metitur Bq, contra Hypoth.

PROP.

## PROP. XIX.

A, 8. B, 12. C, 18. D, 27.
EC, 216.

*Tribus numeris datis A, B, C, conſiderare an poſsit ipſis quartus proportionalis D inveniri.*

Si A metiatur B C per aliquem D, *a* ergo A D = B C; *b* conſtat igitur eſſe A. B :: C. D. Q. E. F. a 9.ax.7. b ex 19 7.

Sin A non metiatur B C, non datur quartus proportionalis; quod oſtendetur, prout in præcedenti.

## PROP. XX.

A, 2. B, 3. C, 5.
D, 30. G - - - -

*Primi numeri plures ſunt omni propoſita multitudine primorum numerorum* A, B, C.

*a* Sit D minimus, quem A, B, C metiuntur. ſi D+1 primus ſit, res patet; ſi compoſitus, *b* ergo aliquis primus, puta G, metitur D+1, qui non eſt aliquis trium A, B, C; nam ſi ita, quum is *c* totum D+1, & *d* ablatum D metiatur, *e* idem reliquam unitatem metietur. Q. E. A. Ergo propoſitorum primorum numerorum multitudo aucta eſt per D+1; vel ſaltem per G. a 38.7. b 33.7. c ſuppoſ. d conſtr. e 12.ax 7.

## PROP. XXI.

$\overset{5}{\text{A}} \ldots\ldots \overset{5}{\text{E}} \ldots\ldots \overset{3}{\text{B}} \ldots \overset{3}{\text{F}} \ldots \overset{2}{\text{C}} \ldots \overset{2}{\text{G}} \ldots \text{D}\ 20.$

*a Si pares numeri quotcunque* AB, BC, CD *componantur, totus* AD *par erit.*

*a* Sume EB = $\frac{1}{2}$ AB & FC = $\frac{1}{2}$ BC, & GD = $\frac{1}{2}$ CD. *b* liquet EB + FC + GD = $\frac{1}{2}$ AD. *c* ergo AD eſt par numerus. Q. E. D. a 6.def.7. b 12. 7. c 6.def.7.

## PROP. XXII.

8 . . . I I I
A........F.B......G.C....H.D..L. E 22.
9 7 5 3

*Si impares numeri quotcunque* AB, BC, CD, DE *componantur, multitudo autem ipſorum ſit par, totus* AE *par erit.*

a 7.def.7. Detracta unitate ex ſingulis imparibus, *a* manebunt AF, BG, CH, DL numeri pares, &
b 21. 9. *b* proinde compoſitus ex ipſis par erit; adde his
c hyp. *c* parem numerum conflatum ex reſiduis unitatibus,
d 21. 9. *d* totus idcirco AE par erit. Q. E. D.

## PROP. XXIII.

7 5 I
A.......B.....C..E.D 15.
3

*Si impares numeri quotcunque* AB, BC, CD *componantur, multitudo autem ipſorum ſit impar; & totus* AD *impar erit.*

Nam dempto CD uno imparium, reliquorum
a 22. 9. aggregatus AC *a* eſt par numerus. huic adde
b 21. 9. CD − 1; *b* totus AE eſt etiam par; quare reſti-
c 7.def.7. tuta unitate totus AD *c* impar erit. Q. E. D.

## PROP. XXIV.

4 5 I
A....B.....D. C 10.
6

*Si à pari numero* AC *par* AB *detrahatur, & reliquus* BC *par erit.*

Nam ſi BD (BC −
a 7.def.7. 1) impar fuerit, *a* erit BC (BD + 1) par. Q. E. D.
b hyp. Sin BD parem dicas, propter AB *b* parem, *c* erit
c 21. 9. AD par; *a* ideoque AC (AD + 1) impar, contra Hypoth. ergo BC eſt par. Q. E. D.

PROP.

## Prop. XXV.

6 1 3
A......D.C...B 10.
7

*Si à pari numero* AB *impar* AC *detrahatur, & reliquus* CB *impar erit.*

Nam AC – 1 (AD) a eſt par. b ergo DB eſt par. c ergo CB (DB – 1) eſt impar. Q.E.D. a 7. def. 7. b 24. 9. c 7. def. 7.

## Prop. XXVI.

4 6 1
A....C......D.B 11.
7

*Si ab impari numero* AB *impar* CB *detrahatur, reliquus* AC *par erit.*

Nam AB – 1 (AD) & CB – 1 (CD) a ſunt pares. b ergo AD – CD (AC) eſt par. Q.E.D. a 7. def. 7. b 24. 9.

## Prop. XXVII.

1 4 6
A.D....C......B 11.
5

*Si ab impari numero* AB *par detrahatur* CB, *reliquus* AC *impar erit.*

Nam AB – 1 (DB) a eſt par; & CB ponitur par. b ergo reliquus CD par eſt. c ergo CD + 1 (CA) eſt impar. Q.E.D. a 7. def. 7. b 24. 9. c 7. def. 7.

## Prop. XXVIII.

A, 3.
B, 4.
$\overline{AB}$, 12.

*Si impar numerus* A *parem numerum* B *multiplicans fecerit aliquem* AB, *factus* AB *par erit.*

Nam AB a componitur ex impari A toties accepto, quoties unitas continetur in B pari. b ergo AB eſt par numerus. a hyp. & 15. def. 7. b 21. 9.

*Schol.*

Eodem modo, ſi A ſit numerus par, erit AB par.

Prop.

## PROP. XXIX.

A, 3.
B, 5.
AB, 15.

*Si impar numerus* A, *imparem numerum* B *multiplicans fecerit aliquem* AB; *factus* AB *impar erit.*

Nam AB *a* componitur ex B impari numero toties accepto, quoties unitas includitur in A etiam impari. *b* ergo A B eſt impar. Q.E.D.

a 15. def. 7.
b 23. 9.

### *Scholium.*

B, 12 (C, 4.
A, 3.

1. *Numerus* A *impar numerum B parem metiens, per numerum parem* C *eum metitur.*

Nam ſi C impar dicatur, quoniam *a* B = AC, erit B impar, contra Hypoth.

a 9. ax. 7.
b 29. 9.

B, 15 (C, 5.
A, 3

2. *Numerus* A *impar numerum* B *imparem metiens, per numerum* C *imparem eum metitur.*

Nam ſi C dicatur par; *a* erit A C, vel B par, contra Hypoth.

a 28. 9.

B, 15 (C, 5.
A, 3

3. *Omnis numerus* (A & C) *metiens imparem numerum* B, *eſt impar.*

Nam ſi utervis A, vel C dicatur par, *a* erit B numerus par, contra Hypoth.

a 28. 9.

## PROP. XXX.

B, 24 (C, 8. D, 12 (E, 4.
A, 3 A, 3

*Si impar numerus* A *parem numerum* B *metiatur, & illius dimidium* D *metietur.*

*a* Sit $\frac{B}{A}$ = C. *b* ergo C eſt numerus par. Sit igitur E = $\frac{1}{2}$C, erit B *c* = CA *d* = 2EA *e* = 2D. *f* ergo EA = D; & *g* proinde $\frac{D}{A}$ = E. Q.E.D.

a hyp.
b 1. Schol. 29. 9.
c 9. ax. 7.
d 1. 2.
e hyp.
f 7. ax. 1.
g 7. ax. 7.

PROP.

### PROP. XXXI.

A, 5. B, 8. C, 16. D - - - *Si impar numerus A ad aliquem numerum B primus ſit; & ad illius duplum C primus erit.*

Si fieri poteſt, aliquis D metiatur A, & C. *a* ergo D metiens imparem A impar erit, *b* ideoque ipſum B paris C ſemiſſem metietur. ergo A, & B non ſunt primi inter ſe, contra Hypoth.

a 3. *ſchol.* 29. 9.
b 30. 9.

*Coroll.*

Sequitur hinc, numerum imparem, qui ad aliquem numerum progreſſionis duplæ primus eſt, primum quoque eſſe ad omnes numeros illius progreſſionis.

### PROP. XXXII.

1. A, 2. B, 4. C, 8. D, 16. *Numerorum A, B, C, D, &c. à binario duplorum unuſquiſque pariter par eſt tantum.*

Conſtat omnes 1, A, B, C, D *a* pares eſſe; atque *b* ∺ nimirum in ratione dupla, & *c* proinde quemque minorem metiri majorem per aliquem ex illis. *d* Omnes igitur ſunt pariter pares. Sed quoniam A primus eſt, *e* nullus extra eos eorum aliquem metietur. Ergo pariter pares ſunt tantum. Q. E. D.

a 6 *def.* 7.
b 20 *def.* 7.
c 11. 9.
d 8. *def.* 7.
e 13. 9.

### PROP. XXXIII.

A, 30. B, 15. D - - - E - - *Si numerus A dimidium B habeat imparem, A pariter impar eſt tantum.*

Quoniam impar numerus B *a* metitur A per 2 parem, *b* eſt B pariter impar. Dic etiam pariter parem. *c* ergo eum par aliquis D per parem E metitur. unde 2 B *d* = A *d* = D E. *e* quare 2. E ::

a *hyp.*
b 9. *def.* 7.
c 8. *def.* 7.
d 9 *ax.* 7.
e 19. 7.

f 6. def. 7 g 20. def. 7. E :: D. B. ergo ut 2 *f* metitur parem E, *g* ſic D par imparem B metitur. Q. E. N.

## PROP. XXXIV.

A, 24. *Si par numerus A, neque à binario duplus ſit, neque dimidium habeat imparem; pariter par eſt, & pariter impar.*

Liquet A eſſe pariter parem, quia dimidium imparem non habet. Quia vero ſi A bifarietur; & rurſus ejus dimidium, & hoc ſemper fiat, tandem incidemus in aliquem *a* imparem (quia non in binarium, quoniam A à binario duplus non ponitur) is metietur A per parem numerum (nam *b* alias ipſe A impar eſſet, contra Hypoth.) ergo A eſt etiam pariter impar. Q. E. D.

a 7. def. 7

b 1. ſch. 19. 9.

## PROP. XXXV.

A ........ 8.

4 . 8

B .... F ........ G 12.

C .................. 18.

9 . 6 . 4 . 8

D ........ H ...... L .... K ...... N 27.

*Si ſint quotcunque numeri deinceps proportionales A, B G, C, D N, detrahantur autem F G à ſecundo, & K N ab ultimo, æquales ipſi primo A; erit ut ſecundi exceſſus B F ad primum A, ita ultimi exceſſus DK ad omnes A, BG, C ipſum antecedentes.*

Ex DN deme NL=BG, & NH=C.

a hyp. b 17. 5. Quoniam D N. C. (H N) *a* :: H N. B G. (L N) *a* :: L N. (B G) A. (K N.) *b* erit dividendo ubique, D H. H N :: H L. L N :: LK. KN. *c* quare DK. C + BG + A :: LK (*d* BF.) KN. (A.) Q. E. D.

c 12. 5. d 3. ax. 1.

*Coroll.*

e 18. 5. Hinc *e* componendo, DN + BG + C. A + BG + C :: BG. A.

PROP.

## Prop. XXXVI.

1. A, 2. B, 4. C, 8. D, 16.
E, 31. G, 62. H, 124. L, 248. F, 496.
M, 31. N, 465.
P---- Q---

*Si ab unitate quotcunque numeri* 1, A, B, C, D, *deinceps exponantur in dupla proportione, quoad totus compoſitus* E *fiat primus, & totus hic* E *in ultimum* D *multiplicatus faciat aliquem* F; *factus* F *erit perfectus.*

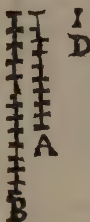
Sume totidem, E, G, H, L etiam in proportione dupla continue; ergo [a] ex æquo A. D :: a 14. 7.
E. L. [b] ergo A L = D E [c] = F. [d] ergo L = $\frac{F}{2}$ b 19. 7. c hyp. d 7. ax. 7.
quare E, G, H, L, F ſunt ∺ in ratione dupla. Sit G − E = M, & F − E = N. [e] ideo M. E :: e 35. 9.
N. E + G + H + L. [f] at M = E. [g] ergo N = f 3. ax 1. g 14. 5.
E + G + H + L. ergo F = 1 + B + h 2. ax. 1.
C + D + E + G + H + L = E + N.
Quinetiam quia D [k] metitur DE (F,) [l] etiam ſinguli 1, A, B, C [m] metientes D, [m] nec non E, k 7. ax. 7. l 11. ax. 7.
G, H, L metiuntur F. Porro nullus alius eun- m 11. 9.
dem F metitur. Nam ſi aliquis, ſit P, qui metiatur F per Q. [n] ergo P Q = F = D E. [o] ergo n 9. ax 7.
E. Q :: P. D. ergo cum A primus numerus o 19. 7.
metiatur D, & [p] proinde nullus alius P eundem metiatur, [q] conſequenter E non metitur Q. qua- p 13. 9. q 20 def. 7.
re cum E primus ponatur, [r] idem ad Q primus erit. [ſ] ergo E & Q in ſua ratione minimi ſunt, r 31. 7. ſ 23. 7.
& [t] propterea E ipſum P ac Q ipſum D æque t 21. 7.
metiuntur. [u] ergo Q eſt aliquis ipſorum A, B, C.
Sit igitur B; ergo cum ex æquo ſit B. D :: E. H; u 13. 7.
[x] ideoque B H = D E = F = P Q. [x] adeoque x 19. 7.
Q. B :: H. P. [y] erit H = P. ergo P eſt etiam y 14. 5.
aliquis ipſorum A, B, C, &c. contra Hypoth.
ergo nullus alius præter numeros prædictos eundem F metietur: [z] proinde F eſt numerus perfe- z 22. def. 7.
ctus. Q. E. D.

LIB.

# LIB. X.

## *Definitiones.*

I. Commenſurabiles magnitudines dicuntur, quas eadem menſura metitur.



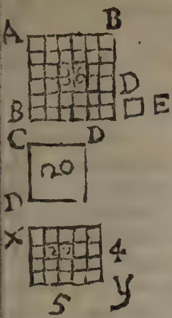

*Commenſurabilitatis nota eſt* ⊔, *ut* A ⊔ B; *hoc eſt*, *linea* A 8 *pedum commenſurabilis eſt lineæ* B 13 *pedum; quia* D *linea unius pedis ſingulas* A *&* B *metitur. Item* $\surd$ 18 ⊔ $\surd$ 50; *quia* $\surd$ 2 *ſingulas* $\surd$ 18, *&* $\surd$ 50 *metitur. Nam* $\surd \frac{18}{2} = \surd 9 = 3$. *&* $\surd \frac{50}{2} = \surd 25 = 5$. *quare* $\surd 18.\ \surd 50 :: 3.\ 5.$

II. Incommenſurabiles autem ſunt, quorum nullam communem menſuram contingit reperiri.

*Incommenſurabilitas ſignificatur nota* '⊔. *ut* $\surd$ 6 '⊔ $\surd$ 25 (5;) *hoc eſt* $\surd$ 6 *incommenſurabilis eſt numero* 5, *vel magnitudini hoc numero deſignatæ; quia harum nulla eſt communis menſura, ut poſtea patebit.*

III. Rectæ lineæ potentia commenſurabiles ſunt, cum quadrata earum idem ſpatium metitur.

*Hu-*




*Hujuſce commenſurabilitatis nota eſt* ⊤, *ut* AB ⊤ CD; *h.e. linea* AB *ſex pedum potentia commenſurabilis eſt lineæ* C D, *quæ exprimitur per* √ 20. *quia ſpatium* E *unius pedis quadrati metitur tam* ABq (36) *quam rectangulum* XY (20,) *cui æquale eſt quadratum lineæ* CD(√20.) *Eadem nota* ⊤ *nonnunquam valet potentia tantum commenſurabilis.*

IV. Incommenſurabiles vero potentia, cum uadratis earum nullum ſpatium, quod ſit communis eorum menſura, contingit reperiri.

*Hujuſmodi incommenſurabilitas denotatur ſic;* ⊤ *v*√ 8; *hoc eſt, numeri vel lineæ* 5, & *v*√ 8 *unt incommenſurabiles potentia; quia harum quarata* 25, & √ 8 *ſunt incommenſurabilia.*

V. Quæ cum ita ſint, manifeſtum eſt cuicunque rectæ propoſitæ, rectas lineas multitudine nfinitas, & commenſurabiles eſſe, & incommenurabiles; alias quidem longitudine & potentia, lias vero potentia ſolum. Vocetur autem proioſita recta linea Rationalis.

*Hujus nota eſt* ῥ.

VI. Et huic commenſurabiles, ſive longitudine & potentia, ſive potentia tantum, Rationaes, ῥ.

VII. Huic vero incommenſurabiles Irrationales vocentur.

*Hæ ſic denotantur* ῥ̸.

VIII. Et quadratum, quod à propoſita recta fit, dicatur Rationale ῥν.

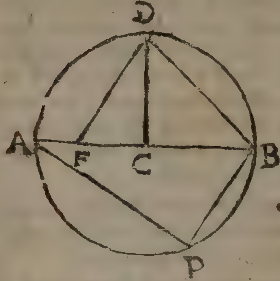
IX. Et huic commenſurabilia quidem Raionalia ῥα.

X. Huic

X. Huic vero incommenſurabilia, Irrationalia dicantur, ρα.

XI. Et rectæ, quæ ipſa poſſunt, Irrationales, ρ.

*Schol.*




*Vt poſtremæ 7 definitiones exemplo aliquo illuſtrentur, ſit circulus* A D B P, *cujus ſemidiameter* C B; *huic inſcribantur latera figurarum ordinatarum, Hexagoni quidem* B P, *Trianguli* A P; *quadrati* B D, *pentagoni* F D. *Itaque ſi juxta 5 defin. ſemidiameter* CB *ſit Rationalis expoſita, numero* 2. *expreſſa, cui reliquæ* BP, AP, BD, FD *comparandæ ſunt,*[a] *erit* BP [a] = BC = 2. *quare* BP *eſt* ρ́ ⊤⊤ B C, *juxta 6. def. Item* A P [b] = √12 (*nam* ABq (16) − BPq (4) = 12) *quare* AP *eſt* ρ́ ⊤⊥ B C, *etiam juxta 6. def. atque* APq (12) *eſt* ρ́ν, *per def. 9. Porro* B D [b] = √ DCq + BCq = √ 8; *unde* BD *eſt* ρ́ ⊤⊥ BC; & BDq ρ́ν. *Denique,* FDq = 10 − √ 20 (*ut patebit ex praxi ad* 10. 13. *tradenda*) *erit* ρ́ν, *juxta* 10 *def.* & *proinde* FD = √:10 − √ 20 *eſt* ρ́, *juxta* 11 *defin.*

a cor. 15 4.
b 67. 1.

## *Poſtulatum.*

POſtuletur, quamlibet magnitudinem toties poſſe multiplicari, donec quamlibet magnitudinem ejuſdem generis excedat.

*Axio-*

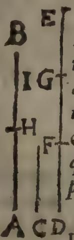
## *Axiomata.*

1. MAgnitudo quotcunque magnitudines metiens, compoſitam quoque ex ipſis metitur.

2. Magnitudo quamcunque magnitudinem metiens, metitur quoque omnem magnitudinem quam illa metitur.

3. Magnitudo metiens totam magnitudinem & ablatam, metitur & reliquam.

### PROP. I.

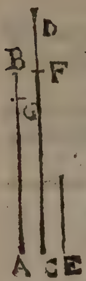



*Duabus magnitudinibus inæqualibus* A B, C *propoſitis, ſi à majore* A B *auferatur majus quam dimidium* (A H) *& ab eo* (HB) *quod reliquum eſt, rurſus detrahatur majus quam dimidium* (H I,) *& hoc ſemper fiat; relinquetur tandem quædam magnitudo* I B, *quæ minor erit propoſita minore magnitudine* C.

*a* Accipe C toties, donec ejus multiplex DE proxime excedat AB; ſintque DF = FG = GE = C. Deme ex A B pluſquam dimidium AH, & à reliquo HB pluſquàm dimidium HI; & ſic deinceps, donec partes AH, HI, IB æque multæ ſint partibus DF, FG, GE. Iam liquet FE, quæ non minor eſt quam $\frac{1}{2}$ DE, majorem eſſe quam HB, quæ minor eſt quam $\frac{1}{2}$ AB $\sqsupset$ DE. Pariterque G E quæ non minor eſt quam $\frac{1}{2}$ FE, major eſt quam IB $\sqsupset \frac{1}{2}$ HB. ergo C, vel GE $\sqsubset$ IB. Q. E. D. *a poſt. 10.*

Idem demonſtrabitur, ſi ex AB auferatur dimidium AH, & ex reliquo HB rurſus dimidium HI, & ita deinceps.

## PROP. II.

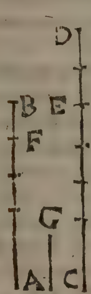


*Si duabus magnitudinibus inæqualibus propoſitis* (AB, CD) *detrahatur ſemper minor* AB *de majore* CD, *alterna quadam detractione, & reliqua minime præcedentem metiatur; incommenſurabiles erunt ipſæ magnitudines.*

Si fieri poteſt, ſit aliqua E communis menſura. Quoniam igitur AB detracta ex CD, quoties fieri poteſt, relinquit aliquam FD ſe minorem, & FD ex AB relinquit GB, & ſic deinceps, *a* tandem relinquetur aliqua GB ⊐ E. ergo E *b* metiens AB, *c* ideoque CF, *b* & totam CD; *d* etiam reliquam FD, metitur. *c* proinde & AG; *d* ergo & reliquam GB, ſeipſa minorem. Q. E. A.

a 1. 10.
b *hyp.*
c 2 *ax.* 10.
d 3. *ax.* 10

## PROP. III.



*Duabus magnitudinibus commenſurabilibus datis,* AB, CD, *maximam earum communem menſuram* FB *reperire.*

Deme AB ex CD, & reliquum ED ex AB, & FB ex ED, donec FB metiatur ED; (quod tandem fiet, *a* quia per Hyp. AB ⊥⊥ CD) erit FB quæſita.

Nam FB *b* metitur ED, *c* ideoque ipſam AF; ſed & ſeipſam, *d* ergo etiam AB, & *c* propterea CE, *d* ideoque & totam CD. Proinde FB communis eſt menſura ipſarum AB, CD. Dic G communem quoq; eſſe menſuram, hac majorem; ergo G metiens AB, & CD, *e* metitur CE, & *f* reliquam ED, *e* ideoque AF, & *f* proinde reliquam FB major minorem. Q. E. A.

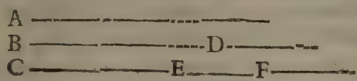
a 2. 10.
b *conſtr.*
c 2 *ax.* 10.
d 1 *ax.* 19.
e 2. *ax.* 10.
f 3. *ax.* 10.

*Corol.*

*Coroll.*

Hinc, magnitudo metiens duas magnitudines, metitur & maximam earum menſuram communem.

## PROP. IV.




*Tribus magnitudinibus commenſurabilibus datis* A, B, C; *maximam earum menſuram communem invenire.*

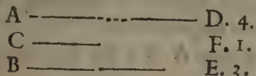
*a* Inveni D maximam communem menſuram duarum quarumcunque A, B; *a* item E ipſarum D & C maximam communem menſuram; erit E quæſita. a 3. 10.

*a* Nam perſpicuum eſt E metiens D & C *b* metiri tres A, *B*, C. Puta aliam F hac majorem eaſdem metiri. *c* ergo F metitur D; *c* proinde & E, ipſorum D, C maximam communem menſuram, major minorem. Q. E. A. b *conſtr* & 2.*ax*.10. c *cor*.3.10.

*Coroll.*

Hinc quoque, magnitudo metiens tres magnitudines, metitur quoque maximam earum communem menſuram.

## PROP. V.




*Commenſurabiles, magnitudines* A, *B inter ſe rationem habent, quam numerus ad numerum.*

*a* Inventa C ipſarum A, B maxima communi menſura; quoties C in A & B, toties 1 continеatur in numeris D & E. *b* ergo C. A :: 1. D; quare inverſe A. C :: D. 1. *b* atqui etiam C. a 3.10. b 20.*def*.7.

c 22. 5. B :: 1. E. *c* ergo ex æquali A. B :: D. E :: N. N. Q. E. D.

### PROP. VI.

E ——————— F. 1. *Si duæ ma-*
A ————————————— C. 4. *gnitudines* A, B
B ——————————— D. 3. *inter ſe propor-*
*tionem habeant, quam numerus* C *ad numerum* D; *commenſurabiles erunt magnitudines* A, B.

a ſch. 10 6. b conſtr. c hyp. d 22. 5. e 5. ax. 7. f 20. def. 7. g conſtr. h 1. def. 10.

Qualis pars eſt 1 numeri C, *a* talis fiat E ipſius A. Quoniam igitur E. A *b* :: 1. C. atque A. B *c* :: C. D; *d* ex æquo erit E. B :: 1. D. ergo quum 1 *e* metiatur numerum D, *f* etiam E metitur B; ſed & ipſum A *g* metitur. *h* ergo A ⊤⊥ B. Q. E. D.

### PROP. VII.

A ——————————— *Incommenſurabiles*
B ————————— *magnitudines* A, B *inter ſe proportionem non habent, quam numerus ad numerum.*

a 6 10. Dic A. B :: N. N. *a* ergo A ⊤⊥ B, contra Hypoth.

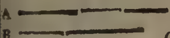
### PROP. VIII.

A ——————————— *Si duæ magnitudine*
B ———————— A, B *inter ſe proportionem non habeant, quam numerus ad numerum, incommenſurabiles erunt magnitudines.*

a 5. 10. Puta A ⊤⊥ B *a* ergo A. B :: N. N, contr: Hypoth.

PROP

## Prop. IX.




E, 4.
F, 3.

*Quæ à rectis lineis longitudine commenſurabilibus fiunt quadrata, inter ſe proportionem habent, quam quadratus numerus ad quadratum numerum: & quadrata inter ſe proportionem habentia, quam quadratus numerus ad quadratum numerum, & latera habebunt longitudine commenſurabilia. Quæ vero à rectis lineis longitudine incommenſurabilibus fiunt quadrata, inter ſe proportionem non habent, quam quadratus numerus ad quadratum numerum: & quadrata inter ſe proportionem non habentia, quam quadratus numerus ad quadratum numerum, neq; latera habebunt longitudine commenſurabilia.*

1. *Hyp.* A. ⊓ B. Dico Aq. Bq :: Q. Q.
Nam [a] ſit A. B :: num. E. num. F. ergo $\frac{Aq}{Bq}\left({}^{b}\ \frac{A}{B}\text{bis}\right)^{c} = \frac{E}{F}\text{bis.}\ {}^{d}= \frac{Eq}{Fq}\ {}^{e}$ ergo Aq. Bq :: Eq. Fq :: Q. Q. Q. E. D.

2. *Hyp.* Aq. Bq :: Eq. Fq :: Q. Q. Dico A ⊓ B. Nam $\frac{A}{B}$ bis $\left({}^{f}\ \frac{Aq}{Bq}\right)^{g} = \frac{Eq}{Fq}\ {}^{h} = \frac{E}{F}$ bis. [i] ergo A. B :: E. F :: N. N. [k] quare A ⊓ B. Q. E. D.

3. *Hyp.* A ⊓ B. Nego eſſe Aq. Bq :: Q. Q. Nam dic Aq. Bq :: Q. Q. Ergo A ⊓ B, ut modo oſtenſum eſt, contra Hypoth.

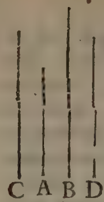
4. *Hyp.* Non Aq. Bq :: Q. Q. Dico A ⊓ B. Nam puta A ⊓ B; ergo Aq. Bq :: Q. Q. ut modo diximus, contra Hypoth.

a *per* 5. 10.
b 20. 6.
c *ſch.* 23. 5.
d 11, 8.
e 11. 5.
f 10. 6.
g *hyp.*
h 11. 8.
i *ſch.* 23. 5.
k 6. 10.

### Coroll.

Lineæ ⊓ ſunt etiam Ⴗ; at non contra. Sed lineæ ⊓ non ſunt idcirco Ⴗ. Lineæ vero Ⴗ ſunt etiam ⊓.

PROP. X.




*Si quatuor magnitudines proportionales fuerint* (C. A :: B. D; ) *prima vero* C *ſecundæ* A *fuerit commenſurabilis*; *& tertia* B *quartæ* D *commenſurabilis erit. Et ſi prima* C *ſecundæ* A *fuerit incommenſurabilis, & tertia* B *quartæ* D *incommenſurabilis erit.*

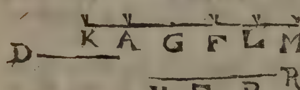
Si C ⊤⊥ A, *a* ideo erit C. A :: N.
a 5. 10. N *b* :: B. D. *b* ergo B ⊤⊥ D. Sin C
b 6. 10. ⊥⊤ A, ergo *c* non erit C. A :: N. N :: B. D.
c 7. 10. *d* quare B ⊥⊤ D. Q. E. D.
d 8. 10.

*LEMMA* 1.

*Duos numeros planos invenire, qui proportionem non habeant, quam quadratus numerus ad quadratum numerum.*

Huic Lemmati ſatisfacient duo quilibet numeri plani non ſimiles, quales ſunt numeri habentes proportionem ſuperparticularem, vel ſuperbipartientem, vel duplam; vel etiam duo quivis numeri primi. vid. Schol. 27. 8.

*LEMMA* 2.




B, 5, C, 3.

*Invenire lineam* HR, *ad quam data recta linea* KM *ſit in ratione datorum numerorum* B, C.

ſch. 10 6. *a* Divide K M in partes æquales æque multas unitatibus numeri *B*. harum tot, quot unitates
3. 1. ſunt in numero C, *b* componant rectam H R. liquet eſſe KM. HR :: B. C.

*LEMMA* 3.

*Invenire lineam* D, *ad cujus quadratum datæ rectæ* K M *quadratum ſit in ratione datorum numerorum* B, C.

Fac

Fac *B*. C *a* :: KM. HR. ac inter KM, & HR *b* inveni mediam proportionalem D. Erit KMq. Dq *c* :: KM. HR *d* :: B. C. a 2 *lem* 10. 10. b 13. 6. c 20. 6. d *conſtr.*

### PROP. XI.

A ——————— B.20.
E — ———— C.16.
D ————

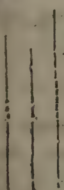
*Propoſitæ rectæ lineæ A invenire duas rectas lineas incommenſurabiles; alteram quidem D longitudine tantum, alteram vero E etiam potentia.*

1. Sume numeros B, C, *a* ita ut non ſit B. C :: Q. Q. *b* fiatque B. C :: Aq. Dq. *c* liquet A ⊐⊏ D. Sed Aq *d* ⊐⊏ Dq. Q. E. F. a 2 *lem.* 10. 10. b 3. *lem* 10. 10. c 9. 10. d 6. 10

2. *d* Fac A. E :: E. D. Dico Aq ⊐⊏ Eq. Nam A. D *e* :: Aq. Eq. ergo cum A ⊐⊏ D, ut prius, *f* erit Aq ⊐⊏ Eq. Q. E. F. d 13. 6. e 10. 6. f 10. 10.

### PROP. XII.

*Quæ (A, B) eidem magnitudini C ſunt commenſurabiles, & inter ſe ſunt commenſurabiles.*



D, 18 : E, 8.
F, 2. G, 3.
H, 5 : I, 4 : K, 6.
A B C

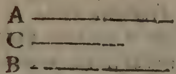
Quia A ⊐⊏ C, & C ⊐⊏ B, *a* ſit A. C :: N. N :: D. E. atque C. *B* :: N. N :: F. G. *b* ſumantur tres numeri H, I, K minimi ∺ in rationibus D ad E, & F ad G. Iam quia A. C *c* :: D. E *c* :: H. I. ac C. B *c* :: F. G. *c* :: I. K. *d* erit ex æquali A. B :: H. K :: N. N. *e* ergo A ⊐⊏ B. Q. E. D. a 5 10. b 4 8. c *conſtr.* d 22. 5. e 6. 10.

*Schol.*

Hinc, omnis recta linea rationali lineæ commenſurabilis, eſt quoque ρ rationalis. Et omnes rectæ rationales inter ſe commenſurabiles ſunt, ſaltem potentia. Item, omne ſpatium rationali ſpatio commenſurabile, eſt quoque rationale; & omnia ſpatia rationalia inter ſe commen- 12. 10 & *def.* 6. *def.* 9.

menſurabilia ſunt. Magnitudines vero, quarum
def 7. & 10. altera eſt rationalis, altera irrationalis, ſunt inter ſe incommenſurabiles.

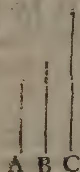
## PROP. XIII.



*Si ſint duæ magnitudines A, B; & altera quidam A eidem C ſit commenſurabilis, altera vero B incommenſurabilis; incommenſurabiles erunt magnitudines A, B.*

a hyp. b 12. 10. Dic B ⊤⊥ A. ergo cum C a ⊤⊥ A, b erit C ⊤⊥ B, contra Hypoth.

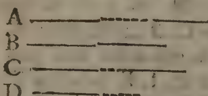
## PROP. XIV.



*Si ſint duæ magnitudines commenſurabiles A, B; altera autem ipſarum A magnitudini cuipiam C incommenſurabilis fuerit; & reliqua B eidem C incommenſurabilis erit.*

a hyp. b 12. 10. Puta B ⊤⊥ C. ergo cum A ⊤⊥ a B, b erit A ⊤⊥ C, contra Hyp.

## PROP. XV.

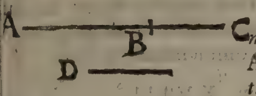


*Si quatuor rectæ lineæ proportionales fuerint (A. B :: C. D;) prima vero A tanto plus poſſit quam ſecunda B, quantum eſt quadratum rectæ lineæ ſibi commenſurabilis longitudine; & tertia C tanto plus poterit, quam quarta D, quantum eſt quadratum rectæ lineæ ſibi longitudine commenſurabilis. Quod ſi prima A tanto plus poſſit quam ſecunda B, quantum eſt quadratum rectæ lineæ ſibi incommenſurabilis longitudine; & tertia C tanto plus poterit, quam quarta D, quantum eſt quadratum rectæ lineæ ſibi longitudine incommenſurabilis.*

a hyp. b 22. 6. c 17. 5. Nam quia A. B a :: C. D. b erit Aq. Bq :: Cq. Dq. c ergo dividendo Aq — Bq. Bq :: Cq — Dq.

Dq.Dq. *d* quare √ : Aq—Bq. B :: √:Cq—Dq. d 22.6.
D. *e* invertendo igitur B. √ : Aq—Bq :: D. √ : e cor.4.5. f 22.5.
Cq—Dq. *f* ergo ex æquali A. √ : Aq—Bq ::
C. √ : Cq—Dq. proinde ſi A ⊔, vel ⊔ √
Aq—Bq, *g* erit ſimiliter C ⊔, vel ⊔ √ : g 10.10.
Cq—Dq. Q. E. D.

## PROP. XVI.



*Si duæ magnitudines commenſurabiles* AB, BC *componantur, & tota magnitudo* AC *utrique ipſarum* AB, BC *commenſurabilis erit: quod ſi tota magnitudo* AC *uni ipſarum* AB, *vel* BC *commenſurabilis fuerit; & quæ à principio magnitudines* AB, BC *commenſurabiles erunt.*

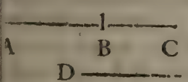
1. *Hyp.* *a* Sit D ipſarum AB, BC communis menſura. *b* ergo D metitur AC. *c* ergo AC ⊔ AB, & BC. Q. E. D. a 3.10. b 1.ax.10. c 1.def.10.

2. *Hyp.* *a* Sit D communis menſura ipſarum AC, AB; *d* ergo D metitur AC—AB (BC;) proinde AB ⊔ BC. Q. E. D. d 3.ax.10.

*Coroll.*

Hinc etiam, ſi tota magnitudo ex duabus compoſita, commenſurabilis ſit alteri ipſarum, eadem & reliquæ commenſurabilis erit.

## PROP. XVII.



*Si duæ magnitudines incommenſurabiles* AB, BC *componantur, & tota magnitudo* AC *utrique ipſarum* AB, BC *incommenſurabilis erit: Quod ſi tota magnitudo* AC *uni ipſarum* AB *incommenſurabilis fuerit, & quæ à principio magnitudines* AB, BC *incommenſurabiles erunt.*

1. *Hyp.*

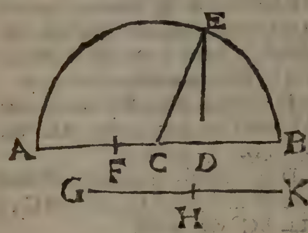
1. *Hyp.* Si fieri poteſt, ſit D ipſarum A C
a 3 ax.10. A B communis menſura. *a* ergo D metitu
b 1.def.10. AC − AB (BC.) *b* ergo AB ⊐⊏ BC, contra
Hypoth.

c 16, 10. 2. *Hyp.* Dic A B ⊐⊏ B C. *c* ergo A C ⊐⊏
AB, contra Hypoth.

*Coroll.*

Hinc etiam, ſi tota magnitudo ex duabu compoſita, incommenſurabilis ſit alteri ipſarum eadem & reliquæ incommenſurabilis erit.

## PROP. XVIII.

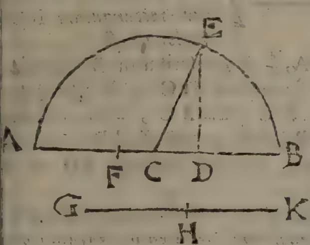



*Si fuerint duæ rectæ lineæ inæquales* A B, G K; *quartæ autem parti quadrati, quod fit à minori* G K *æquale parallelogrammum* ADB *ad majorem* AB *applicetur, deficiens figura quadrata, & in partes* AD, DB *longitudine commenſurabiles ipſam dividat; major* A B *tanto plus poterit quam minor* GK, *quantum eſt quadratum rectæ lineæ* F D *ſibi longitudine commenſurabilis.* *Quod ſi major* A B *tanto plus poſſit, quam minor* G K, *quantum eſt quadratum rectæ lineæ* F D *ſibi longitudine commenſurabilis; quartæ autem parti quadrati, quod fit à minori* GK, *æquale parallelogrammum* A D B *ad majorem* A B *applicetur, deficiens figura quadrata, in partes* AD, DB *longitudine commenſurabiles ipſum dividet.*

a 10. 1. *a* Biſeca GK in H; & *b* fac rectang. ADB =
b 28 6. GHq: abſcinde AF = DB. Eſtque ABq *c* =
c 8. 2. 4 ADB *d* (4 GHq, vel GKq) + FDq. Iam
d conſtr. & 4. 2. primo

rimo, Si AD ⊔⊔ DB, erit AB *e* ¯⊔⊔ DB *e* ¯⊔⊔ e 16. 10.
DB *f* (AF + DB, vel AB − FD) *g* ergo f *conſtr.* g *cor.* 16 10
B ⊔⊔ FD. Q. E. D. Sin ſecundo; AB ¯⊔⊔ h *cor* 16 10.
D, *h* erit ideo AB ⊔⊔ AB − FD (2 DB) k 12. 10.
ergo AB ⊔⊔ DB. *l* quare AD ⊔⊔ DB. l 16 10.
Q. E. D.

PROP. XIX.



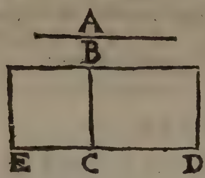

*Si fuerint duæ rectæ lineæ inæquales, AB, GK; quartæ autem parti quadrati, quod ſit à minore GK, æquale parallelogrammum ADB ad majorem AB applicetur, deficiens figura quadrata; & in partes incommenſurabiles longitudine AD, DB, ipſam AB dividat; major AB tanto plus poterit, quam minor GK, quantum eſt quadratum rectæ lineæ FD, ſibi longitudine incommenſurabilis. Quod ſi major AB tanto plus poſſit, quam minor GK, quantum eſt quadratum rectæ lineæ FD ſibi longitudine incommenſurabilis; quartæ autem parti quadrati, quod ſit à minore GK, æquale parallelogrammum ADB ad majorem AB applicetur, deficiens figura quadrata; in partes longitudine incommenſurabiles AD, DB ipſam AB dividet.*

Facta puta, & dicta eadem, quæ in præcedenti. Itaque primo, Si AD ¯⊔⊔ DB, *a* erit propterea AB ¯⊔⊔ DB; *b* quare AB ¯⊔⊔ 2 DB (AB − FD) *c* ergo AB ¯⊔⊔ FD. Q. E. D. a 17. 10. b 13. 10. c *cor.* 17 10.

Secundo, Si AB ¯⊔⊔ FD; *c* ergo AB ¯⊔⊔ AB − FD (2 DB;) *d* quare AB ¯⊔⊔ DB, & proinde AD ¯⊔⊔ DB. Q. E. D. d 13. 10. e 17. 10.

PROP.

## Prop. XX.



*Quod ſub rationalibus longitudine commenſurabilibus rectis lineis* BC, CD, *ſecundum aliquem prædictorum modorum, continetur rectangulum* BD, *rationale eſt.*

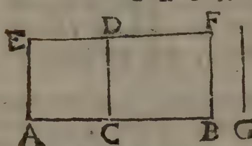
a 46. 1. b 1. 6. c *hyp.* d 10. 10. e *hyp* & 9. *def.* 10. f 12. 10.

Exponatur A, ρ̊. & a deſcribatur BE quadratum ex BC. Quoniam DC. CE (BC) b :: BD. BE. & DC c ⊐⊏ BC ; d erit rectang. BD ⊐⊏ quad. BE. ergo quum quad. BE e ⊐⊏ Aq; f erit BD ⊐⊏ Aq. proinde rectang. BD eſt ρ̊. Q. E. D.

Not. *Tria ſunt genera linearum rationalium inter ſe commenſurabilium. Aut enim duarum linearum rationalium longitudine inter ſe commenſurabilium altera æqualis eſt expoſitæ rationali; aut neutra rationali expoſitæ æqualis eſt, longitudine tamen ei utraque eſt commenſurabilis; aut denique utraque expoſitæ rationali commenſurabilis eſt ſolum potentia. Hi ſunt modi illi, quos innuit præſens theorema.*

In numeris, ſit BC, $\sqrt{8}$ ($2\sqrt{2}$) & CD, $\sqrt{18}$ ($3\sqrt{2}$,) erit rectang. BD $= \sqrt{144} = 12$.

## Prop. XXI.



*Si rationale* DB *ad rationalem* DC *applicetur, latitudinem* CB *efficit rationalem, & ei* DC *ad quam applicatum eſt* DB, *longitudine commenſurabilem.*

a 1. 6. b *hyp.* c *ſch.* 12. 10. d 10. 10.

Exponatur G, ρ̊. & deſcribatur DA quadratum ex BC. quoniam BD. DA a :: BC. CA; atque, BD DA b ſunt ρ̊æ, c ideoque ⊐⊏ ; d erit

BC

⊏⊥ CA. at CD (CA) *b* eſt ρ̇. *e* ergo BC ρ̇. Q. E. D. *e ſch.* 12. 10.

In numeris, ſit rectang. DB, 12; & DC, √ 8. t CB, √ 18. atqui √18 = 3 √ 2. & √ 8 = 2 √ 2.

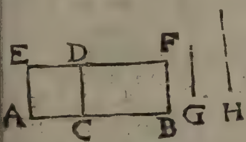
## *LEMMA.*



*Duas rectas rationales potentia ſolum commenſurabiles invenire*

Sit A expoſita ρ̇. *a* Sume B ⊤ A, *b* & C ⊤ B. liquet B, & C eſſe quæſitas. a 11. 10. b *ſch.* 12. 10.

## PROP. XXII.




*Quod ſub rationalibus* DC, CB *potentia ſolum commenſurabilibus rectis lineis continetur rectangulum* DB, *irrationale eſt; & recta linea* H *ipſum potens, irrationalis; vocetur autem Media.*

Sit G expoſita ρ̇. & deſcribatur DA quadratum ex DC; ſitque Hq = DB. Quoniam AC. CB *a* :: DA. DB. *b* atque AC ⊏⊥ CB, *c* erit DA ⊏⊥ DB (Hq). *d* atqui Gq ⊏⊥ DA. *e* ergo Hq ⊏⊥ Gq. *f* ergo H eſt ρ̇. Q. E. D. vocetur autem Media. quia AC. H :: H. CB.

a 1. 6.
b *hyp.*
c 10. 10.
d *hyp. & 9. def.* 10.
e 13. 10.
f 11. 10.

In numeris, ſit DC, 3; & CB, √ 6. erit rectangulum DB (Hq) √ 54. quare H eſt υ√ 54.

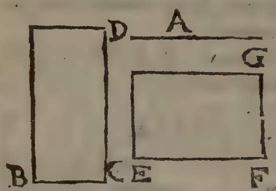
Mediæ nota eſt μ, Medii vero μν; pluraliter μα.

## *SCHOL.*

Omne rectangulum, quod poteſt contineri ſub duabus rectis rationalibus potentia ſolum commenſurabilibus, eſt Medium; quamvis contineatur ſub duabus rectis irrationalibus: atque omne

omne Medium poteſt contineri ſub duabus rect[is] rationalibus potentia tantum commenſurabi[li]bus, ut exemp. gr. $\sqrt{24}$ eſt $\mu\nu$. quia continet[ur] ſub $\sqrt{3}$, & $\sqrt{8}$, qui ſunt $\dot\rho$ ⊐. etſi poſſet co[n]tineri ſub $v\sqrt{6}$, & $v\sqrt{96}$ irrationalibus; na[m] $\sqrt{94} = v\sqrt{576} = v\sqrt{6}$ in $v\sqrt{96}$.

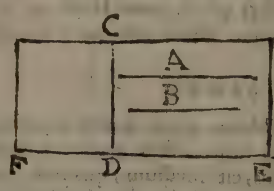
## PROP. XXIII.



*Quod (BD) à media A fit, a[d] rationalem B[C] applicatum, latitudinem CD r[a]tionalem efficit & ei BC, a[d] quam applicatu[m] eſt BD longitudine incommenſurabilem.*

a ſch 22. 10. Quoniam A eſt $\mu$, ᵃ erit Aq rectangulo al[i]
b 1 ax. 1. cui (EG) æquale contento ſub EF, & F[G]
c 14. 6. $\dot\rho$ ⊐. ᵇ ergo BD = EG. ᶜ quare BC. EF :: FG
d 22. 6. CD. ᵈ ergo BCq. EFq :: FGq. CDq. ſed BCq
e hyp. & EFq ᵉ ſunt $\dot\rho\alpha$, ᶠ ideoque ⌐⊐. ᵍ ergo FGq ⌐⊐
f ſch. 12 10.
g 10. 10
h ſch. 12 10. CDq. Ergo quum FG ſit $\rho$, ʰ erit CD $\dot\rho$. Por-
k 1. 6. ro, quia EF. FG ᵏ :: EFq. EG (BD); o[b]
l 10. 10. EF ⌐⊐ FG, ˡ erit EFq ⌐⊐ BD. verum EF[q]
m ſch. 12. 10. ᵐ ⌐⊐ CDq. ⁿ ergo rectang. BD ⌐⊐ CDq
n 13. 10.
o 1. 6. quum igitur CDq. BD ᵒ :: CD. BC. ᵖ erit CD
p 10. 10. ⌐⊐ BC. ergo, &c.

## PROP. XXIV.



*Mediæ A commenſurabili[s] B, media eſt.*

a 11. 6. Ad CD ᵃ fac rectang. CE = Aq; ᵃ & rectang. CF = Bq. Quoniam

b hyp. Aq (CE) eſt $\mu\nu$, ᵇ & CD $\rho$, ᶜ erit latitudo
c 23. 10 DE

E $\dot{\rho}$ ⊓ CD. Quoniam vero CE. CF *d* :: d 1. 6.
D. DF, & CE *e* ⊔ CF, *f* erit ED ⊔ DF. e *hyp.* f 10. 10.
ergo DF eſt $\dot{\rho}$ ⊔ CD. *h* ergo rectang. CF g 12. & 13 10.
Bq) eſt $\mu\nu$. & proinde B eſt $\mu$. Q. E. D. h 22. 10.

Nota *quod ſignum* ⊓ *plerumque valet poten-a tantum commenſurabile, ut in hac demonſtratio-e, & in præced. &c. quod intellige, ut ex uſu erit, juxta citationem.*

*Coroll.*

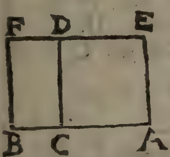
Hinc liquet ſpatium medio ſpatio commenſu-abile medium eſſe.

*LEMMA.*

*Duas rectas medias* A, B *longitudine commenſurabiles; item duas* A, C *potentia tantum commenſurabiles invenire.*

*a* Sit A $\mu$ quævis; ſume B ⊔ A; *c* & C ⊓ A. a lem 22. 10. & 13. 6 b 2. lem. 10. 10. c 3 lem. 10. 10.
Factum eſſe liquet.

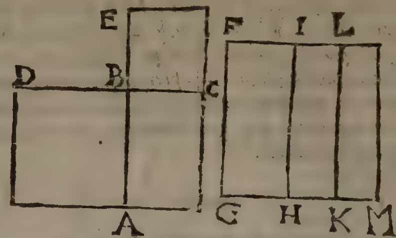
PROP. XXV.



*Quod ſub* DC, CB *mediis longitudine commenſurabilibus rectis lineis continetur rectangulum* DB, *medium eſt.* d *conſtr.* & 24. 10.

Super DC conſtruatur quadratum DA. Quoniam a 1. 6.
AC. (DC) CB *a* :: DA. DB. & DC ⊔ CB; b 10. 10.
*b* erit DA ⊔ DB. *c* ergo DB eſt $\mu\nu$. Q. E. D. c 24. 10.

## PROP. XXVI.




*Quod ſub mediis potentia tantum commenſurabilibus rectis lineis* A B, BC *continetur rectangulum* AC, *vel rationale eſt, vel medium.*

Super rectas AB, BC *a* deſcribe quadrata AD, CE. atque ad FG ρ̇, *b* fac rectangula FH = AD, *b* & IK = AC, *a b* & LM = CE.

Quadrata AD, CE, hoc eſt, rectangula FH, LM *c* ſunt μα, & ⊥; ergo eandem habentes rationem GH, KM ſunt *d* ρ̇, & *e* ⊥. *f* ergo GH×KM eſt ρ̇ν. atqui quia AD, AC, CE hoc eſt FH, IK, LM *g* ſunt ∺; & *h* proinde GH, HK, KM etiam ∺, *k* erit HKq = GH × KM; *l* ergo HK eſt ρ̇, vel ⊥, vel ⊤ IF (GF); ſi ⊥, *m* ergo rectang. IK vel AC eſt ρ̇ν. Sin ⊤. *m* ergo AC eſt μν. Q. E. D.

a 46. 1.
b Cor. 16. 6.
c hyp. & 24. 20.
d 23. 10.
e 10. 10.
f 20. 10.
g ſch. 22. 6.
h 1. 6.
k 17. 6.
l 12. 10.
m 20. 10
m 22. 10.

### LEMMA A.




*Si* A, & E *ſint* ⊤ *tantum;*

*Erunt primo,* Aq, Eq, Aq + Eq, Aq — Eq *a* ⊥.

*Erunt ſecundo,* Aq, Eq, Aq + Eq, Aq — Eq ⊤ AE, & 2 AE. Nam A. E *b* :: Aq. AE *b* :: AE. Eq. ergo cum A *c* ⊤ E. *d* erit Aq ⊤ AE, *e* & 2 AE. item Eq *d* ⊤ A E, *e* & 2 AE. *c* quare cum Aq + Eq ⊥ Aq, & Eq; & Aq — Eq ⊤ Aq, &

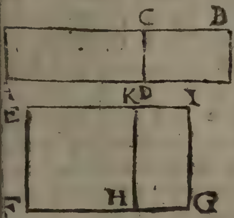
a hyp. & 16. 10.
b 1, 6.
c hyp.
d 10. 10.
e 14. 10.

Eq

[.] ferunt Aq+Eq, *f* & Aq−Eq ⊓ AE, & AE. f 14. 10.

*Hinc erunt tertio*, Aq, Eq, Aq+Eq, Aq−Eq, AE *g* ⊓ Aq+Eq+2AE; & Aq+Eq−2AE. & Aq+Eq+2AE ⊓ Aq+Eq−2AE. (Q. A−E.) g 14, 16, & 17 10. h cor. 7. 10.

## PROP. XXVII.

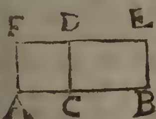


*Medium* AB *non ſuperat medium* AC *rationali* DB.

Ad EF ϱ *a* fac EG = AB, *a* & EH = AC. Rectangula AB, AC, hoc eſt, EG, EH *b* ſunt μα, *c* ergo FG, & FH ſunt ϱ ⊤ EF. a cor. 16. 6. b hyp. c 23. 10.

[It]aque ſi KG, *d* id eſt DB ſit ϱν, *e* erit HG ⊓ K; *f* quare HG ⊤ FH. *g* ergo FGq ⊓ FHq. [Se]d FH eſt ϱ. *h* ergo FG eſt ϱ. verum prius [er]at FG ϱ. Quæ repugnant. d 3. ax. 1. e 21. 10. f 13. 10. g lem. 16 10. h ſch. 12. 10.

### SCHOL.



1. *Rationale* AE *ſuperat rationale* AD *rationali* CE.

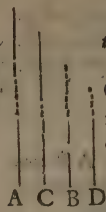
Nam AE *a* ⊓ AD; *b* ergo AE ⊓ CE. *c* quare CE eſt ϱν. Q. E. D. a hyp. b cor. 16. 10 c ſch. 12. 10



2. *Rationale* AD *cum rationali* CF *facit rationale* AF.

Nam AD *a* ⊓ CF; *b* quare AF ⊓ AD, & CF. *c* proinde AF eſt ϱν. Q. E. D. a ſch 12. 10. b 16 10 c ſch. 12. 10

## PROP. XXVIII.



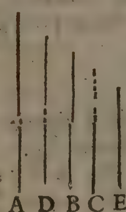
*Medias invenire* (C, *&* D) *quæ rationale* CD *contineant*.

a lem. 21. 10.
b 13. 6.
c 12. 6.
d 22. 10.
e conſt.
f 10. 10.
g 24 10.
h 17. 6.
h ſch. 12. 10.

*a* Sume A, & B $\acute{\rho}$ ⊤. *b* fac A. C :: C. B. *c* atque A. B :: C. D. Dico factum. Nam A B (C q) *d* eſt $\mu\nu$; *d* unde C eſt $\mu$. quum vero A. B *e* :: C. D, *f* erit C ⊤ D. *g* ergo D eſt $\mu$. porro permutando A. C :: B. D. *e* hoc eſt C. B :: B. D. *h* ergo Bq = CD. atqui Bq *e* eſt $\acute{\rho}\nu$. *h* ergo CD eſt $\acute{\rho}\nu$. Q. E. F.

In numeris, ſit A, $\surd$ 2; & B, $\surd$ 6. ergo C eſt $v\surd$ 12. fac $\surd$ 2. $\surd$ 6 :: $v\surd$ 12. D. vel $v\surd$ 4. $v\surd$ 36 :: $v\surd$ 12. D. erit D, $v\surd$ 108. atqui $v\surd$ 12 in $v\surd$ 108 = $v\surd$ 1296 = $\surd$ 36 = 6. ergo CD eſt 6. item C. D :: 1. $\surd$ 3. quare C ⊤ D.

## PROP. XXIX.



*Medias invenire potentia tantum commenſurabiles* D, *&* E, *quæ medium* DE *contineant*.

a lem 21. 10.
b 13. 6.
c 12. 6.
d 17. 6.
e 22. 10.
f conſtr.
g 10. 10.
h 24 10
k conſtr. & cor. 4 5.
l 16. 6.
m 22. 6.

*a* Sume A, B, C $\acute{\rho}$ ⊤. Fac A. D *b* :: D. B. *c* & B. C :: D. E. Dico factum.

Nam AB *d* = Dq & AB *e* eſt $\mu\nu$. ergo D eſt $\mu$. & B *f* ⊤ C. *g* ergo D ⊤ E. *h* ergo E eſt $\mu$. porro B. C *f* :: D. E, & permutando B. D :: C. E *k* hoc eſt. D. A :: C. E. *l* ergo DE = AC. Sed AC *m* eſt $\mu\nu$. ergo DE eſt $\mu\nu$. Q. E. D.

In numeris ſit A, 20; & B, $\surd$ 200; & C, $\surd$ 80. Ergo D eſt $\surd\surd$ 80000; & E $v\surd$ 12800. Ergo DE = $\surd\surd$ 1024000000 = $\surd$ 32000. & D. E :: $\surd$ 10. 2. quare D ⊤ E.

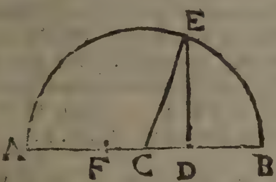
*SCHOL.*

| | |
|---|---|
| , 6. | C, 12. |
| , 4. | D, 8. |
| B, 24. | CD, 96. |
| , 6. | C, 5. |
| , 4. | D, 8. |
| B, 24. | CD, 40. |

*Invenire duos numeros planos ſimiles vel diſsimiles.*

Sume quoſcunque quatuor numeros proportionales, A.B :: C.D. liquet AB, & CD eſſe ſimiles planos. Planos autem diſſimiles quotcunque reperies ope ſcholii 27. 8.

*LEMMA.*




1. *Duos numeros quadratos* (DEq & CDq) *invenire, ita ut compoſitus ex ipſis* (CEq) *quadratus etiam ſit.*

Sume AD, DB numeros planos ſimiles (quorum ambo pares ſint, vel ambo impares) nimirum AD, 24. & DB, 6. Horum ſumma, (AB) eſt 30; differentia (FD) 18, cujus ſemiſſis (CD) eſt 9. *a* Habent vero plani ſimiles AD, DB unum medium numerum proportionalem, nempe DE. patet igitur ſingulos numeros CE, CD, DE rationales eſſe; proinde CEq (*b* CDq + DEq) eſt numerus quadratus requiſitus. a 18. 8. b 47. 1.

Facile itaque invenientur duo numeri quadrati, quorum exceſſus ſit quadratus, vel non quadratus numerus. nempe ex eadem conſtructione, erit CEq − CDq = DEq. c 3 ax. 1.

Quod ſi AD, DB ſint numeri plani diſſimi-

 les,

les, non erit media proportionalis (DE) nu-
merus rationalis; proinde quadratorum CEq
CDq exceſſus (DEq) non erit numerus qua-
dratus.

## LEMMA 2.

2. *Duos numeros quadratos* B, C *invenire, ita ut compoſitus ex ipſis* D, *non ſit quadratus. item quadratum numerum* A *dividere in duos numeros* B, C *non quadratos.*

A, 3. B, 9. C, 36. D, 45.

1. Sume numerum quemlibet quadratum B ſitque C = 4B; & D = B + C. Dico factum.

Nam B eſt Q. ex conſtr. item quia B. C :: 1. 4 :: Q. Q. *a* erit C etiam quadratus. Sed quoniam B + C. (D) C :: 5. 4 :: non Q. Q. *b* non erit D numerus quadratus. Q. E. F.

a 24. 8.
b cor. 24 8.

A, 36. B, 24. C, 12. D, 3. E, 2. F, 1.

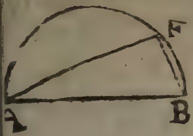
2. Sit A numerus quivis quadratus. Accipe D, E, F numeros planos diſſimiles, ſitque D = E + F. fac D. E :: A. B. & D. F :: A. C. Dico factum.

Nam quia D. E + F :: A. B + C. & D = E + F *a* erit A = B + C. Iam dic B quadratum eſſe *b* ergo A & B, & *c* proinde D & E, ſunt numeri plani ſimiles, contra Hypoth. idem abſurdum ſequetur, ſi C dicatur quadratus. ergo &c.

a 14. 5.
b 21. def 7.
c 26, 8.

PROP.

PROP. XXX.



C .... E ..... D

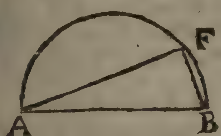
*Invenire duas rationales* AB, AF *potentia tantum commenſurabiles, ita ut major* AB *plus poſsit, quam minor* AF, *quadrato rectæ lineæ* BF *longitudine ſibi commenſurabilis.*

Exponatur AB, ρ'. *a* Sume CD, CE numeros uadratos, ita ut CD − CE (ED) ſit non Q. Fiatque CD. ED :: ABq. AFq. In circulo per AB diametrum deſcripto *c* aptetur AF, ucaturque BF. Sunt AB, AF, quas petis. a 1.*lem.*29. 10. b 3.*lem.*10. 10. c 1.4.

Nam ABq. AFq *d* :: CD. ED. *e* ergo ABq ⊏⊐ AFq. verum AB eſt ρ'. *f* ergo AF eſt ρ'. ſed uia CD eſt Q: at ED non Q: *g* erit AB ⊏⊐ AF. porro, ob ang. *h* rectum AFB, eſt ABq = AFq + BFq; cum igitur ABq. AFq :: CD. ED. per converſionem rationis erit ABq. BFq::CD.CE::Q.Q. *l* ergo AB ⊏⊐ BF. Q.E.F. d *conſtr.* e 6. 10. f *ſch.*12.10. g 9. 10. h 31.3. k 47. 1. l 9. 10.

In numeris; ſit AB, 6; CD, 9, CE, 4; quare ED, 5. Fac 9. 5 :: 36. (Q: 6) AFq. erit AFq 20. proinde AF √ 20. ergo BFq = 36 − 20 = 16. quare BF eſt 4.

PROP. XXXI.



C ......... E .... D

*Invenire duas rationales* AB, AF *potentia tantum commenſurabiles, ita ut major* AB *plus poſsit, quam minor* AF, *quadrato rectæ lineæ* BF *ſibi longitudine incommenſurabilis.*

Exponatur AB, ρ'. *a* accipe numeros CE, ED quadratos, ita ut CD = CE + ED ſit non Q. & in reliquis imitare conſtructionem præcedentis. Dico factum. a 2.*lem.*29. 10.

 Nam,

Nam, ut ibi, AB, AF ſunt ῥ ⊓. item ABq.
BFq :: CD. ED. ergo cum CD ſit non Q
b9. 10. *b* erunt AB, BF ⊔. Q. E. F.

In numeris, ſit AB, 5. CD, 45. CE = 36
ED = 9. Fac 45. 9 :: 25 (ABq.) 5 (AFq)
ergo AF = √ 5. proinde BFq = 45 − 25 =
20. quare BF = √ 20.

## PROP. XXXII.

A ————————
B ———————
C ——————
D ————

*Invenire duas medias C, D potentia tantum commenſurabiles; quæ rationale CD contineant, ita ut major C plus poſſit, quam minor D quadrato rectæ lineæ ſibi longitudine commenſurabilis.*

a 30. 10. *a* Accipe A, & B ῥ ⊓; ita ut √Aq − Bq ⊔
b 13. 6. A. *b* Fiatque A. C :: C. B, *c* atque A. B :: C.
c 12. 6. D. Dico factum.
d conſtr.
e 21. 10. Nam quia A, & *d* B ſunt ῥ ⊓, *e* erit C (*f* √
f 17. 6. AB) μ. item *g* ideo C ⊓ D. *h* ergo D etiam
g 10. 10. μ. porro quia A. B *d* :: C. D; & permutatim A.
h 24. 10. C :: B. D :: C. B; & Bq *d* eſt ῥν, erit CD
k 17. 6. *k* (Bq) ῥν. Denique quia √ Aq − Bq *d* ⊔
l 15. 10. A, *l* erit √ Cq − Dq ⊔ C. ergo, &c. Sin √
Aq − Bq ⊔ Aq, erit √ Cq − Dq ⊔ C.

In numeris, ſit A, 8; B, √ 48 (√ : 64 − 16)
ergo C = √ AB = υ√ 3072. & D = υ√ 1728
quare CD = υ√ 5308416 = √ 2304.

## PROP. XXXIII.

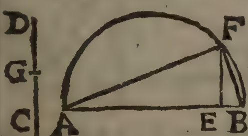
A ————————
D ———————
B ——————
C —————
E ———

*Invenire duas medias D, E potentia ſolum commenſurabiles, quæ medium DE contineant, ita ut major D plus poſſit, quam minor E, quadrato rectæ lineæ ſibi longitudine commenſurabilis.*

Sume

ᵃ Sume A, & C ρ', ⊥; ita ut √ Aq − Cq ⊔ a 30. 10.
A. ᵇ ſume etiam B ⊥ A, & C; & fac A.D ᶜ :: b lem. 21. 10.
D. B ᵈ :: C.E. Erunt D, & E quæſitæ. c 13. 6.
d 12. 6.
Nam quoniam A, & C ᵉ ſunt ρ', ᵉ & B ⊥ e conſtr.
A & C, ᶠ erit B ρ', & D (√ AB) ᵍ erit μ. f ſch. 12. 10.
Quia vero A. D :: C. E. erit permutando A. g 22. 10.
C :: D. E. ergo cum A ⊥ C, ʰ erit D ⊥ E. h 10. 10.
ergo E eſt μ. porro, ˡ quia D. B :: C. E; ˡ & k 24. 10.
BC eſt μν, etiam DE ei ᵐ æquale eſt μν. deniq; l 22. 10.
propter A. C :: D. E. ᵉ quia √ Aq − Cq ⊔ m 16. 6
A, ⁿ erit √ Dq − Eq ⊔ D. ergo, &c. Sin √ n 15. 10.
Aq − Cq ⊔ A, erit √ Dq − Eq ⊔ Eq.
In numeris, ſit A, 8; C, √ 48; B, √ 28. erit
D ᵒ √ 3072; & E ᵘ √ 588. quare D. E :: 2. √ 3.
& DE = √ 1344.

## PROP. XXXIV.




*Invenire duas rectas lineas* AF, BF *potentia incommenſurabiles, quæ faciant compoſitum quidem ex ipſarum quadratis rationale, rectangulum vero ſub ipſis contentum, medium.*

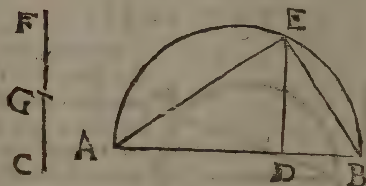
ᵃ Reperiantur AB, CD ρ' ⊥; ita ut √ ABq − a 31. 10.
CDq ⊔ AB. ᵇ biſeca CD in G. ᶜ fac rectang. b 10. 1.
AEB = GCq. Super AB diametrum duc ſe- c 28. 6.
micirculum AFB. erige perpendicularem EF. d 12. 6.
duc AF, BF. Hæ ſunt quæ indagandæ erant. e cor. 8. 6. & 17. 6.
Nam AE. BE ᵈ :: BA x AE. AB x BE. Sed f 7. 5.
BA x AE ᵉ = AFq; ᵉ & AB x BE = FBq. ᶠ ergo g 19. 10.
AE. EB :: AFq. FBq. ergo cum AE ᵍ ⊔ h 10. 10.
EB, ʰ erit AFq ⊔ FBq. Quinetiam ABq k 31. 3. & 47. 1.
(ᵏ AFq + FBq) ˡ eſt ρ'ν. denique EFq ˡ = l conſtr.
AEB ˡ = CGq. ᵐ ergo EF = CG. ergo CD x m 1. ax. 1.
AB = 2 EF x AB. atqui CD x AB ⁿ eſt μν. n 22. 10.
ᵒ ergo AB x EF, ᵖ vel AF x FB, eſt μν. Q. E. D. o 24. 10.
p ſch. 22. 6.

*Explicatio per numeros.*

Sit AB, 6. CD, $\sqrt{12}$. quare CG $= \sqrt{\frac{12}{4}} = \sqrt{3}$. Eſt vero AE $= 3 + \sqrt{6}$. & EB $= 3 - \sqrt{6}$. & unde AF erit $\sqrt{: 18 + 216}$. Et FB, $\sqrt{: 18 - \sqrt{216}}$. item AFq + FBq eſt 36, & AF FB $= \sqrt{108}$.

Cæterum AE invenitur ſic. Quia BA (6) AF :: AF. AE; erit 6 AE = AFq = AEq + 3 (EFq.) ergo 6 AE − AEq = 3. pone 3 + e = AE. ergo 18 + 6e − 9 − 6e − ee, hoc eſt 9 − ee = 3. vel ee = 6. quare e $= \sqrt{6}$ proinde AE $= 3 + \sqrt{6}$.

## PROP. XXXV.




*Invenire duas rectas lineas* AE, EB *potentia incommenſurabiles, quæ faciant compoſitum quidem ex ipſarum quadratis medium, rectangulum vero ſub ipſis contentum, rationale.*

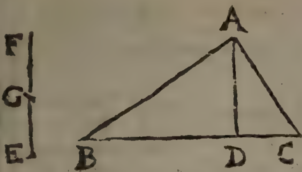
a 32. 10. *a* Sume AB, & CF μ ⊤, ita ut AB × CF ſit ῥν, atque $\sqrt{}$ ABq − CFq ⊓⊔ AB. & reliqua fiant, ut in præcedenti. erunt AE, EB, quas petis.

Nam, ut iſthic oſtenſum eſt, AEq ⊓⊔ EB. *c* item ABq (AEq + EBq) eſt μν. & denique AB × CF *b* eſt ῥν, idcirco & AB × DE, *d* hoc eſt, AE × EB, eſt ῥν. ergo, &c.

b conſtr.
c ſchol. 12. 10
d ſchol. 22. 6.

PROP.

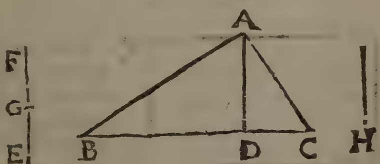
PROP. XXXVI.




*Invenire duas rectas lineas BA, AC potentia incommenſurabiles, quæ faciant & compoſitum ex ipſarum quadratis medium, & rectangulum ſub ipſis comprehenſum medium, incommenſurabileque compoſito ex ipſarum quadratis.*

*a* Accipe BC & EF μ ⊤; ita ut BC x EF ſit μν. & √ Bcq − EFq ⊔ BC. & reliqua fiant, ut in præcedentibus. Erunt BA, AC exoptata. Nam, ut prius, BAq ⊔ ACq; item BAq + ACq eſt μν. & BA x AC eſt μν. Denique BC *b* ⊔ EF, atque *c* ideo BC ⊔ EG; eſtque BC. EG *d* :: BCq. BC x EG, (BC x AD, vel BA x AC.) *e* ergo BCq (ABq + ACq) ⊔ BA x AC. ergo, &c. a 33. 10. b conſtr. c 13. 10. d 1. 6. e 14. 10.

*Schol.*




*Invenire duas medias longitudine & potentia incommenſurabiles.*

*a* Sume BC μ. ſitque BA x AC μν, & ⊔ BCq (BAq + ACq.) *b* Fac BA. H :: H. AC. Sunt BC, & H μ ⊤. Nam BC eſt μ. *a* & BA x AC (*c* Hq) eſt μν. quare H eſt etiam μ. a 36. 10. b 13. 6. c 17. 6.

d 24.10. μ. d item B A x A C ⊔⊔ B Cq; ergo Hq ⊔⊔ BCq. ergo, &c.

*Principium ſenariorum per compoſitionem.*

### PROP. XXXVII.




*Si duæ rationales* AB, BC *potentia tantum commenſurabiles componantur, tota* AC *irrationalis eſt; vocetur autem ex binis nominibus.*

a hyp. b lem.26.10. c 11.def.10. Nam quia A B a ⊔⊔ B C, b erit A Cq ⊔⊔ ABq. Sed AB a eſt ρ'. c ergo AC eſt ρ'. Q. E. D.

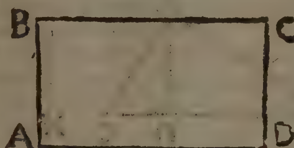
### PROP. XXXVIII.




*Si duæ mediæ* A B, B C *potentia tantum commenſurabiles componantur, quæ rationale contineant, tota* AC *irrationalis eſt; vocetur autem ex binis mediis prima.*

a hyp. b lem.26.10 c 11.def.10. Nam quoniam AB a ⊔⊔ BC, b erit ACq ⊔⊔ AB x BC, ρ'r. c ergo AC eſt ρ'. Q. E. D.

### LEMMA.

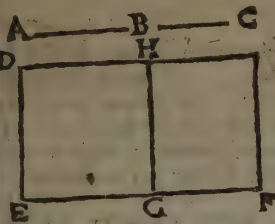



*Quod ſub linea rationali* AB, *& irrationali* B C *continetur rectangulum* A C, *irrationale eſt.*

a hyp. b 21.10. Nam ſi rectang. A C dicatur ρ'ν; quum AB ſit ρ'; b erit latitudo B C etiam ρ'. contra Hyp.

PROP.

PROP. XXXIX.



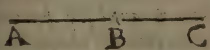

*Si duæ mediæ AB, BC potentia tantum commenſurabiles componantur, quæ medium contineant; tota AC irrationalis erit; vocetur autem ex binis mediis ſecunda.*

Ad expoſitam DE ρ^c^ *a* fac rectang. DF = ACq; *b* & DG = ABq + BCq.

Quoniam ABq *c* ⊔ BCq, *d* erit ABq + BCq, hoc eſt DG ⊔ ABq; ſed ABq *e* eſt μν. *e* ergo DG eſt μ. verum rectang. ABC ponitur μν; *e* ideoque 2 ABC (*f* HF) eſt μν; *g* ergo EG, & GF ſunt ρ^c^. quia vero DG *h* ⊔ HF; atque DG. HF :: *e* EG. GF *l* erit EG ⊔ GF. *m* ergo tota EF eſt ρ'. *n* quare rectang DF eſt ρ^c^ν. *o* ergo √ DF, id eſt AC, eſt ρ'. Q.E.D.

a cor 16 6. b 47. 1. & 11. 6. c hyp. d 16. 10. e 14. 10. f 4. 2. g 23. 10. h lem. 26. 10. k 1. 6. l 10. 10. m 37. 10. n lem. 38, 10. o 11. def. 10.

PROP. XL.



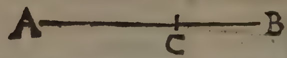

*Si duæ rectæ lineæ AB, BC potentia tantum commenſurabiles componantur, quæ faciant compoſitum quidem ex ipſarum quadratis rationale, quod autem ſub ipſis continetur, medium; tota recta linea* AC, *irrationalis erit: vocetur autem major.*

Nam quia ABq + BCq *a* eſt ρ^c^ν, & *b* ⊔ 2 ABC *c* μν, & proinde ACq (*d* ABq + BCq + 2 ABC) *e* ⊔ ABq + BCq ρν, *f* erit AC ρ'. Q.E.D.

a hyp. b ſch. 12. 10. c hyp. & 24. 10. d 4. 2. e 17. 10. f 11. def. 10.

PROP.

## PROP. XLI.

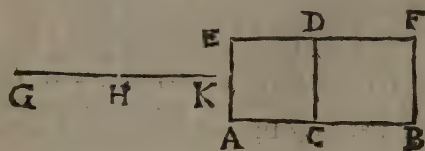


*Si duæ rectæ lineæ* AC, CB *potentia incommenſurabiles componantur, quæ faciant compoſitum quidem ex ipſarum quadratis medium, quod autem ſub ipſis continetur, rationale; tota recta linea* AB *irrationalis erit: vocetur autem rationale ac medium potens.*

a hyp. & ſch.12.10.
b ſch.12.10.
c hyp.
d 17.10.
e 11.def.10.

Nam 2 rectang. ACB, *a* $\rho'\nu$ *b* ⊏⊐ ACq + CBq *c* $\mu\nu$. *d* ergo 2 ACB *d* ⊏⊐ ABq. quare *e* AB eſt $\varrho'$. Q. E. D.

## PROP. XLII.



*Si duæ rectæ lineæ* GH, HK *potentia incommenſurabiles componantur, quæ faciant & compoſitum ex ipſarum quadratis medium, & quod ſub ipſis continetur medium, incommenſurabileque compoſito ex quadratis ipſarum; tota recta linea* GK *irrationalis erit: vocetur autem bina media potens.*

a hyp.
b 23.10.
c 4.2.
d 1.6.
e 10.10.
f 37.10.
g lem 38.10.
h 11.def.10.

Ad expoſitam FB $\rho'$, fiant rectang. AF = GKq, & CF = GHq + HKq. Quoniam GHq + HKq (CF) *a* eſt $\mu\nu$; latitudo CB *b* erit $\rho'$. Item quia 2 rectang. GHK (*c* AD) *a* eſt $\mu\nu$, etiam AC *b* erit $\rho'$. Porro quia rectang. AD *a* ⊏⊐ CF, *d* atque AD. CF :: AC. CB, *e* erit AC ⊏⊐ CB. *f* Quare AB eſt *g* $\rho'$. ergo rectang. AF, id eſt, GKq eſt $\rho'\nu$. *h* proinde GK eſt $\rho'$. Q. E. D.

PROP.

PROP. XLIII.



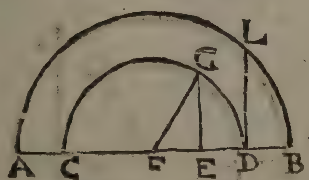

*Quæ ex binis nominibus* A B, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Si fieri poteſt, binomium A B alibi in E ſecetur in alia nomina A E, E B. Liquet A B ſecari utrobique inæqualiter, quia AD ⊓ DB, & AE ⊓ EB.

Quoniam rectangula ADB, AEB *a* ſunt μα; *a* & ſingula ADq, DBq, AEq, EBq ſunt ῥα; *b* adeoque ADq + DBq, *b* & AEq + EBq etiam ῥα, *b* idcirco ADq + DBq − : AEq + EBq. *c* hoc eſt, 2 AEB − 2 ADB eſt ῥν. *d* ergo AEB − ADB ῥν. ergo μν ſuperat μν per ῥν. *e* Q.E.A.

a 37. 10.
b ſch. 27. 10
c ſch. 5. 2.
d ſch. 12. 10
e 27. 10.

PROP. XLIV.



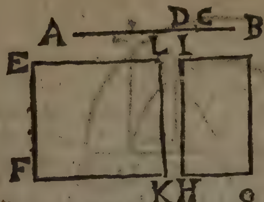

*Quæ ex binis mediis prima* A B, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Puta AB dividi in alia nomina AE, EB. quo poſito, ſingula ADq, DBq, EBq, *a* ſunt μα; *a* & rectangula ADB, AEB, eorumque dupla, ſunt ῥα. *b* ergo 2 A E B − 2 A D B, *c* hoc eſt ADq + DBq − : AEq + EBq eſt ῥν. *d* Q. E. A.

a 38. 10.
b ſch. 27. 10.
c ſch 5. 2.
d 27. 10.

PROP.

## PROP. XLV.

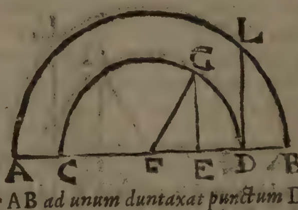



*Quæ ex binis mediis ſecunda* AB; *ad unum duntaxat punctum* C *dividitur in nomina* AC, CB.

Dic alia eſſe nomina AD, DB. Ad expoſitam EF ρ′, fac rectang. EG = ABq. & EH = ACq + CBq; item EK = ADq + DBq.

a 39. 10. Quoniam ACq, CBq *a* ſunt μν ⊓; *b* erit
b 16 & 24. 10. ACq + CBq (EH) μν. *c* ergo latitudo FH
eſt ρ′, *a* quin & rectang. ACB, *d* ideoque 2 ACB
c 23. 10. *e* (IG) eſt μν: *c* ergo HG, eſt etiam ρ′. Cum
d 24. 10. igitur EH *f* ⊓ IG, *g* atque EH. IG :: FH.
e 4. 2. HG; *h* erunt FH, HG ⊓. *k* ergo FG eſt bino-
f lem. 26. 10. mium; cujus nomina FH, HG. Simili argu-
g 1. 6. mento FG eſt bin. cujus nomina FK, KG, contra
h 10. 10. 43. hujus.
k 37. 10.

## PROP. XLVI.




*Major* AB *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

a 40. 10. Concipe alia nomina AE, EB. quo poſito re-
b ſch. 27. 10. ctangula ADB, AEB *a* μα; *a* & tam ADq +
c ſch. 9. 6. DBq, quam AEq + EBq ſunt ρα. *b* ergo ADq
d 27. 10. + DBq − : AEq + EBq, *c* hoc eſt 2 AEB −
2 ADB eſt ρ′ν. *d* Q. F. N.

PROP.

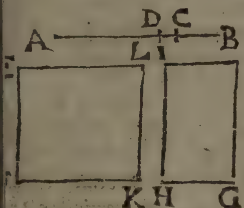
PROP. XLVII.




*Rationale ac medium potens* AB, *ad unum* [illegible]*ntaxat punctum* D *dividitur in nomina* AD, DB.

Dic alia nomina AE, EB. *a* ergo tam AEq a 41. 10.
- EB, quam ADq + DBq ſunt μα. *a* & re-
angula AEB, ADB, ſunt ρτα. *b* ergo 2 AEB b ſch. 27. 10.
2 ADB, *c* hoc eſt, ADq + DBq — : AEq + c ſch 5. 2.
Bq eſt ρτν. Q. E. A. d 27. 10.

PROP. XLVIII.




*Bina media potens* AB, *ad unum duntaxat punctum* C *dividitur in nomina* AC, CB.

Vis AB dividi in alia nomina AD, DB. Ad expoſi-
am EF ρτ, fiant rectang. EG = ABq, & EH =
Cq + CBq, & EK = ADq + DBq. Quo-
iam ACq + CBq, nempe EH, *a* eſt μν, *b* erit a 42. 10.
atitudo FH ρτ. Item quia 2 ACB, *c* hoc eſt, b 23. 10.
G, eſt *a* μν, *b* erit HG etiam ρτ. Ergo cum EH c 4. 2.
⌐⊏ IG, atque EH. IG *d* :: FH. HG, *e* erit d 1. 6.
H ⌐⊏ HG. *f* ergo FG eſt bin. cujus nomina e 10. 10.
H. HG. Eodem modo ejuſdem nomina eruat f 37. 10.
K, KG; contra 43 hujus.

*Definitiones ſecundæ.*

Expoſita rationali, & quæ ex binis nominibus, diviſa in nomina; cujus majus nomen plus poſſit quam minus, quadrato rectæ lineæ ſibi longitudine commenſurabilis;

I. Siquidem majus nomen expoſitæ rationali com-

commenſurabile ſit longitudine, vocetur tota ex binis nominibus prima.

II. Si vero minus nomen expoſitæ rationali longitudine ſit commenſurabile, vocetur ex binis nominibus ſecunda.

III. Quod ſi neutrum ipſorum nominum ſit longitudine commenſurabile expoſitæ rationali, vocetur ex binis nominibus tertia.

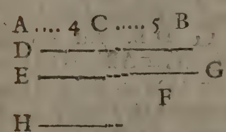
Rurſus, ſi majus nomen plus poſſit quam minus, quadrato rectæ lineæ ſibi longitudine incommenſurabilis;

IV. Si quidem majus nomen expoſitæ rationali commenſurabile ſit longitudine, vocetur ex binis nominibus quarta.

V. Si vero minus nomen, vocetur quinta.

VI. Quod ſi neutrum ipſorum nominum, vocetur ſexta.

## PROP. XLIX.




*Invenire ex binis nominibus primam,* E G.

a ſch.29. 10. *a* Sume AB, AC numeros quadra-
b 1. lem. 10. 10. tos, quorum exceſſus CB non Q. exponatur D $\rho$.
c 3 lem. 10. 10. *b* accipe quamvis E F ⊥ D. *c* fac A B. C B :: EFq. FGq. erit EG bin. 1.

d conſtr. Nam E F *d* ⊥ D. *e* ergo E F $\rho$. *f* item
e 6. def 10. E Fq ⊥ F Gq. *g* ergo F G eſt etiam $\rho$. item
f 6. 10. g ſch. 12. 10 *d* quia EFq. FGq :: AB. CB :: Q. non Q. *h* erit
h 9. 10. E F ⌐⊥ F G. denique quia per converſionem rationis E Fq. E Fq − F Gq :: AB. AC :: Q. Q.
k 9. 10. *k* erit E F ⊥ √ EFq − FGq. *l* ergo E G eſt
l 1. def. 48. 10. bin. 1. Q. E. F.

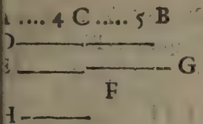
*Explicatio per numeros.*

Sit D, 8. E F, 6. A B, 9. C B, 5. quare cum

9. 5.

: 5 :: 36: 20. erit FG, √.20. proinde EG eſt 6 + √. 20.

## P r o p. L.



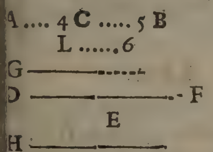

*Invenire ex binis nominibus ſecundam,* E G.

Accipe AB, & AC numeros quadratos, quorum exceſſus CB ſit non Q. Sit D expoſita ρ̇. ſume FG ˉ⊔ D. Fac CB. AB :: FGq. EFq. Erit EG quæſita. Proba ut praecedentem

Nam FG ⊔ D, quare FG eſt ρ̇. item EFq ⊔ FGq. ergo EF eſt etiam ρ̇. item quia FGq. EFq :: CB. AB :: non Q. Q. eſt FG ˉ⊔ EF. denique quia CB. AB :: FGq. EFq, inverſeque AB. CB :: EFq. FGq, erit ut in præcedenti, EF ⊔ √ EFq − FGq. *a* è quibus EG eſt bin. 2. Q. E. F. a 2.def.48.10.

*In numeris,* ſit D, 8; FG 10; AB, 9; CB, 5. erit EF, √ 180. quare EG eſt 10 + 180.

## P r o p. LI.




*Invenire ex binis nominibus tertiã,* DF.

*a* Sume numeros AB, AC quadratos, quorum exceſſus CB non Q. Sitq; L numerus non Q, proxime major quam CB, nempe unitate, vel binario. ſit G expoſita ρ̇. *b* Fac L. AB :: Gq. DEq. *b* & AB. CB :: DEq. EFq. erit DF bin. 3. a ſch.29.10. b 3.lem.10.10.

Nam quia DEq *c* ⊔ Gq, *d* eſt DE ρ̇. item Gq. DEq :: L. AB :: non Q. Q. *e* ergo G ˉ⊔ DE. item quia DEq *e* ⊔ EFq, *d* etiam EF eſt ρ̇. quinetiam quia DEq. EFq :: AB. CB :: Q. non Q. *f* eſt DE ˉ⊔ EF. porro, quia per c conſtr. 10. d ſch.12.10. e 6.10. f 9.10.

conſtr. & ex æquali Gq. EFq :: L.CB :: non Q.
g ſch.27.8. Q. (nam g L, & CB non ſunt ſimiles plani numeri) b erit G etiam ⊓⊔ EF. denique ut in
h 9.10. præced. √ DEq − EFq ⊓⊔ DE. k ergo DF eſt
k 3 def.48.10. bin.3. Q. E. F.

*In numeris*, ſit AB, 9; CB, 5; L, 6; G, 8. erit DE, √ 96 & EF, √ $\frac{480}{9}$. quare DF = √ 96 + √ $\frac{480}{9}$.

## PROP. LII.

A ... 3 C ...... 6 B
G ————-----
D ——————————F
E
H ————

*Invenire ex binis nominibus quartam, DF.*

a ſch.29.10. a Sume quemvis numerum quadratum AB, a quem divide in A C, C B non quadratos. ſit G expoſita ρ'. b accipe DE ⊓⊔
b 2.lem 10. G. c Fac AB. CB :: DEq. EFq. erit DF bin.4.
c 3.lem.10.10. Nam ut in 49. hujus, D F oſtendetur bin. item, quia per conſtr. & converſionem rationis DEq. DEq − EFq :: AB. AC :: Q. non Q.
d 9.10. d erit DE ⊓⊔ √ DEq − EFq. e ergo DF eſt
e 4 def.48 10. bin. 4. Q. E. F.

*In numeris*, ſit G, 8; DE, 6. erit EF √ 24 ergo DF eſt 6 + √ 24.

## PROP. LIII.

A ... 3 C ...... 6 B
G ————-·--
D ————·———— F
E
H F

*Invenire ex binis nominibus quintam, D F.*

Accipe quemvis numerum quadratum AB cujus ſegmenta A C, CB ſint non Q. ſit G expoſita ρ'. ſume EF ⊓⊔ G. fac CB. AB :: EFq. DEq. erit DF bin.5.

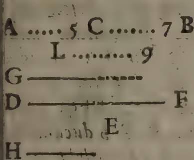
Nam ut in 50. hujus, erit DF bin. & quia per conſtr. & invertendo D Eq. E Fq :: AB.
a 9.10 CB, ideoque per converſionem rationis DEq.
b 5.def.48.10. DEq − EFq :: AB. AC :: Q. non Q. a erit

D

DE ⊐⊏ $\surd$ DEq − EFq. [b] ergo DF eſt bin. 4. Q. E. F.

*In numeris*, ſit G, 7; EF, 6. erit DE $\surd 54$. quare DF eſt $6 + \surd 54$.

## PROP. LIV.



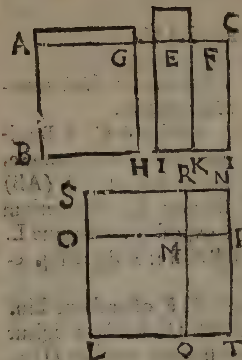

*Invenire ex binis nominibus ſextam.*

Accipe AC, CB primos numeros utcunque, ſic ut AC + CB (AB) ſit non Q. ſume etiam quemvis L num. Q. ſit G expoſ. ρ'. [a] fiatque L. AB :: Gq. DEq. atque AB. CB :: DEq. EFq. erit DF. bin. 6. [a 3. lem. 10. 10.]

Nam ut in 51. hujus, DF oſtendetur bin. item quod DE, & EF ⊐⊏ G. denique igitur quia per conſtr. & converſionem rationis DEq. DEq − EFq :: AB. AC :: non Q. Q. (Nam AB primus eſt ad AC, [b] ideoque ei diſſimilis) [c] ergo DE ⊐⊏ $\surd$ DEq − EFq. [d] ergo DF eſt bin. 6. Q. E. F. [b ſch. 27. 8. c 9. 10. d 6. def. 48. 10.]

*In numeris*, ſit G, 6; DE $\surd 48$. erit EF $\surd 28$. quare DF eſt $\surd 48 + \surd 28$.

## LEMMA.




Sit AD rectang gulum, cujus latu AC ſecetur inæqua liter in E; biſectumqu ſit ſegmentum minu EC in F; atque a AE, a fiat rectang AGE = EFq; perqu G, E, F b ducantur a AB parallela GH EI, FK: c Fiat auten quadratum LM = rectang. AH, atque a OMP productam c ſi at quadratum MN = GI; rectæque LOS LQT, NRS, NPT producantur.

a 28. 6.
b 31. 1.
c 14. 2.

Dico 1. MS, MT ſunt rectangula. Nam ob quadratorum angulos OMQ, RMP rectos a erit QMR recta linea. b ergo anguli RMO QMP recti ſunt. quare pgra MS, MT ſunt re ctangula.

a ſch. 15. 1.
b 13. 1.

c 2. ax. 2. 2. *Hinc patet* LS c = LT; *& proinde* LN eſſ *quadratum.*

3. *Rectangula* SM, MT, EK, FD æquali *ſunt.* Nam quia rectang. AGE d = EFq, e eri AE. EF :: EF. GE. f ideoque AH. EK :: EK GI. hoc eſt per conſtr. LM. EK :: EK. MN g verum LM. SM :: SM. MN. ergo EK h = SM k = FD l = MT.

d hyp
e 17. 6.
f 1. 6.
g ſch. 22. 6.
h 9. 5.
k 36. 1.
l 43. 1.
m 2. ax. 1.
n 16. 10.

4. *Hinc* LN m = AD.

5. *Quia* EC *biſecta eſt in* F, n *patet* EF, FC EC ⊐ *eſſe.*

o 18. & 16. 10.

6. Si AE ⊤ EC, & AE ⊐ √ AEq - ECq, o erunt AG, GE, AE ⊐. item, qu

AG

G. GE :: AH. GI *p* erunt AH, GI; hoc eſt p 10. 10.
M, MN ⊤⊥. item iiſdem poſitis,

7. OM ⊤⊥ MP. Nam per Hyp. AE, ⊤⊥
C, *q* ergo EC ⊤⊥ GE. *q* quare EF ⊤⊥ GE. q 14. 10.
d EF. GE :: EK. GI. *r* ergo EK ⊤⊥ GI, r 10. 10.
oc eſt SM ⊤⊥ MN. atqui SM. MN :: OM.
IP. *r* ergo OM ⊤⊥ MP.

8. Sin ponatur AE ⊤⊥ √ AEq − ECq,
patet AG, GE, AE eſſe ⊤⊥. unde LM ⊤⊥ f 19. & 17.
MN. nam AG. GE :: AH. GI :: LM. MN. 10.

*His bene perſpectis, facile ſex ſequentes Propoſi-
iones expediemus.*

## PROP. LV.

*Si ſpatium* AD *contineatur ſub rationali* AB, *& ex binis nominibus prima* AC, (AE + EC;) *recta linea* OP *ſpatium potens irrationalis eſt, quæ ex binis nominibus appellatur.*

Suppoſitis iis, quæ in lemmate proxime præcedenti deſcripta, & demonſtrata ſunt, liquet rectam OP poſſe ſpatium AD. *a* item AG, GE, AE ſunt ⊤⊥. ergo cum AE *b* ſit ϱ ⊤⊥ AB, *c* erunt AG, & GE, ϱ ⊤⊥ AB. *d* ergo rectangula AH, GI, hoc eſt quadrata LM, MN ſunt ϱ. ergo OM, MP ſunt ϱ *e* ⊤⊥. *f* proinde OP eſt bin. Q. E. D.

a hyp. & lem. 54. 10.
b hyp.
c ſch. 12. 10.
d 20. 10.
e lem. 54. 10.
f 37. 10.

*In numeris*, ſit AB, 5; AC, 4 + √ 12. quare rect ang. AD = 20 + √ 300 = quadr. LN. ergo OP eſt √ 15 + √ 5; nempe bin. 6.

## PROP. LVI.

*Si ſpatium* AD *contineatur ſub rationali* AB, *& ex binis nominibus ſecunda* AC (AE + EC;) *recta linea* OP *ſpatium* AD *potens, irrationalis eſt, quæ ex binis mediis prima appellatur.*

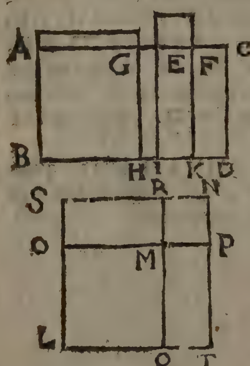
Rurſus adhibito lemmate ad 54. hujus, erit OP = √AD. *a* item AE, AG, GE ſunt ⊐⊏. ergo quum AE *b* ſit $\dot{\rho}$, ⊐⊏ AB, *c* erunt AG, GE etiam $\dot{\rho}$ ⊐⊏ AB. ergo rectangula AH, GI; hoc eſt OMq, MPq *d* ſunt μα. *e* quinetiam OM ⊐⊏ MP. denique EF ⊐⊏ EC, & EC *f* ⊐⊏ AB. *g* quare EF eſt $\dot{\rho}$ ⊐⊏ AB. *g* ergo EK; hoc eſt SM, vel OMP eſt $\dot{\rho}$ν. *h* Proinde OP eſt 2 μ prima. Q. E. D.

a hyp. & lem. 54. 10.
b hyp.
c ſch. 12, 10.
d 22. 10.
e lem. 54 10.
f hyp. 12. 10.
g 20. 10.
h 38. 10.

*In numeris*, ſit AB, 5; & AC, √48 : + 6. ergo rectang. AD = √ : 1200 + 30 = OPq. ergo OP eſt *v*√675 + *v*√75; nempe bimed. 1,

*Vide Schem.* 57.

## PROP. LVII.




*Si ſpatium* AD *contineatur ſub rationali* AB, *& ex binis nominibus tertia* AC (AE + EC;) *recta linea* OP *ſpatium* AD *potens, irrationalis eſt, quæ ex binis mediis ſecunda dicitur.*

Ut prius, OPq = AD. item rectangula AH, GI, hoc eſt OMP, MPq ſunt μα. *a* item EK, vel OMP eſt μγ. *b* ergo OP eſt bimed. 2.

a hyp. & 22. 10.
b 39. 10.

*In*

*In numeris*, ſit AB, 5; AC, $\sqrt{32}+\sqrt{34}$. quare AD eſt $\sqrt{800}+\sqrt{600}=OPq$. proinde OP eſt $v\sqrt{450}+v\sqrt{50}$; hoc eſt bimed. 2.

## PROP. LVIII.




*Si ſpatium* AD *contineatur ſub rationali* AB, *& ex binis nominibus quarta* AC (AE + EC;) *recta linea* OP *ſpatium potens, irrationalis eſt, quæ vocatur major.*

Nam iterum, OMq [a] ⊏⊐ MPq. rectang. vero AI, hoc eſt OMq + MPq [b] eſt $\acute{\rho}\nu$. [c] item EK, vel OMP eſt $\mu\nu$. [d] ergo OP ($\sqrt{AD}$) eſt major. Q. E. D.

a lem. 54. 10.
b hyp. & 20. 10.
c hyp. & 22. 10.
d 40. 10.

*In numeris*, ſit AB 5; & AC, $4+\sqrt{8}$. ergo rectang. AD eſt $20+\sqrt{200}$. quare OP eſt $\sqrt{:}\ 20+\sqrt{200}$.

## PROP. LIX.

*Si ſpatium* AD *contineatur ſub rationali* AB, *& ex binis nominibus quinta* AC; *recta linea* OP *ſpatium* AD *potens, irrationalis eſt, quæ rationale & medium potens appellatur.*

Rurſus OMP ⊏⊐ MPq. rectang. vero AI, vel OMq + MPq eſt $\mu\nu$. [a] item rectang. EK, vel OMP eſt $\acute{\rho}\nu$. [b] ergo OP ($\sqrt{AD}$) eſt potens $\acute{\rho}\nu$, & $\mu\nu$. Q. E. D.

a ut in præc.
b 41. 10.

*In numeris*, ſit AB, 5; & AC $2+\sqrt{8}$. ergo rectang. AD $=10+\sqrt{200}=OPq$. quare OP eſt $\sqrt{:}\ 10+\sqrt{200}$

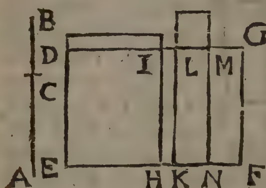
## PROP. LX.

*Si ſpatium* A D *contineatur ſub rationali* A B *& ex binis nominibus ſexta* BC (AE + EC; *recta linea* OP *ſpatium* AD *potens, irrationalis eſt, quæ bina media potens appellatur.*

Ut ſæpe prius, OMq ⊓ MPq. & OMq + MPq eſt μν. & rectang. (EK) OMP etiam μν.
a 42. 10. *a* ergo OP = √ AD eſt potens 2 μα. Q. E. D.

*In numeris*, ſit AB, 5; AC, √ 12 + √ 8; ergo rectang. AD, vel OPq eſt √ 300 + √ 200. proinde OP eſt √: √ 300 + √ 200.

### LEMMA.




*Sit recta* A B *inæqualiter ſecta in* C, *ſitque* A C *majus ſegmentum; & cuivis* D E *applicentur rectangula,* DF = ABq, & D H = ACq, & I K = CBq. *ſitque* LG *biſecta in* M, *ducaturque* M N *parall.* GF.

Dico 1. *Rectang.* A C B = L N, vel M F.
a 4.2. & 3. *a* Nam 2 ACB = L F.
ax. 1. 2. D L ⊏ LG. nam DK (ACq + CBq)
b 7. 2. *b* ⊏ LF (2 ACB) ergo cum DK, LF ſint æque
c 1. 6. alta, *c* erit DL ⊏ LG.
d 16. 10. 3. Si AC ⊔ CB, *d* erit rectang. DK ⊓ ACq, & CBq.

4. *Item*, D L ⊓ L G. nam ACq + CBq *e* ⊓ 2 A C B : hoc eſt D K ⊓ LF. ſed DK.
e lem.26.10. LF *e* :: DL. LG. *f* ergo DL ⊓ LG.
f 10. 10. 5. *Ad hæc*, DL ⊓ √ DLq − LGq. Nam
g 1. 6. ACq. A C B *g* :: A C B. CBq. hoc eſt D H. LN ::

N :: LN. IK. *c* quare DI. LM :: LM. IL. h 17. 6.
ergo DI x IL = LMq. ergo cum ACq *k* ˉ⊓⊔ k hyp
Bq. hoc eſt DH ˉ⊓⊔ IK, & *l* proinde DI ˉ⊓⊔ l 10 10
L, *m* erit DL ˉ⊓⊔ √ DLq − LGq. Q. E. D. m 18, 10.

6. *Sin ponatur* ACq ˉ⊓⊔ CBq, *n erit* DL ˉ⊓⊔ n 19. 10.
√ DLq − LGq.

*Hoc lemma præparationis vicem ſubeat pro 6. ſequentibus propoſitionibus.*

## PROP. LXI.

*Quadratum ejus quæ ex binis nominibus* ( AC + CB ) *ad rationalem* DE *applicatum, facit latudinem* DG *ex binis nominibus primam.*

Suppoſitis iis, quæ in lemmate proxime antecedenti deſcripta & demonſtrata ſunt. Quoniam AC, CB *a* ſunt ῤ ⊓⊔, *b* erit rectang. DK ⊓⊔ ACq; *c* ergo DK eſt ῤν. *d* ergo DL ⊓⊔ DE ῤ. rectang. vero ACB, ideoque 2 ACB (LF) *e* eſt μν. *f* ergo latitudo LG eſt ῤ ˉ⊓⊔ DE. *g* ergo etiam DL ˉ⊓⊔ LG. *h* item DL ⊓⊔ √ DLq − LGq. ex quibus *k* ſequitur DG eſſe bin. 1. Q. E. D.

a hyp. b lem 60. 10. c ſch. 12. 10. d 21. 10. e 22. & 24. 10. f 23. 10. g 13. 10. h lem 60. 10. k 1. def. 48. 10.

## PROP. LXII.

*Quadratum ejus, quæ ex binis mediis prima* ( AC + CB ) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus ſecundam.*

Rurſus adhibito lemmate proxime præcedenti; Rectang. DK ⊓⊔ ACq. *a* ergo DK eſt μν. *b* ergo latitudo DK eſt ῤ ˉ⊓⊔ DE. Quia vero rectang. ACB, ideoque LF ( 2 ACB ) eſt ῤν, *d* erit LG ῤ ⊓⊔ DE. *e* ergo DL, LG ſunt ˉ⊓⊔. *f* item DL ⊓⊔ √ DLq − LGq. *g* ex quibus patet DG eſſe bin. 2. Q. E. D.

a 24. 10. b 23. 10. c hyp. & ſch. 12. 10. d 21. 10. e 13. 10. f lem 60. 10. g 2 def. 48. 81.

PROP.

## PROP. LXIII.

*Quadratum ejus, quæ ex binis mediis ſecunda* (AC+CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus tertiam.*

Ut in præced. DL eſt ρ̂ ⊓ DE. porro quia
a hyp. & 24. 10. rectang. ACB, ideoque LF (2 ACB) *a* eſt
b 23. 10. μν, *b* erit LG ρ̂ ⊓ DE. *c* quinetiam DL ⊓
c lem. 60. 10. LG. *c* itemque DL ⊓ $\sqrt{}$ DLq — LGq. *d* er-
d 3. def. 48. 10. go DG eſt bin. 3. Q. E. D.

## PROP. LXIV.

*Quadratum Majoris* (AC+CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus quartam.*

a hyp. & ſch. 12. 10. Rurſus ACq + CBq, hoc eſt DK *a* eſt ρ'ν.
b 21. 10. *b* ergo DL eſt ρ̂ ⊓ DE. item ACB, ideoque
c hyp & 24. 10. LF (2 ACB) *c* eſt μν. *d* ergo LG eſt ρ' ⊓
d 23. 10. DE. *e* proinde etiam DL ⊓ LG. denique
e 13, 10. quia AC ⊓ BC, *f* erit DL ⊓ DLq —
f lem 60. 10. LGq. *g* unde DG. eſt bin. 4. Q. E. D.
g 4. def. 48 10.

## PROP. LXV.

*Quadratum ejus, quæ rationale ac medium poteſt,* (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus quintam.*

a 23. 10. Iterum, DK eſt μν. *a* ergo DL eſt ρ' ⊓
b 21. 10. DE. item LF eſt ρ'ν. *b* ergo LG eſt ρ' ⊓ DE.
c 13. 10. *c* ergo DL ⊓ LG. *d* item DL ⊓ $\sqrt{}$ DLq —
d lem 60. 10. LGq. *e* proinde DG eſt bin. 5.
e 5. def. 48. 10.

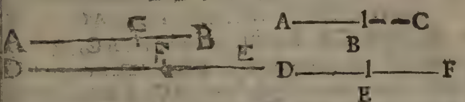
## PROP. LXVI.

*Quadratum ejus, quæ bina media poteſt* (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus ſextam.*

Ut

Ut prius, DL & LG ſunt ρ̂ ⊔ DE. a hyp.
Quia vero ACq + CBq (DK) *a* ⊔ ACB, b 14. 10.
ideoque DK ⊔ LF (2 ACB) eſtque DK. c 1. 6.
LF *c* :: DL. LG. *d* erit DL ⊔ LG. *e* denique d 10. 10.
DL ⊔ √ DLq – LGq. *f* ex quibus liquet e lem. 60. 10.
DG eſſe bin. 6. Q. E. D. f 6. def. 48. 10.

## *LEMMA.*




*Sint* AB, DE ⊔; *fiatque* AB. DE :: AC DF.

Dico 1. AC ⊔ DF. ut patet ex 10. 10. item CB ⊔ FE. *a* quia AB. DE :: CB. FE. a 19. 5.

2. AC. CB :: DF. FE. Nam AC. DF :: AB. DE :: CB. FE. ergo permutando AC. CB :: DF. FE.

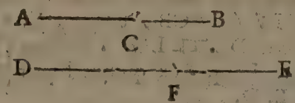
3. *Rectang.* ACB ⊔ DFE. Nam ACq. ACB *b* :: AC. CB *c* :: DF. EF :: DFq. DFE. quare permutando ACq. DFq :: ACB. DFE. ergo cum ACq ⊔ DFq, *d* erit ACB ⊔ DFE. b 1. 6. c prius. d 10. 10.

4. ACq + CBq ⊔ DFq + FEq. Nam quia ACq. CBq *e* :: DFq. FEq. erit componendo ACq + CBq. CBq :: DFq + FEq. FEq. ergo cum CBq ⊔ FEq, *f* erit ACq + CBq ⊔ DFq + FEq. e 22. 6. f 10. 10.

5. Hinc, ſi AC ⊔, vel ⊔ CB, *g* erit pariter DE ⊔, vel ⊔ EF. g 10. 10.

PROP.

## PROP. LXVII.




*Ei, quæ ex binis nominibus* (AC + CB) *longitudine commenſurabilis* DE, *& ipſa ex binis nominibus eſt, atq; ordine eadem.*

Fac AB. DE :: AC. DF. *a* ſunt AC, DF
a lem. 66. 10. ⊓⊔; *a* & CB, FE ⊓⊔. quare cum AC, & CB
b hyp. *b* ſint ρ' ⊓⊦, *c* erunt DF, FE ρ' ⊓⊦. ergo DE
c lem. 66. 10. & ſch. 12. 10. eſt etiam bin. Quia vero AC. CB *a* :: DF. FE. ſi AC ⊓⊔, vel ⊓⊔ √ ACq − BCq,
d 15. 10. *d* etiam ſimiliter DF ⊓⊔, vel ⊓⊔ √ DFq −
e 12. 10. & 14. 10. FEq. item ſi AC ⊓⊔, vel ⊓⊔ ρ' expoſ. *e* erit ſimiliter DF ⊓⊔, vel ⊓⊔ ρ' expoſ. at ſi CB ⊓⊔ vel ⊓⊔ ρ', *e* erit pariter FE ⊓⊔, vel ⊓⊔ ρ'. Sin
g Per def. 48. 10. vero utraque AC, CB ⊓⊦ ρ', erit utraq; etiam DF, FE ⊓⊦ ρ'. *g* Hoc eſt, quodcunque binomium fuerit AB, erit DE ejuſdem ordinis. Q. E. D.

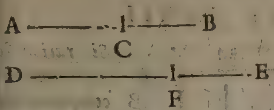
## PROP. LXVIII.

*Ei, quæ ex binis mediis* (AC + CB) *longitudine commenſurabilis* DE, *& ipſa ex binis mediis eſt, atque ordine eadem.*

a 12. 6. *a* Fiat AB. DE :: AC. DF. *b* ergo AC ⊓⊔
b lem 66. 10. DF, & CB ⊓⊔ FE. ergo cum AC & CB
c hyp. *c* ſint μ, *d* etiam DF, & FE erunt μ. & cum
d 24. 10. AC *c* ⊓⊦ CB, *e* erit FD ⊓⊦ FE. *f* ergo DE
e 10. 10. f 38. 10. eſt 2μ. Si igitur rectang. ACB ſit ρ'ν, quia
g ſch. 12. 10. DFE *b* ⊓⊔ ACB, *g* etiam DFE eſt ρ'ν; et ſi
h 24. 10. illud μν, *h* hoc etiam erit μν. *k* Id eſt, ſive AB
k 38. vel 39. 10. ſit bimed. 1. ſive bimed. 2. erit DF ejuſdem ordinis. Q. E. D.

PROP.

## PROP. LXIX.




*Majori* (AC + CB) *commenſurabilis* DE, & *ipſa Major eſt.*

Fac AB. DE :: AC. DF. Quoniam AC [a] ⊤⊢ CB, [b] erit DF ⊤⊢ FE. item ACq + CBq [a] eſt ρ'ν; proinde cum DFq + FEq [b] ⊓⊥ ACq + CBq, [c] etiam DFq + FEq eſt ρ'ν. denique rectang. ACB [a] eſt μν. [d] ergo rectang. DFE eſt μν (quia DFE [b] ⊓⊥ ACB.) [e] Quare DE eſt major. Q. E. D.

a hyp. b lem.66.10. c ſch.12.10. d 24.10. e 40.10.

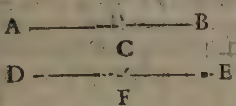
## PROP. LXX.

*Rationale ac medium potenti* (AC + CB) *commenſurabilis* DE, & *ipſa rationale ac medium potens eſt.*

Iterum fac AB. DE :: AC. DF. Quia AC [a] ⊤⊢ CB, [b] etiam DF ⊤⊢ FE. item quia ACq + CBq [a] eſt μν, [c] erit DFq + FEq μν. denique quia rectang. ACB [c] eſt ρ'ν, [d] etiam DFE eſt ρ'ν. [e] ergo DE eſt potens ρ'ν, ac μν. Q. E. D.

a hyp. b lem 66 10. c 24 10. d ſch.12.10. e 41. 10.

## PROP. LXXI.



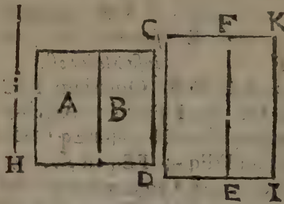

*Bina media potenti* (AC + CB) *commenſurabilis* DE, & *ipſa bina media potens eſt.*

Divide DE, ut in præced. Quia ACq [a] ⊓⊥ CBq, [b] erit DFq ⊓⊥ FEq. item quia ACq + CBq [a] eſt μν, [c] erit DFq + FEq etiam μν. pariterque quia ACB [a] eſt μν, [d] etiam DFE eſt μν. denique quia ACq + CBq [a] ⊓⊥ ACB, [e] erit

a hyp b lem 65.10. c 24 10. d 24. 10.

e 14. 10. *e* erit DFq + FEq ⊔ DFE. *f* è quibus ſequi-
f 42. 10. tur DE eſſe potentem 2 μσ. Q. E. D.

PROP. LXXII.



*Si rationale A, & medium B componantur, quatuor irrationales fiunt; vel ea quæ ex binis nominibus, vel quæ ex binis mediis prima, vel major, vel rationale ac medium potens.*

Nimirum ſi Hq = A + B, erit H una 4 linearum, quas theorema deſignat. Nam ad CD
a cor. 16. 6. expoſitum $\rho^{c}$, *a* fiat rectang. CE = A; item FI
b 2. ax. 1. = B; *b* ideoque CI = Hq. Quoniam igitur A
c 21. 10. eſt $\rho^{c}\nu$, etiam CE eſt $\rho^{c}\nu$. *c* ergo latitudo CF
eſt $\rho^{c}$ ⊔ CD. & quia B eſt μν, erit FI μν.
d 23. 10. *d* ergo FK eſt $\rho^{c}$ ⊔ CD. *e* ergo CF, FK ſunt
e 13. 10. $\rho^{c}$ ⊔. Tota igitur CK *f* eſt bin. Si igitur A
f 37. 10 ⊏ B, hoc eſt CE ⊏ FI, *g* erit CF ⊏ FK. er-
g 1. 6. go ſi CF ⊔ √ CFq − FKq, *h* erit CK bin.
h 1. def. 48. 10. 1. & proinde H = √ CI *k* eſt bin. Si ponatur
k 55. 10. CF ⊔ √ CFq − FKq, *l* erit CK bin. 4.
l 4. def. 48 10 quare H (√ CI) *m* eſt major. Sin A ⊐ B;
m 58. 10. *g* erit CF ⊐ FK; proinde ſi FK ⊔ √ FKq −
n 2. def. 48. 10. CFq, *n* erit CK bin. 2. *o* quare H eſt 2 μ prima. denique ſi FK ⊔ √ FKq − CFq, *p* erit
o 56. 10. CK bin. 5. *q* unde H erit potens $\rho^{c}\nu$ ac μν.
p 5 def. 48. 10. Q. E. D.
q 59. o.

PROP.

## P R O P. LXXIII.




*Si duo media A, B, inter ſe incommenſurabilia componantur, duæ reliquæ irrationales fiunt; vel ex binis mediis ſecunda, vel bina media potens.*

Nempe H potens A + B eſt una dictarum irrationalium. Nam ad CD expoſ. $\rho^c$, fac rectang. CE = A, & FI = B. unde Hq = CI. Quoniam igitur CE, & FI [a] ſunt $\mu\alpha$, [b] erunt latitudines CF, FK $\overset{c}{\rho}$ $\sqcup$ CD. item quia CE [a] $\sqcup$ FI; eſtque CE. FI [c] :: CF. FK, [d] erit CF $\sqcup$ FK. [e] ergo CK eſt bin 3. nempe, ſi CF $\sqcup$ $\surd$ CFq − FKq. unde H = $\surd$ CI [f] erit 2 $\mu$ 2ª. Sin vero CF $\sqcup$ $\surd$ CFq − FKq, [g] erit CK bin. 6. & [h] proinde H eſt potens 2 $\mu\alpha$. Q. E. D.

a *hyp.*
b 13. 10.
c 1. 6.
d 10. 10.
e 3. *def.* 48. 10.
f 57. 10.
g 6. *def.* 48. 10
h 60. 10.

### *Principium Senariorum per detractionem.*

## P R O P. LXXIV.

D E F

*Si à rationali* D F *rationalis* D E *auferatur, potentia tantum commenſurabilis exiſtens toti* DF; *reliqua* EF *irrationalis eſt: vocetur autem apotome.*

Nam EFq [a] $\sqcup$ DEq; ſed DEq [b] eſt $\rho^c\gamma$. [c] ergo EF eſt $\rho'$. Q. E. D.

*In numeris*, ſit DF, 2; DE, $\surd$ 3. EF erit 2 − $\surd$ 3.

a *lem* 26. 10.
b *hyp.*
c 10. & 11. *def.* 10.

P R O P.

## PROP. LXXV.

D E F

*Si à media* DF *media* DE *auferatur, potentia tantum commenſurabili exiſtens toti* DF, *quæ cum tota* DF *rationale contineat; reliqua* EF *irrationalis eſt; vocetur autem mediæ apotome prima.*

a ſch. 16. 10. Nam EFq *a* ⊐ rectang. FDE. ergo cum
b hyp. FDE *b* ſit ῥν; *c* erit EF ῥ. Q. E. D.
c 20. & 11. def. 10. *In numeris*, ſit DF v√54; & DE v√24. ergo EF eſt v√54 − v√24.

## PROP. LXXVI.

D E F

*Si à media* DF *media* DE *auferatur, potentia tantum commenſurabilis exiſtens toti* DF, *quæ cum tota* DF *medium contineat; reliqua* EF *irrationalis eſt; vocetur autem mediæ apotome ſecunda.*

a hyp. Quia DFq, & DEq *a* ſunt μα ⊐;
b 16. 10. *b* erit DFq + DEq ⊐ DEq. *c* quare DFq
c 24. 10. + DEq eſt μν. item rectang. FDE, *c* ideoque
d cor. 7. 2. 2 FDE *a* eſt μν. ergo EFq (*d* DFq + DEq −
e 27. 10. 2 FDE) *e* eſt ῥ'ν quare EF eſt ῥ'. Q. E. D.

*In numeris*, ſit DF, v√18; & DE, v√8. erit EF v√18 − v√8.

## PROP. LXXVII.

A B C

*Si à recta linea* AC *recta auferatur* AB, *potentia incommenſurabilis exiſtens toti* BC, *quæ cum tota* AC *faciat compoſitum quidem ex ipſarum quadratis rationale, quod autem ſub ipſis continetur medium; reliqua* BC *irrationalis eſt: vocetur autem minor.*

a hyp. Nam ACq + ABq *a* eſt ῥν. at rectang. ACB
b ſch. 12. 10. *a* eſt μν. *b* ergo 2 CAB ⊐ ACq + ABq
c 7. 2. d 17. 10. (2 *c* CAB + BCq;) *d* ergo ACq + ABq ⊐
e 11. def. 10. BCq. *e* ergo BC eſt ῥ'. Q. E. D.

*In*

*In numeris*, ſit AC, $\sqrt{:}\ 18. + \sqrt{108}$. AB $\sqrt{:}\ 18 - \sqrt{108}$. ergo BC eſt $\sqrt{:}\ 18 + \sqrt{:}\ 108. - \sqrt{:}\ 18 - \sqrt{108}$.

## PROP. LXXVIII.

D———E———F *Si à recta linea* DF *recta auferatur* DE *potentia incommenſurabilis exiſtens toti* DF, *quæ cum tota* DF *faciat compoſitum quidem ex ipſarum quadratis medium, quod autem ſub ipſis continetur, rationale; reliqua* EF *irrationalis eſt: vocetur autem cum rationali medium totum efficiens.*

Nam 2 FDE *a* eſt ῥν. *b* & DFq + DEq eſt μν. *c* ergo 2 FDE ⊓⊔ DFq + DEq *d* (2 FDE + EFq) *e* ergo EF eſt ῥ'. Q. E. D. a hyp. & ſch. 12.10. b hyp. c ſch. 12.10. d 7. 2. e ſch. 12.10. & 11. def. 10.

*In numeris* ſit DF, $\sqrt{:}\ \sqrt{216} + \sqrt{72}$. DE, $\sqrt{:}\ \sqrt{216} - \sqrt{72}$. ergo EF eſt $\sqrt{:}\ \sqrt{216} + \sqrt{72} - \sqrt{:}\ \sqrt{216} - \sqrt{72}$.

## PROP. LXXIX.

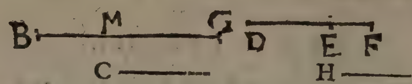



*Si à recta* DF *recta auferatur* DE, *potentia incommenſurabilis exiſtens toti* DF, *quæ cum tota faciat & compoſitum ex ipſarum quadratis, medium; & quod ſub ipſis continetur, medium, incommenſurabileque compoſito ex quadratis ipſarum, reliqua irrationalis eſt: vocetur autem cum medio medium totum efficiens.*

Nam 2 FDE, & DFq + DEq *a* ſunt μν; *b* ergo EFq (*c* DFq + DEq − 2 FDE) eſt ῥ'ν. *d* proinde EF eſt ῥ'. Q. E. D. a hyp. & 24. 10. b 27. 10. c cor. 7. 2. d 11. def. 10.

Exempl. gr. ſit DF, $\sqrt{:}\ \sqrt{180} + \sqrt{60}$. DE, $\sqrt{:}\ \sqrt{180} - \sqrt{60}$. EF erit $\sqrt{:}\ \sqrt{180} + \sqrt{60} - \sqrt{:}\ \sqrt{180} - \sqrt{60}$.

## LEMMA.




*Si idem ſit exceſſus inter primam magnitudinem* BG, *& ſecundam* C (MG) *qui inter tertiam magnitudinem* DF, *& quartam* H (EF;) *erit & viciſſim idem exceſſus inter primam magnitudinem* BG, *& tertiam* DE, *qui inter ſecundam* C, *& quartam* H.

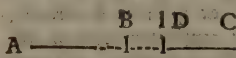
a hyp.
a 15.ax,1.

Nam quia a æqualibus BM, DE adjectæ ſunt æquales MG, EF, a hoc eſt C, H; erit exceſſus totorum BG, DF, b æqualis exceſſui adjectorum, C, H. Q. E. D.

### Coroll.

Hinc, quatuor magnitudines Arithmetice proportionales, viciſſim erunt Arithmetice proportionales.

## PROP. LXXX.



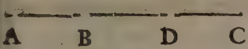

*Apotomæ* AB *una tantum congruit recta linea rationalis* BC, *potentia tantum commenſurabilis exiſtens toti* AB.

a 22. 10.
b 24. 10.
c cor.7. 2.
d lem.79.10.
e hyp. & 27. 10.
f ſch.12. 10.
g 27. 10.

Si fieri poteſt, alia BD congruat. a ergo rectangula ACB, ADB; b ideoq; eorum dupla ſunt $\mu\alpha$. cum igitur ACq + BCq − 2 ACB c = ABq c = ADq + DBq − 2 ADB. ergo viciſſim ACq + BCq −: ADq + BDq d = 2 ACB −: 2 ADB. Sed ACq + BCq −: ADq + BDq e eſt $\rho^{\tau}\nu$. f ergo 2 ACB −: 2 ADB eſt $\rho^{\tau}\nu$. Q. E. A.

PROP.

## Prop. LXXXI.

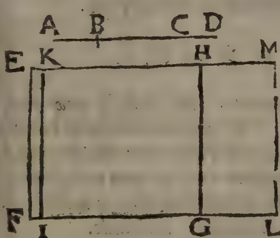



*Mediæ Apotomæ primæ* AB *una tantum congruit recta linea media* BC, *potentia ſolum commenſurabilis exiſtens toti, & cum tota rationale continens.*

Dic etiam BD congruere. igitur quoniam tam ACq, & BCq; quam ADq, & BDq *a* ſunt μα ⊏⊐. *b* etiam ACq + BCq, & ADq + BDq erunt μα. *c* ſed rectangula ACB, ADB; *d* adeoque 2 ACB, & 2 ADB ſunt ρ´α. *e* ergo 2 ACB −: 2 ADB; *f* hoc eſt ACq + BCq −: ADq + BDq eſt ρ´ν. *g* Q.E.A.

a *hyp.* b 16. & 24. 10. c *hyp.* d *ſch.* 12. 10. e *ſch.* 27. 10. f 7. 2. & *lem.* 79. 10. g 27. 10.

## Prop. LXXXII.




*Mediæ Apotomæ ſecundæ* AB *una tantum congruit recta linea media* BC, *potentia ſolum commenſurabilis exiſtens toti, & cum tota medium continens.*

Si fieri poteſt, congruat alia BD. Ad EF ρ´ fiant rectang. EG = ACq + BCq; item rectang. EL = ADq + BDq. Item EI = ABq. Jam 2 ACB + ABq = ACq + BCq = EG, ergo cum EI = ABq, *a* erit KG = 2 ACB. porro ACq, & BCq *b* ſunt μα ⊏⊐. *c* Ergo EG (ACq + BCq) eſt μν. *d* ergo latitudo EH ρ´ ⊏⊐ EF. *e* Quinetiam rectang. ACB; *f* ideoque 2 ACB (KG) eſt μν. *d* ergo KH eſt etiam ρ´ ⊏⊐ EF. denique quia ACq + BCq, id eſt, EG, *g* ⊏⊐ 2 ACB (KG) eſtque

a 4. 2. & 3. *ax.* 1. b *hyp.* c 24. 10. d 23. 10. e *hyp.* f 24. 10. g *lem.* 26. 10.

h 1.6. k 30. 10. l 74. 10.

EG.KG :: b EH. KH k erit EH ⊏⊐ KH. l ergo EK eſt aptome, cujus congruens KH. ſimili argumento erit KM ejuſdem EK congruens; contra 80 hujus.

## PROP. LXXXIII.

A B D C *Minori* AB, *una tantum congruit recta linea* (BC) *potentia incommenſurabilis exiſtens toti, & cum tota faciens compoſitum quidem ex ipſarum quadratis rationale; quod autem ſub ipſis continetur medium.*

a hyp. b lem. 97. 10. c ſch. 27. 10. d 27. 10.

Puta alium BD congruere. Cum igitur ACq + BCq, & ADq + BDq a ſint ῥᾱ, eorum exceſſus (2 b ACB —: 2 ADB) c eſt ῥν, d Q.E.A; quia ACB, & ADB ſunt μα per hypoth.

## PROP. LXXXIV.

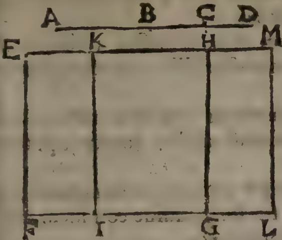
A B D C *Ei* (AB,) *quæ cum rationali medium totum facit, una tantum congruit recta linea* BC, *potentia incommenſurabilis exiſtens toti, & cum tota faciens compoſitum quidem ex ipſarum quadratis medium; quod autem ſub ipſis continetur, rationale.*

a hyp. b ſch. 12. 10. c lem 79. 10. d ſch. 27. 10.

Dic aliam BD etiam congruere. a ergo rectangula ACB, ADB. b ideoque 2 ACB, & 2 ADB ſunt ῥᾱ. ergo 2 ACB —: 2 ADB; c hoc eſt, ACq + BCq —: ADq + BDq d eſt ῥν. Q.E.A: quum ACq + BCq, & ADq + BDq ſint μα per hypoth.

PROP.

## Prop. LXXXV.




*Si (AB,) quæ cum medio medium totum facit una tantum congruit recta linea BC potentia incommenſurabilis exiſtens toti, & cum tota faciens & compoſitum ex ipſarum quadratis medium, & quod ſub ipſis continetur, medium, incommenſurabileque compoſito ex ipſarum quadratis.*

Suppoſitis iis quæ facta & oſtenſa ſunt in 82 hujus; liquet EH, & KH eſſe ρ[c] ⊓⊔ EF. Porro igitur quia ACq + CBq, hoc eſt, rectang. EG [a] ⊓⊔ ACB, [b] ideoque EG ⊓⊔ 2 ACB (KG) eſtque EG. KG :: [c] EH. KH; erit EH ⊓⊔ KH. ergo EK eſt apotome, cujus congruens KH. Haud aliter KM eidem apotomæ EK. congruere oſtendetur; contra 80 hujus.

a hyp.
b 14. 10.
c 1. 6.

## *Definitiones tertiæ.*

EXpoſita rationali, & apotoma, ſi tota plus poſſit quam congruens quadrato rectæ lineæ ſibi longitudine commenſurabilis;

I. Si quidem tota expoſitæ rationali longitudine ſit commenſurabilis, vocetur apotome prima.

II. Si vero congruens expoſitæ rationali longitudine ſit commenſurabilis, vocetur apotome ſecunda.

III. Quod ſi neque tota, neque congruens expoſitæ rationali ſit longitudine commenſurabilis, vocetur apotome tertia.

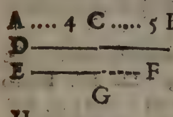
Rurſus, ſi tota plus poſſit quam congruens quadrato rectæ ſibi longitudine incommenſurabilis;

IV. Si quidem tota expoſitæ rationali ſit longitudine commenſurabilis, vocetur apotome quarta.

V. Si vero congruens expoſitæ rationali ſit longitudine commenſurabilis, vocetur apotome quinta.

VI. Quod ſi neque tota, neque congruens, expoſitæ rationali ſit longitudine commenſurabilis, vocetur apotome ſexta.

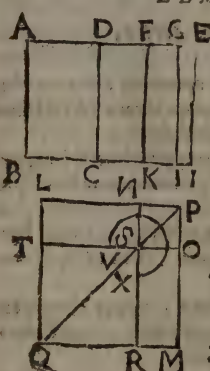
PROP. LXXXVI, 87, 88, 89, 90, 91.




*Invenire apotomen primam, ſecundam, tertiam, quartam, quintam, ſextam.*

Apotomæ inveniuntur, ſubductis minoribus binomiorum nominibus ex majoribus. Exemp. gr. Sit $6 + \sqrt{20}$, bin. 1. erit $6 - \sqrt{20}$, apot. 1. &c. Quare de earum inventione plura repetere nihil eſt neceſſe.

*LEMMA.*



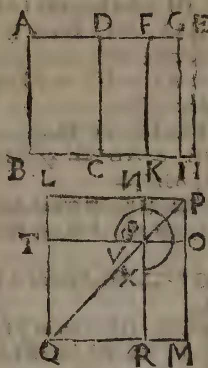

*Sit rectangulum* AC *ſub rectis* AB, AD. *producatur* AD *ad* E, *&* *biſecetur* DE *in* F. *ſitque rectang.* AGE = FEq. *&* *compleantur rectangula* AI, DK, FH. *Fiant vero quadratum* LM = AH, *&* *quadratum* NO = GI, *producanturque* NSR, OST.

Dico primo, rectangul. AI = LM + NO = TOq + SOq. ut patet ex conſtr.

Se-

Secundo, *Rectang.* DK = LO. Nam quia
rectang. A G E [a] = FEq, [b] ſunt A G, F E, G E a conſtr.
∺, [c] adeoque A H, FI, GI ∺; [a] hoc eſt, LM, b 17. 6.
FI, NO ∺. atqui LM, LO, NO [d] ſunt ∺; c 1. 6.
ergo FI = [e] LO [f] = DK = [g] NM. d ſch. 22. 6.
Tertio, *Hinc*, AC = AI − DK − FI = e 9. 5.
LM + NO − LO − NM = TR. f 36. 1.
Quarto, [h] *Liquet* DF, FE, DE eſſe ⊔. g 43. 1.
Quinto, *Si* AE ⊓ DE, & AE ⊔ √ AEq h 16. 10.
− DEq, [k] *erunt* AG, GE, AE ⊔. k 18. 10. & 10. 10.
Sexto, *Item, quia* AE [l] ⊔ DE, [m] *erunt* AE, l hyp.
FE ⊔. [n] *ideoque* AI, FI; *hoc eſt*, LM + NO m 13. 10.
& LO *ſunt* ⊔.
Septimo, *Item quia* AG * ⊔ GE, [n] *erunt* AH n 1. 6. & 10. 10.
GI, *hoc eſt*, LM, NO ⊔. * prius.
Octavo, *Sed quia* AE [l] ⊔ DE, [o] *erunt* FE, o 14. 10.
GE ⊔, [n] *ideoque rectang.* FI ⊔ GI, *hoc eſt* LO p 2. 6.
⊔ NO. *quare cum* LO. NO [p] :: TS, SO. [q] *erunt* q 10. 10.
TS. SO ⊔.
Nono, *Sin ponatur* AE ⊔ √ AEq − DEq; r 19. 10. & 17. 10.
[r] *erunt* AG, GE, AE ⊔.
Decimo, [ſ] *Quare rectang.* AH, GI, *hoc eſt* ſ 1. 6. & 19. 10.
TOq, SOq *erunt* ⊔.

## PROP. XCII.



*Si ſpatium* AC *contineatur ſub rationali* AB, *& Apotoma prima* AD (AE − DE;) *recta linea* TS *ſpatium* AC *potens, apotome eſt.*

Adhibe lemma proxime antecedens pro præparatione ad demonſtrationem hujus. Igitur TS = √ AC. item AG, GE, AE ſunt ⊤⊥; ergo cum AE ⊤⊥ *a* AB ρ'; *b* erunt AG, & GE ⊤⊥ AB. *c* ergo rectangula AH & GI, hoc eſt TOq & SOq ſunt ρ'α. *d* item TO, SO ſunt ρ' ⊤|⊥, *e* proinde TS eſt apotome. Q. E. D.

*a hyp.*
*b 12. 10.*
*c 20. 10.*
*d lem. 91. 10.*
*e 74 10.*

## PROP. XCIII.

*Vide Schem. præced.*

*Si ſpatium* AC *contineatur ſub rationali* AB, *& apotoma ſecunda* AD (AE − DE;) *recta linea* TS *ſpatium* AC *potens; mediæ eſt apotome prima.*

Rurſus juxta lemma antecedens, AG, GE, AE ſunt ⊤⊥. cum igitur AE *a* ſit ρ' ⊤|⊥ AB; *b* erunt AE, GE etiam ρ' ⊤|⊥ AB. *c* ergo rectangula AH, GI, hoc eſt TOq, SOq, ſunt μα; *d* item TO ⊤⊥ SO. Denique quia DE *e* ⊤⊥ AB. ρ'. *f* erit rectang. DI, ejuſque ſemiſſis DK, vel LO, hoc eſt TOS ρ' ν *g* è quibus ſequitur TS (√ AC) eſſe mediæ apot. 1. Q. E. D.

*a hyp.*
*b 13. 10.*
*c 22. 10.*
*d lem. 74 10.*
*e hyp.*
*f 20. 10.*
*g 75. 10.*

PROP.

## P R O P. XCIV.

*Vide idem.*

*Si ſpatium* AC *contineatur ſub rationali* AB, *& apotoma tertia* AD (AE − DE;) *recta linea* TS *ſpatium* AC *potens, mediæ eſt apotome ſecunda.*

Ut in præcedenti TO, & SO ſunt $\mu$. Quoniam igitur DE *a* eſt $\rho^{c}$ ⊔ AB, *b* erit rectang. DI, *c* ideoque DK; vel TOS $\mu\nu$. *d* ergo TS = √ AC eſt mediæ apot. 2. Q. E. D.

*a hyp. b 22. 10. c 14. 10. d 76. 10.*

## P R O P. XCV.

*Vide idem.*

*Si ſpatium* AC *contineatur ſub rationali* AB, *& apotoma quarta* AD (AE − DE) *recta linea* TS *ſpatium* AC *potens, minor eſt.*

Rurſus TO *a* ⊔̸ SO. Quoniam igitur AE *b* eſt $\rho^{c}$ ⊔ AB, *c* erit AI, (TOq + SOq) $\rho^{c}\nu$. atqui ut prius rectang. TOS eſt $\mu\nu$. *d* ergo TS = √ AC eſt minor. Q. E. D.

*a lem 91. 10. b hyp. c 20 10. d 77. 10.*

## P R O P. XCVI.

*Vide idem.*

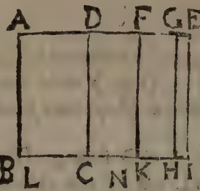
*Si ſpatium* AC *contineatur ſub rationali* AB, *& apotoma quinta* AD (AE − DE;) *recta linea* TS *ſpatium* AC *potens, eſt quæ cum rationali medium totum efficit.*

Rurſus enim TO ⊔̸ SO. itaque cum AE *a* ſit $\rho^{c}$ ⊔ AB, *b* erit AI, hoc eſt TOq + SOq $\mu\nu$. Sed prout in 93 rectang. TOS eſt $\rho^{c}\nu$. *c* proinde TS = √ AC eſt quæ cum $\rho^{c}\nu$ facit totum $\mu\nu$. Q. E. D.

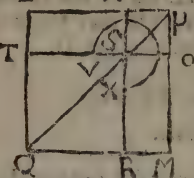
*a hyp. b 22. 10. c 78. 10.*

P R O P.

## PROP. XCVII.




*Si ſpatium* A C *contineatur ſub rationali* A P, *& apotoma ſexta* A D (A E − D E;) *recta linea* T S *ſpatium* A C *potens, eſt quæ cum medio medium totum efficit.*

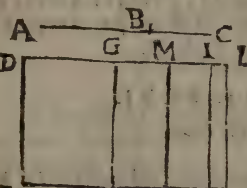



Itidem, ut ſæpe prius, TO ⊓+ SO. item ut in 96, TOq + SOq eſt μν. rectang. vero TOS eſt ῥν, ut in 94. *a* denique TOq + SOq ⊓+ TOS. *b* ergo TS = √ AC eſt quæ cum μν facit totum μν. Q. E. D.

a lem 91. 10.
b 79. 10.

### LEMMA.

* cor. 166.




*Ad rectam quamvis* D E * *applicentur rectang.* DF = ABq, *&* DH = ACq, *&* IK = BCq; *& ſit* GL *biſecta in* M; *ductaque ſit* M N *parall.* GF.

Erit primo, *Rectang.* DK = ACq + BCq, *ut conſtructio indicat.*

Secundo, *Rectang.* ACB = GN, *vel* MK. Nam DK *a* = ACq + BCq *b* = 2 ACB + ABq. at ABq *a* = DF. ergo GK *c* = 2 ACB. & *d* proinde GN, vel MK = ACB.

a conſtr.
b 7. 2.
c 3. ax. 1.
d 7. ax. 1.

Tertio, *Rectang.* DIL = MLq. Nam quia ACq. ACB *e* :: ACB. BCq; hoc eſt DH. MK

e 1. 6

MK :: MK. IK , *e* erit DI. ML :: ML. IL. *f* ergo DIL = MLq. f 17. 6.

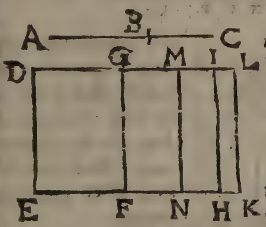
Quarto, *Si ponatur* AC ⊤ BC, *erit* DK ⊥ ACq. Nam ACq + BCq (DK) *g* ⊥ ACq. g 16. 10.

Quinto, *Item* , DL ⊥ √ DLq − GLq. Nam quia DH (ACq) ⊥ IK (BCq) *h* erit DI ⊥ IL. *k* ergo √ DLq − GLq ⊥ DL. h 10. 10. k 18. 10.

Sexto , *Item* DL ⊥ GL. Nam ACq + BCq ⊥ *l* 2 ACB; hoc eſt, DK ⊥ GK. *m* ergo DL ⊥ GL. l lem. 26. 10. m 10. 10.

Septimo, *Sin ponatur* AC ⊤ BC , *n erit* DL ⊥ √ DLq − GLq. n 19. 10.

## PROP. XCVIII.




*Quadratum apotomæ* AB (AC − BC) *ad rationalem* DE *applicatum , facit latitudinem* DG *apotomen primam.*

Fac ut in lemmate proxime præcedenti.

Quoniam igitur AC , BC *a* ſunt ρ ⊤ , *b* erit DK (ACq + BCq) ⊥ ACq ; *c* ergo DK eſt ρ*v*. *d* quare DL eſt ρ ⊥ DE. *e* item rectang. GK (2 ACB) eſt μ*v*. *f* ergo GL eſt ρ ⊥ DE. *g* proinde DL ⊥ GL ; *h* ſed DLq ⊥ GLq. *k* ergo DG eſt apotome, & *l* quidem prima (quia *m* AC ⊤ BC , & propterea DL ⊥ √ DLq − GLq.) Q. E. D.

a hyp. b lem 97. 10. c ſch 12. 10. d 21. 10. e 22, & 24. 10. f 23. 10. g 13. 10. h ſch. 12. 10. k 74 10. l 1 def. 85. 10. m lem. 97. 10.

PROP.

## PROP. XCIX.

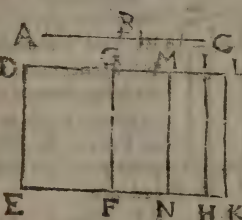
### *Vide Schema ſubſequens.*

*Quadratum mediæ apotomæ primæ* AB ( AC — BC ) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen ſecundam.*

a hyp.
b lem 97. 10.
c 24. 10.
d 23. 10.
e hyp. & ſch. 12. 10
f 21. 10.
g 13. 10.
h ſch. 12. 10.
k 74. 10.
l lem 97. 10.
m 2. def. & 85. 10.

Rurſus ( ſuppoſito lemmate præcedenti ) quia AC, & BC *a* ſunt $\mu$ ⊓⊔ *b*, erit DK ( ACq + BCq ) ⊓⊔ ACq; *c* quare DK eſt $\mu\nu$. *d* ergo DL eſt $\rho$ ⊓⊔ DE. *e* item GK ( 2. ACB ) eſt $\mu\nu$. *f* ergo GL eſt $\rho$ ⊓⊔ DE; *g* quare DL ⊓⊔ GL. *h* Sed DLq ⊓⊔ GLq. *k* ergo DG eſt apotome. quia vero DL *l* ⊓⊔ $\surd$ DLq — GLq, *m* erit DG apotome ſecunda. Q. E. D.

## PROP. C.




*Quadratum mediæ apotomæ ſecundæ* AB ( AC — BC ) *ad rationalem* DE *applicatum; facit latitudinem* DG *apotomen tertiam.*

a 23. 10.
b lem. 16. 10.
c 1. 6. & 10. 10.
d ſch. 12. 10.
e 74. 10.
f 3. def. 85. 10.
g lem. 97. 10.

Iterum DK eſt $\mu\nu$; *a* quare DL eſt $\rho$ ⊓⊔ DE. item GK eſt $\mu\nu$. *a* unde GL eſt $\rho$ ⊓⊔ DE; *b* item DK ⊓⊔ GK, *c* quare DL ⊓⊔ GL; *d* at DLq ⊓⊔ GLq. *e* ergo DG eſt apot. & quidem *f* 3ª. *g* quia DL ⊓⊔ $\surd$ DLq — GLq. Q. E. D.

## PROP. CI.

### *Vide Schema præced.*

*Quadratum minoris* AB ( AC — BC ) *ad rationalem*

*ionalem* DE *applicatum, facit latitudinem* DG *apotomen quartam.*

Ut prius, ACq + BCq, hoc eſt DK eſt ῥ̔ν; ergo DL eſt ῥ̔ ⊐⊏ DE. at rectang ACB, idemque GK (2 ACB) * eſt μν, *b* quare GL eſt ῥ̔ ˘⊐⊏ DE. *c* ergo DL ˘⊐⊏ GL. *d* at DLq ⊐⊏ GLq. quia vero * ACq ˘⊐⊏ BCq, *e* erit DL ˘⊐⊏ √ DLq − GLq: *f* ergo DG conditiones habet apotomæ quartæ. Q. E. D. a 21. 10. * hyp. b 13. 10. c 13. 10. d ſch. 12. 10. e lem 97 10 f 4. def 85. 10.

P R O P. C I I.

*Vide Schem. præced.*

*Quadratum ejus* AB (AC − BC,) *quæ cum rationali medium totum efficit, ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen quintam.*

Rurſus enim, DK eſt μν, *a* quare DL eſt ῥ̔ ⊐⊏ DE. item GK eſt ῥ̔ν, *b* unde GL eſt ῥ̔. ⊐⊏ DE. *c* ergo DL ˘⊐⊏ GL, *d* ſed DLq ⊐⊏ GLq. porro, DL *e* ˘⊐⊏ √ DLq − GLq. ex quibus, DG *f* eſt apot. quinta. Q. E. D. a 23. 10. b 21. 10. c 13. 10. d ſch 12 10. e lem 97. 10. f 5 def 85. 10.

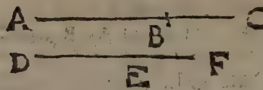
P R O P. C I I I.

*Vide Schema idem.*

*Quadratum ejus* AB (AC − BC,) *quæ cum medio medium totum efficit, ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen ſextam.*

Haud aliter, quam antea, DK, & GK ſunt μα; *a* quare DL & GL ſunt ῥ̔ ⊐⊏. DE. item DK *b* ˘⊐⊏ GK, *c* quare DL ˘⊐⊏ GL. *d* ergo DG eſt apot. *b* cum igitur ACq ˘⊐⊏ BCq, ideoque DL ˘⊐⊏ √ DLq − GLq, *e* erit DG. apot. ſexta. Q. E. D. a 23. 10. b hyp & lem. 97. 10. c 10 10. d 74 10. e 6 def 85. 10.

## PROP. CIV.



*Recta linea* DE *apotomæ* AB (AC — BC) *longitudine commenſurabilis, & ipſa apotome eſt, atque ordine eadem.*

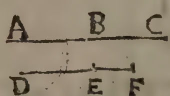
### LEMMA.

*Sit* AB. DE :: AC. DF. & AB ⊔ DE.
Dico AC + BC ⊔ DF + EF.

Nam AC. BC *a* :: DF. EF. ergo componendo AC + BC. BC :: DF + EF. EF. ergo per-
a lem 66 10. b 10.10. mutando AC + BC. DF + EF :: BC. EF. *a* at BC ⊔ EF. *b* ergo AC + BC ⊔ DF + EF. Q. E. D.

a 12 6 b lem.103. 10. c hyp. d 67. 10. e Præfinitiones ad 85. 10. *a* Fac AB. DE :: AC. DF. *b* igitur AC + BC ⊔ DF + EF. ergo cum AC + BC *c* binomium ſit, *d* erit DF + EF ejuſdem ordinis binomium: *e* quare DF — EF ejuſdem ordinis apotome eſt, cujus AC — BC. Q. E. D.

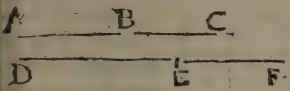
## PROP. CV.



*Recta linea* DE *mediæ apotomæ* AB (AC — BC) *commenſurabilis, & ipſa mediæ apotome eſt, atque ordine eadem.*

a 11. 6 b lem. 103. 10. c 68 10 d 75, & 76. 10. Iterum *a* fac AB. DE :: AC. DF. *b* quare AC + BC ⊔ DF + EF. *c* ergo DF + EF eſt bimed. ejuſdem ordinis, cujus AC + BC. *d* proinde & DF — EF mediæ apotome erit ejuſdem claſſis, cujus AC — BC. Q. E. D.

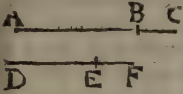
PROP. CVI.




*Recta linea DE Minori AB (AC – BC) commenſurabilis, & ipſa minor eſt.*

Fiat AB. DE :: AC. DF. a eſtque AC + BC ⊔ DF + EF. atqui AC + BC b eſt Major, c ergo DF + EF quoque Major eſt. d & proinde DF – EF eſt Minor. Q. E. D. a lem. 103. 10. b hyp. c 69. 10. d 77. 10.

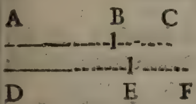
PROP. CVII.




*Recta linea DE commenſurabilis ei AB (AC – BC) quæ cum rationali medium totum efficit, & ipſa cum rationali medium totum efficiens eſt.*

Nam ad modum præcedentium oſtendemus DF + EF eſſe potentem ῥ.ν, & μ.ν. a ergo DF – EF eſt ut dicitur. a 78. 10.

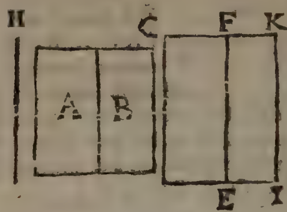
PROP. CVIII.




*Recta linea DE commenſurabilis ei AB (AC – BC) quæ cum medio medium totum efficit, & ipſa cum medio medium totum efficiens eſt.*

Nam, ad normam præcedentium, erit DF + EF potens 2 μα. a ergo DF – EF erit ut in propoſ. a 79. 10.

## PROP. CIX.




*Medio B à rationali A + B detracto, recta linea H, quæ reliquum ſpatium A poteſt, una ex duabus irrationalibus fit, vel apotome; vel Minor.*

a 3. ax. 1.
b hyp. & conſtr.
c 21. 10.
d 23. 10.
e 13. 10.
f 74. 10.
g 1. def. 85. 10.
h 92. 10.
k 4. def. 85. 10.
l 95. 10.

Ad CD ῥ, fac rectang. CI = A + B; & FI = B. quare CE *a* = A: (Hq) Quoniam igitur CI *b* eſt ῥν, *c* erit CK ῥ ⊓ CD. ſed quia FI *b* eſt μν, *d* erit FK ῥ ⊓ CD. *e* unde CK ⊓ FK. *f* ergo CF eſt apotome. Si igitur CK ⊓ √ CKq − FKq. *g* erit CF apot. prima; *h* quare √ CE (H) eſt apotome. ſin CK ⊓ √ CKq − FKq, *k* erit CF apot. quinta. & proinde H (√ CE) *l* erit Minor. Q. E. D.

## PROP. CX.

*Vide Schem. præced.*

*Rationali B à medio A + B detracto; aliæ duæ irrationales fiunt, vel mediæ apotome prima, vel cum rationali medium totum efficiens.*

a 3. ax. 1.
b hyp. & conſtr.
c 23. 10.
d 21. 10.
e 13. 10.
f 74. 10.
g 2. def. 85. 10.
h 93. 10.
k 5. def. 85. 10.
96. 10.

Ad CD expoſ. ῥ fiant rectang. CI = A + B; & FI = B, *a* unde CE = A = Hq. Quoniam igitur CI *b* eſt μν: *c* erit CK ῥ ⊓ CD. ſed quia FI *b* eſt ῥν, *d* erit FK ῥ ⊓ CD. *e* unde CK ⊓ FK. *f* ergo CF eſt apot. *g* nempe ſecunda; ſi CK ⊓ √ CKq − FKq, *h* quare H (√ CE) eſt mediæ apot. prima. Sin vero CK ⊓ √ CKq − FKq, *k* erit CF apot. quinta. & proinde H (√ CF) erit faciens μν cum ῥν. Q. E. D.

PROP.

## Prop. CXI.

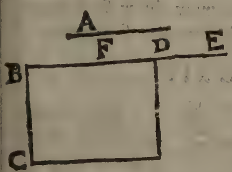
### *Vide Schema idem.*

*Medio B à medio A + B detracto, quod ſit incommenſurabile toti A + B; reliquæ duæ irrationales ſunt, vel mediæ apotome ſecunda, vel cum medio medium totum efficiens.*

Ad CD $\rho^c$ fiant rectang. CI = A + B; & FI = B, *a* quare CE = A = Hq. Quoniam igitur CI eſt μν. *b* erit CK $\rho^c$ ⊐⊏ CD. eodem modo erit FK $\rho^c$ ⊐⊏ CD. item quia CI *c* ⊐⊏ FI, *d* erit CK ⊐⊏ FK; *e* quare CF eſt apotome, *f* tertia ſcilicet, ſi CK ⊐⊏ √ CK − FKq, *g* unde H (√ C E) erit mediæ apot. ſecunda. verum ſi CK ⊐⊏ √ CKq − FKq, *h* erit C F apot. ſexta. *k* quare H erit faciens μν cum μ. Q. E. D.

a 3 *ax.* 1.
b 23. 10.
c *hyp.*
d 10. 10.
e 74 10
f 3 *def.* 85. 10.
g 94. 10.
h 6 *def.* 85. 10.
k 97. 10.

## Prop. CXII.




*Apotome A non eſt eadem, quæ ex binis nominibus.*

Ad expoſ. B C $\rho$, fiat rectang. C D = Aq. Ergo cum A ſit apotome, *a* erit B D apot. prima. ejus congruens ſit DE. *b* quare BE, D E ſunt $\rho^c$ ⊐⊏. *c* & B E ⊐⊏ B C. Vis A eſſe bin. ergo BD eſt bin. 1. ejus nomina ſint BF, FD; ſitque BF ⊏ FD; *d* ergo BF, FD ſunt $\rho^c$ ⊐⊏; & B F *e* ⊐⊏ BC. ergo cum BC ⊐⊏ BE, *f* erit BE ⊐⊏ FB. *g* ergo BE ⊐⊏ FE. *h* ergo FE eſt $\rho$. item quia BE ⊐⊏ DE, *k* erit FE ⊐⊏ DE. *l* quare FD eſt apotome, *l* adeoque FD eſt $\dot{\rho}$. ſed oſtenſa eſt $\rho$: quæ repugnant. ergo A male dicitur binomium. Q. E. D.

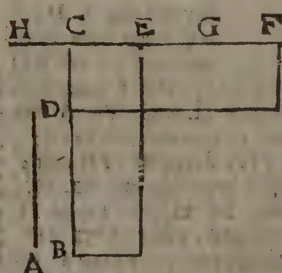
a 98. 10.
b 74 10.
c 1. *def* 85. 10.
d 37. 10.
e 1. *def.* 48. 10.
f 12. 10.
g *cor.* 16. 10.
h *ſch.* 12. 10.
k 14 10.
l 74 10.

*Nomina 13 linearum irrationalium inter ſe differentium.*

1. Media.
2. Ex binis nominibus, cujus 6 ſpecies.
3. Ex binis mediis prima.
4. Ex binis mediis ſecunda.
5. Major.
6. Rationale ac medium potens.
7. Bina media potens.
8. Apotome, cujus etiam 6 ſpecies.
9. Mediæ apotome prima.
10. Mediæ apotome ſecunda.
11. Minor.
12. Cum rationali medium totum efficiens.
13. Cum medio medium totum efficiens.

*Cum latitudinum differentiæ arguant differentias rectarum, quarum quadrata ſunt applicata ad aliquam rationalem, ſitque demonſtratum in præcedentibus, latitudines quæ oriuntur ex applicationibus quadratorum harum 13 linearum inter ſe differre, perſpicue ſequitur has 13 lineas inter ſe differre.*

PROP. CXIII.



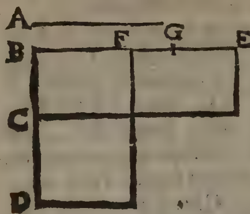

*Quadratum rationalis* A *ad eam, quæ ex binis nominibus* BC (BD + DC) *applicatum, latitudinem facit apotomen* EC, *cujus nomina* EH, CH *commenſurabilia ſunt nominibus* BD, DC *ejus, quæ ex binis nominibus* &

*& in eadem proportione* (EH. BD :: CH. DC;) *& adhæc, apotome* EC *quæ fit, eundem habet ordinem, quem ea* BC, *quæ ex binis nominibus.*

Ad DC minus nomen [a] fac rectang. DF = Aq = BE: quare BC. CD [b] :: FC. CE. ergo dividendo BD. DC :: FE. EC. cum igitur BD [c] ⊏ DC, [d] erit FE ⊏ EC. ſume EG = BC; fiatque FG. GE :: EC. CH. Erunt EH, CH nomina apotomæ EC; quibus conveniunt ea, quæ in theoremate propoſita ſunt. Nam componendo FE. GE. (EC) :: EH. GH. ergo FH. EH [e] :: EH. CH [f] :: FE. EC [f] :: BD. DC. quare cum BD [g] ⊓ DC, [h] erit EH ⊓ CH; [h] & FHq ⊔ EHq. ergo, quia FHq. EHq [k] :: FH. CH. [h] erit FH ⊔ CH, [l] ideoque FC ⊔ CH. Porro CD [g] eſt ρ́, & DF (Aq) [g] eſt ρ́ν, [m] ergo FC eſt ρ́ ⊔ CD, quare etiam CH eſt ρ́ ⊔ CD. [n] igitur EH CH ſunt ρ́, ac ⊓ ut prius. [o] ergo EC eſt apotome, cui congruit CH. porro EH. CH [f] :: BD. DC, ideo permutando EH. BD :: CH. DC. unde quia CH [f] ⊔ DC, [p] erit EH ⊔ BD. quinimo pone BD ⊔ $\sqrt{BDq - DCq}$; [q] erit ideo EH ⊔ $\sqrt{EHq - CHq}$. item ſi BD ⊔ ρ́ expoſ. erit EH ⊔ eidem ρ́; [s] hoc eſt ſi BC ſit bin. 1. [r] erit EC apot. prima. Similiter ſi DC ⊔ ρ́ expoſ. [t] erit CH ⊔ eidem ρ́, [u] hoc eſt ſi BC ſit bin. 2. [a] erit EC apot. 2. & ſi hæc bin. 3. illa erit apot. 3, &c. Sin BD ⊔ $\sqrt{BDq - DCq}$, [y] erit EH ⊔ $\sqrt{EHq - CHq}$; ſi igitur BC ſit bin. 4, vel 5, vel 6. erit EC ſimiliter apot. 4, vel 5, vel 6. Q. E. D.

a cor. 16. 6.
b 14. 6.
c hyp.
d 14. 5.
e 12. 5.
f Prius.
g hyp.
h 10. 10.
k cor. 20. 6.
l 16. 10.
m 21. 10.
n ſch. 12. 10.
o 74. 10.
p 10. 10.
q 15. 10.
r 12. 10.
s 1. def. 48. 10.
t 1. def. 85. 10.
u 2. def. 48. 10.
x 2. def. 85. 10.
y 15. 10.

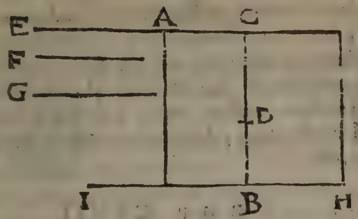
## PROP. CXIV.



*Quadratum rationalis* A *ad apotomen* BC (BD—DC) *applicatum, facit latitudinem* BE *eam, quæ ex binis nominibus; cujus nomina* BE, GE *commenſurabilia ſint apotomæ* BC *nominibus* BD DC, *& in eadem proportione; & adhuc, quæ ex binis nominibus fit* (BE,) *eundem habet ordinem, quem ipſa apotome* BC.

a cor.16.6. *a* Fac rectang. DF = Aq; & BE. FE *b* ::
b 12. 6. EG. GF. Quoniam igitur DF = Aq = CE,
c 14.6. *c* erit BD. BC :: BE. BF. ergo per converſionem rationis BD. CD :: BE. FE :: EG. GF ::
d 19. 5. *d* BG. EG. ſed BD *e* ⊤⊤ CD. *f* ergo BG ⊤⊤
e hyp. GE. ergo quia BGq. GEq *g* :: BG. GF. *h* erit
f 10 10. B G ⊤⊥ G F. *k* ideoque BG ⊤⊥ BF. porro
g cor.20. 6. BD *e* eſt ρ́, & rectang. DF (Aq) *e* eſt ρ́ν. *l* er-
h 10. 10. go BF eſt ρ́ ⊤⊥ BD. *m* ergo etiam BG eſt ρ́ ⊤⊥
k cor.16. 10. BD. *n* ergo BG, GE ſunt ρ́ ⊤⊤. *o* quare BE
l 21. 10. eſt bin. denique igitur quia BD. CD :: BG.
m 12. 10. GE; & permutando BD. BG :: CD. GE; ſitque
n ſch.12.10. BD ⊤⊥ BG; *p* erit CD ⊤⊥ GE. ergo ſi CB ſit
o 37. 10. apot. prima; erit BE bin. 1. &c ut in anteceden-
p 10, 10. ti. ergo, &c.

## PROP. CXV.




*Si ſpatium* AB *contineatur ſub apotoma* AC (CE − AE,) *& ea, quæ ex binis nominibus* CB; *cujus nomina* CD, DB *commenſurabilia ſint apotomæ nominibus* CE, AE, *& in eadem proportione* (CE. AE :: CD. DB.;) *recta linea* F *ſpatium* AB *potens, eſt rationalis.*

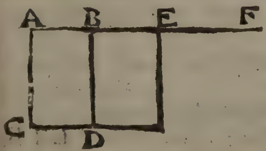
Sit G quævis ρ̇; & fiat rectang. CH = Gq. *a* erit igitur BH (HI − IB) apotome; & HI *a* ⊔ CD *b* ⊔ CE, *a* & BI ⊔ DB; *a* atque HI. BI :: CD. DB *b* :: CE, EA. ergo permutando HI. CE :: BI. EA. *c* ergo BH. AC :: HI. CE :: BI. EA. ergo cum HI *d* ⊔ CE, *e* erit BH ⊔ AC. *f* ergo rectang. HC ⊔ BA. Sed HC (Gq) *b* eſt ρ̇v. *g* ergo BA (Fq) eſt ρ̇v. proinde F eſt ρ̇. Q. E. D.

a 113.10.
b hyp.
c 19.5.
d 12.10.
e 10.10.
f 1.6.& 10.10
g ſch.12.10.

*Coroll.*

Hinc, fieri poteſt, ut ſpatium rationale contineatur ſub duabus rectis irrationalibus.

## PROP. CXVI.

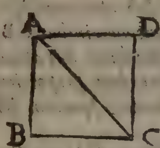



*A media* AB *fiunt infinitæ irrationales* BE, EF, &c. *& nulla alicui antecedentium eſt eadem.*

Sit AC expoſ.

a lem. 38. 10. b 11. 10. $\hat{\rho}$. ſitque AD ſpatium ſub AC, AB. a ergo AD eſt $\rho'\nu$. Sume BE = $\sqrt{}$ AD. b ergo BE eſt $\rho'$, nulli priorum eadem. nullum enim quadratum alicujus priorum applicatum ad $\hat{\rho}$, latitudinem efficit mediam. compleatur rectang. DE; a erit DE $\rho'\nu$; & b proinde EF ($\sqrt{}$ DE) erit $\rho'$; & nulli priorum eadem. nullum enim priorum quadratum ad $\hat{\rho}$ applicatum, latitudinem efficit ipſam BE. ergo, &c.

## PROP. CXVII.




*Propoſitum ſit nobis oſtendere, in quadratis figuris* BD, *diametrum* AC *lateri* AB *incommenſurabilem eſſe.*

a 47. 1. b cor. 24. 8. c 9. 10. Nam ACq. ABq a :: 2. 1 b :: non Q. Q. c ergo AC $\top\!\top$ AB. Q. E. D.

Celebratiſſimum eſt hoc theorema apud veteres philoſophos, adeo ut qui hoc neſciret, eum Plato non hominem eſſe, ſed pecudem diceret.

LIB.

# LIB. XI.

## *Definitiones.*

I. SOlidum eſt, quod longitudinem, latitudinem, & craſſitudinem habet.

II. Solidi autem extremum eſt ſuperficies.

III. Linea recta eſt ad planum recta, cum ad rectas omnes lineas, à quibus illa tangitur, quæque in propoſito ſunt plano, rectos angulos efficit.

IV. Planum ad planum rectum eſt, cum rectæ lineæ, quæ communi planorum ſectioni ad rectos angulos in uno plano ducuntur, alteri plano ad rectos ſunt angulos.

V. Rectæ lineæ ad planum inclinatio eſt, cum à ſublimi termino rectæ illius lineæ ad planum deducta fuerit perpendicularis; atque à puncto quod perpendicularis in ipſo plano effecerit, ad propoſitæ illius lineæ extremum, quod in eodem eſt plano, altera recta linea fuerit adjuncta; eſt, inquam, angulus acutus inſiſtente linea, & adjuncta comprehenſus.

VI. Plani ad planum inclinatio, eſt angulus acutus rectis lineis contentus, quæ in utroque planorum ad idem communis ſectionis punctum ductæ, rectos cum ſectione angulos efficiunt.

VII. Planum ad planum ſimiliter inclinatum eſſe dicitur, atque alterum ad alterum, cum dicti inclinationum anguli inter ſe fuerint æquales.

VIII. Parallela plana ſunt, quæ inter ſe non conveniunt.

IX. Similes ſolidæ figuræ ſunt, quæ ſimilibus planis continentur, multitudine æqualibus.

X. Æquales & ſimiles ſolidæ figuræ ſunt, quæ

quæ ſimilibus planis multitudine & magnitudine æqualibus continentur.

XI. Solidus angulus eſt plurium quam duarum linearum, quæ ſe mutuo contingunt, nec in eadem ſunt ſuperficie, ad omnes lineas inclinatio.

*Aliter.*

Solidus angulus eſt, qui pluribus quam duobus planis angulis in eodem non conſiſtentibus plano, ſed ad unum punctum conſtitutis, continetur.

XII. Pyramis eſt figura ſolida, planis comprehenſa, quæ ab uno plano ad unum punctum conſtituuntur.

XIII. Priſma eſt figura ſolida, quæ planis continetur, quorum adverſa duo ſunt & æqualia, & ſimilia, & parallela; alia vero parallelogramma.

XIV. Sphæra eſt, quando ſemicirculi manente diametro, circumductus ſemicirculus in ſeipſum rurſus revolvitur unde moveri cœperat, circumaſſumpta figura.

*Coroll.*

Hinc radii omnes à centro ad ſuperficiem ſphæræ inter ſe ſunt æquales.

XV. Axis autem ſphæræ, eſt quieſcens illa recta linea, circum quam ſemicirculus convertitur.

XVI. Centrum ſphæræ eſt idem quod & ſemicirculi.

XVII. Diameter autem ſphæræ, eſt recta quædam linea per centrum ducta, & utrinque à ſphæræ ſuperficie terminata.

XVIII. Conus eſt, quando rectanguli trianguli manente uno latere eorum, quæ circa rectum angulum, circumductum triangulum in ſeipſum rurſus revolvitur unde moveri cœperat, circumaſſumpta figura. Atque ſi quieſcens recta linea

linea æqualis ſit reliquæ, quæ circa rectum angulum continetur, orthogonius erit conus; ſi vero minor, amblygonius; ſi vero major, oxygonius.

XIX. Axis autem coni, eſt quieſcens illa linea, circa quam triangulum vertitur.

XX. Baſis vero coni eſt circulus qui à circumducta recta linea deſcribitur.

XXI. Cylindrus eſt, quando rectanguli parallelogrammi manente uno latere eorum, quæ circa rectum angulum, circumductum parallelogrammum in ſeipſum rurſus revolvitur unde cœperat moveri, circumaſſumpta figura.

XXII. Axis autem cylindri, eſt quieſcens illa recta linea, circum quam parallelogrammum convertitur.

XXIII. Baſes vero cylindri ſunt circuli à duobus adverſis lateribus, quæ circumaguntur, deſcripti.

XXIV. Similes coni & cylindri ſunt, quorum & axes, & baſium diametri proportionales ſunt.

XXV. Cubus eſt figura ſolida ſub ſex quadratis æqualibus contenta.

XXVI. Tetraedrum eſt figura ſolida ſub quatuor triangulis æqualibus & æquilateris contenta.

XXVII. Octaedrum eſt figura ſolida ſub octo triangulis æqualibus & æquilateris contẽta.

XXVIII. Dodecaedrum eſt figura ſolida ſub duodecim pentagonis æqualibus & æquilateris & æquiangulis contenta.

XXIX. Icoſaedrum eſt figura ſolida ſub viginti triangulis æqualibus & æquilateris contenta.

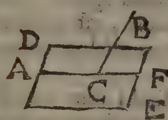
XXX. Parallelepipedum eſt figura ſolida ſex figuris quadrilateris, quarum quæ ex adverſo parallelæ ſunt, contenta.

XXXI. So-

XXXI. Solida figura in ſolida figura dicitur inſcribi, quando omnes anguli figuræ inſcriptæ conſtituuntur vel in angulis, vel in lateribus, vel denique in planis figuræ, cui inſcribitur.

XXXII. Solida figura ſolidæ figuræ viciſſim circumſcribi dicitur, quando vel anguli, vel latera, vel denique plana figuræ circumſcriptæ tangunt omnes angulos figuræ, circum quam deſcribitur.

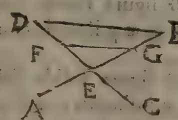
PROP. I.




*Rectæ lineæ pars quædam* A C *non eſt in ſubjecto plano, quædam vero* CB *in ſublimi.*

Producatur A C in ſubjecto plano uſque ad F. vis CB eſſe in directum ipſi AC; ergo duæ rectæ A B, A F habent commune ſegmentum A C.
a 10. ax. 1. *a* Q. F. N.

PROP. II.

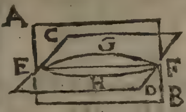



*Si duæ rectæ lineæ* A B, C D *ſe mutuo ſecent, in uno ſunt plano: atque triangulum omne* DEB *in uno eſt plano.*

Puta enim trianguli DEB partem EFG eſſe in uno plano, partem vero FDGB in altero. ergo rectæ ED pars EF eſt in ſubjecto plano, pars vero FD in ſublimi, *a* Q. E. A. ergo triangulum EDB in uno eſt plano; proinde & rectæ ED, EB;
a 1. 11. *a* quare & totæ A B, D C in uno plano exiſtunt. Q. E. D.

PROP.

## PROP. III.

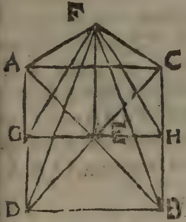



*Si duo plana A B, C D ſe mutuo ſecent, communis eorum ſectio E F eſt recta linea.*

Si E F communis ſectio non eſt recta linea, *a* ducatur in plano A B recta EGF, *a* & in plano CD recta EHF. duæ igitur rectæ EGF, EHF claudunt ſpatium. *b* Q. E. A.

a 1.poſt.1.
b 14.ax.1.

## PROP. IV.



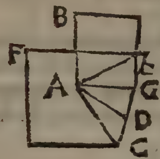

*Si recta linea E F rectis duabus lineis A B, C D ſe mutuo ſecantibus in communi ſectione E ad rectos angulos inſiſtat: illa ducto etiam per ipſas plano A C B D ad angulos rectos erit.*

Accipe EA, EC, EB, ED æquales, & junge rectas A C, C B, B D, A D. per E ducatur quævis recta G H; junganturque FA, FC, FD, FB, FG, FH. Quoniam A E *a* = EB; & D E *a* = E C; & ang. A E D *b* = C E B, *c* erit A D. = C B. *c* pariterque AC = DB. *d* ergo A D parall. C B, *d* & A C parall. DB. *e* quare ang. GAE = EBH. *e* & ang. AGE = EHB. ſed & AE *f* = EB *g* ergo GE = EH, & *g* AG = BH. quare ob angulos rectos, ex hyp. & proinde pares ad E, *h* baſes FA, FC, FB, FD æquantur. Triangula igitur ADF, FBC ſibi mutuo æquilatera ſunt, *k* quare ang. DAF = CBF. ergo in triangulis AGF, FBH latera FG, FH *l* æquantur; & proinde etiam triangula FEG, FEH ſibi mutuo æquilatera ſunt. *m* ergo anguli FEG, FEH æquales ac *n* propterea recti ſunt. Eodem modo F E cum omni-

a conſtr.
b 15. 1.
c 4. 1.
d ſch 34. 1.
e 29. 1.
f conſtr.
g 26. 1.
h 4. 1.
k 8. 1.
l 4. 1.
m 8. 1.
n 10. def. 1.

omnibus in plano A D B C per E ductis rectis
a 3. def. 11. lineis rectos angulos conſtituit, a ideoque eidem
plano recta eſt. Q. E. D.

## PROP. V.

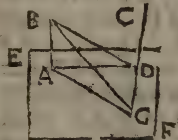



*Si recta linea* AB *rectis tribus lineis* AC, AD, AE *ſe mutuo tangentibus in communi ſectione ad rectos angulos inſiſtat; illæ tres rectæ in uno ſunt plano.*

Nam AC, AD *a* ſunt in uno plano FC; *a* item AD, AE ſunt in uno plano BE. vis diverſa eſſe hæc plana; ſit igitur eorum interſectio *b* recta AG. Quoniam igitur BA ex hypoth. perpendicularis eſt rectis AC, AD, eadem *c* plano FC, *d* ideoque rectæ AG perpendicularis eſt. ergo (ſiquidem & *a* AB eſt in eodem cum AC, AE plano) anguli BAG, BAE recti, & proinde pares ſunt, pars & totum. Q. E. A.

a 2. 11. b 3. 11. c 4. 11. d 3. def. 11.

## PROP. VI.



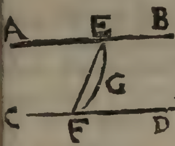

*Si duæ rectæ lineæ* AB, DC *eidem plano* EF *ad rectos ſint angulos; parallelæ erunt illæ rectæ lineæ* AB, DC.

Ducatur AD, cui in plano EF perpendicularis ſit DG = AB; junganturque BD, BG, AG. Quia in triangulis BAD, ADG anguli DAB, ADG *a* recti ſunt; atque AB *b* = DG; & AD communis eſt; *c* erit BD = AG; quare in triangulis AGB, BGD ſibi mutuo æquilateris ang. BAG *d* = BDG; quorum BAG rectus cum ſit, erit BDG etiam rectus. atqui ang. GDC rectus ponitur; ergo recta GD tribus DA, DB, CD recta eſt; *e* quæ ideo in uno ſunt plano, *f* in quo AB exiſtit; cum

a hyp. b conſtr. c 4. 1. d 8. 1. e 5. 11. f 2. 11.

um igitur AB, & CD ſint in uno plano, & anuli interni BAD, CDA recti ſint, *g* erunt AB, CD parallelæ. Q. E. D. g 18. 1.

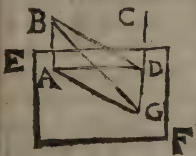
## Prop. VII.




*Si duæ ſint parallelæ rectæ lineæ* AB, CD, *in quarum utraque ſumpta ſint quælibet puncta* E, F; *illa linea* E F, *quæ ad hæc puncta adjungitur, in eodem eſt cum parallelis plano* ABCD.

Planum in quo A B, C D, ſecet aliud planum per puncta E, F. ſi jam E F non eſt in plano ABCD, illa communis ſectio non erit. Sit ergo E G F. *a* hæc igitur recta eſt linea. duæ ergo rectæ EF, EGF ſpatium claudunt. *b* Q. E. A. a 3. 11. b 14 ax. 1.

## Prop. VIII.

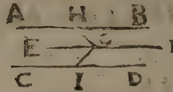



*Si duæ ſint parallelæ rectæ lineæ* A B, C D, *quarum altera* A B *ad rectos cuidam plano* EF *ſit angulos; & reliqua* C D *eidem plano* E F *ad rectos angulos erit.*

Adſcita præparatione & demonſtratione ſextæ hujus; anguli G D A & G D B recti ſunt. *a* ergo GD recta eſt plano per AD, DB (*b* in quo etiam A B, C D exiſtunt.) *c* ergo G D ipſi C D eſt perpendicularis; atqui ang. CDA etiam *d* rectus eſt. *e* ergo CD plano EF recta eſt. Q. E. D. a 4. 11. b 7. 11. c 3 def. 11. d 29. 1. e 4. 11.

PROP.

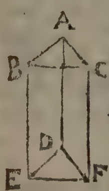
### PROP. IX.




*Quæ* (AB, CD) *eidem rectæ lineæ* EF *ſunt parallelæ, ſed non in eodem cum illa plano, hæ quoque ſunt inter ſe parallelæ.*

In plano parallelarum AB, EF duc HG perpendicularem ad EF. item in plano parallelarum
a 4. 11. EF, CD duc IG perpendicularem ad EF. *a* ergo
b 8. 11. EG recta eſt plano per HG, GI; eidemque
c 6. 11 plano *b* rectæ ſunt AH, & CI. *c* ergo AH, & CI parallelæ ſunt. Q. E. D.

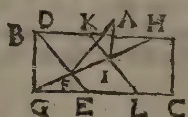
### PROP. X.




*Si duæ rectæ lineæ* AB, AC *ſe mutuo tangentes ad duas rectas* ED, DF *ſe mutuo tangentes ſint parallelæ, non autem in eodem plano, illæ angulos æquales* (BAC, EDF) *comprehendent.*

Sint AB, AC, DE, DF æquales inter ſe, & ducantur AD, BC, EF, BE, CF. Cum AB, DE
a *hyp. & conſtr.* *a* ſint parallelæ & æquales, *b* etiam BE, AD
b 33. 1. parallelæ ſunt, & æquales. Eodem modo CF,
c 2. *ax.* 1. & 30 1. AD parallelæ ſunt, & æquales. *c* ergo etiam BE, FC ſunt parallelæ & æquales. Æquantur ergo
d 33. 1. BC, EF. Cum igitur triangula BAC, EDF ſibi
e 8. 1. mutuo æquilatera ſint, anguli BAC, EDF *e* æquales erunt. Q. E. D.

### PROP. XI.



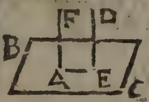

*A dato puncto* A *in ſublimi ad ſubjectum planum* BC *perpendicularem rectam lineam* AI *ducere.*

In plano BC duc quamvis DE, ad quam ex A *a* duc perpendicularem AF. ad eandem per F in

in plano BC *b* duc normalem FH. tum ad FH demitte perpendicularem AI. erit AI recta plano BC. a 12. 1. b 11. 1.

Nam per I *c* duc KIL parall. DE. Quia DE *d* recta eſt ad AF, & FH, *e* erit DE recta plano FA; adeoque & KL eidem plano *f* recta eſt. *g* ergo ang. KIA rectus eſt. atqui ang. AIF etiam *h* rectus eſt. *l* ergo AI plano BC recta eſt. Q. E. D. c 31. 1. d conſtr. e 4. 11. f 8. 11. g 3 def. 11. h conſtr. l 4. 11.

## PROP. XII.

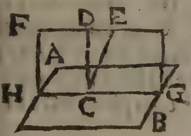



*Dato plano BC, à puncto A, quod in illo datum eſt, ad rectos angulos rectam lineam AF excitare.*

A quovis extra planum puncto D *a* duc DE rectam plano BC; & juncta DA *b* duc AF parall. DE. *c* perſpicuum eſt AF plano BC rectam eſſe. Q. E. F. a 11. 11. b 31. 1. c 8. 11.

Practice perficiuntur hoc, & præcedens problema, ſi duæ normæ ad datum punctum applicentur, ut patet ex 4. 11.

## PROP. XIII.

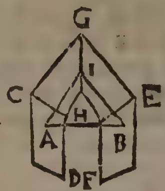



*Dato plano AB, à puncto D, quod in illo datum eſt, duæ rectæ lineæ CD, CE ad rectos angulos non excitabuntur ab eadem parte.*

Nam utraque CD, CE plano AB *a* recta eſſet, eædemque adeo parallelæ forent, quod parallelarum definitioni repugnat, a 6. 11.

## PROP. XIV.

*valet hac converſa.*

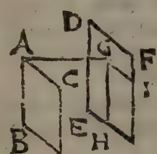



*Ad quæ plana* C D, F E, *eadem recta linea* A B *recta eſt; illa ſunt parallela.*

Si negas, plana C D, F E concurrant, ita ut communis ſectio ſit recta G H; ſume in hac quodvis punctum I, ad quod in propoſitis planis ducantur rectæ I A, I B. unde in triangulo I A B, duo anguli IAB, IBA *a* recti ſunt. *b* Q. E. A.

*a hyp. & 3. def. 11.*
*b 17. 1.*

## PROP. XV.




*Si duæ rectæ lineæ* A B, A C *ſe mutuo tangentes, ad duas rectas* D E, D F *ſe mutuo tangentes ſint parallelæ, non in eodem conſiſtentes plano; parallela ſunt, quæ per illa dicuntur, plana* BAC, EDF.

Ex A *a* duc A G rectam plano E F. *b* Sintque G H, G I parallelæ ad D E, D F. *c* erunt hæ parallelæ etiam ad A B, A C. Cum igitur anguli IGA, HGA *d* ſint recti, *e* erunt etiam CAG, BAG recti. *f* ergo GA recta eſt plano BC; atqui eadem recta eſt plano E F. *h* ergo plana BC, EF ſunt parallela. Q. E. D.

*a 11. 11.*
*b 31. 1.*
*c 30. 1.*
*d 3. def. 11.*
*e 29. 1.*
*f 4. 11.*
*g conſtr.*
*h 14. 11.*

PROP.

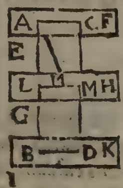
## PROP. XVI.




*Si duo plana parallela* A B, C D, *plano quopiam* HEIGF *ſecentur, communes illorum ſectiones* E H, GF *ſunt parallelæ.*

Nam ſi dicantur non eſſe parallelæ, cum ſint in eodem plano ſecanti, convenient alicubi, puta in I. quare cum totæ HEI, FGI *a* ſint in planis AB, CD productis, etiam hæc convenient, contra hypoth. a 1. 11.

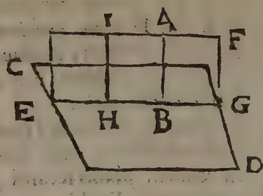
## PROP. XVII.




*Si duæ rectæ lineæ* ALB, CMD *parallelis planis* EF, GH, IK *ſecentur, in eaſdem rationes ſecabuntur* (AL. LB :: CM. MD.)

Ducantur in planis EF, IK rectæ AC, BD. item AD occurrens plano GH in N; junganturque NL, NM. Plana triangulorum ADC, ADB faciunt ſectiones BD, LN; & AC, NM *a* parallelas. ergo AL. LB *b* :: AN. ND *b* :: CM. MD. Q. E. D. a 16. 11. b 2. 6.

## PROP. XVIII.

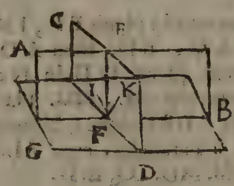



*Si recta linea* AB *plano cuipiam* CD *ad rectos ſit angulos; & omnia, quæ per ipſam* AB *plana* (EF, &c.) *eidem plano* CD *ad rectos angulos erunt.*

Ductum ſit per AB planum aliquod EF, faciens cum plano CD ſectionem EG; è cujus aliquo puncto H, in plano EF *a* ducatur HI parall. AB. *b* erit HI recta plano CD; pariterque aliæ quævis ad EG perpendiculares. *c* ergo planum EF plano CD rectum eſt; eademque ratione quævis alia plana per AB ducta plano EF recta erunt. Q. E. D.

a 31. 1.
b 8. 11.
c 4. *def.* 11.

## PROP. XIX.

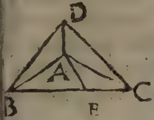



*Si duo plana* AB, CD, *ſe mutuo ſecantia, plano cuidam* GH *ad rectos ſint angulos, communis etiam illorum ſectio* EF *ad rectos eidem plano* (GH) *angulos erit.*

Quoniam plana AB, CD ponuntur recta plano GH, patet ex 4. def. 11. quod ex puncto F in utroque plano AB, CD duci poſſit perpendicularis plano GH; quæ *a* unica erit, & propterea eorundem planorum communis ſectio. Q. E. D.

a 13. 11.

PROP. XX.

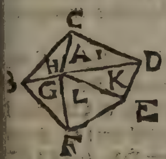



*Si ſolidus angulus* ABCD *tribus angulis planis* BAD, DAC, BAC *contineatur; ex his duo quilibet, utut aſſumpti, tertio ſunt majores.*

Si tres anguli ſunt æquales, patet aſſertio; ſi inæquales, maximus eſto BAC. ex quo *a* aufer AE=BAD; & fac AD=AE; ducanturque EC, BD, DC. a 23. 1.

Quoniam latus BA commune eſt, & AD *b*= AE; & ang. BAE *b*= BAD; *c* erit BE = BD. Sed BD + DC *d* ⊏ BC. *e* ergo DC ⊏ EC. cum igitur AD *b*= AE, & latus AC commune eſt, & DC ⊏ EC *f*, erit ang. CAD ⊏ EAC. *g* ergo ang. BAD + CAD ⊏ BAC. Q. E. D. b conſt. c 4. 1. d 20. 1. e 5. ax. 1. f 25. 1. g 4. ax. 1.

PROP. XXI.

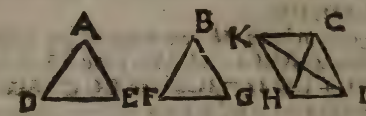



*Omnis ſolidus angulus ſub minoribus, quam quatuor rectis angulis planis, continetur.*

Eſto ſolidus angulus A; planis angulis illum componentibus ſubtendantur rectæ BC, CD, DE, EF, FB in uno plano exiſtentes. Quo facto conſtituitur pyramis, cujus baſis eſt polygonum BCDEF, vertex A, totque cincta triangulis quot plani anguli componunt ſolidum A. Jam vero quia duo anguli ABF, ABC *a* majores ſunt uno FBC, & duo ACB, ACD majores uno BCD, & ſic deinceps, erunt triangulorum G, H, I, K, L circa baſim anguli ſimul ſumpti omnibus ſimul angulis baſis B, C, D, E, F majores. *b* ſed anguli baſeos una cum quatuor rectis faciunt bis tot rectos, quot ſunt latera, ſive quot triangula. *c* Ergo omnes triangulorum circa baſim anguli una a 20. 11. b ſch. 32. 1. c 4. ax. 1.

cum 4 rectis conficiunt amplius quam bis to
rectos quot ſunt triangula. ſed iidem anguli ci
ca baſim una cum angulis qui componunt ſol
a 31. 1. dum, componunt *a* bis tot rectos quot ſunt tr
angula. liquet ergo angulos ſolidum angulum
componentes quatuor rectis eſſe minore
Q. E. D.

## PROP. XXII.

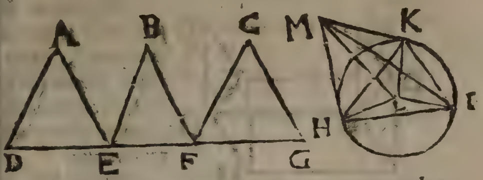



*Si fuerint tres anguli plani* A, B, HCI; *quoru duo utlibet aſſumpti reliquo ſint majores; compr bendant autem ipſos rectæ lineæ æquales* AD, A FB, &c. *fieri poteſt, ut ex rectis lineis* DE, F HI, *æquales illas rectas connectentibus triangulu conſtituatur.*

a 22. 1. Ex iis *a* conſtitui poteſt triangulum, ſi du quælibet reliqua majores exiſtant; ſed ita ſe h
b 23. 1. habet. Nam *b* fac ang. HCK = B, & CK = CI
c 4. 1. ducanturque HK, IK. *c* ergo KH = FG. & qui
d hyp. ang. KCI *d* ⊏ A; erit KI ⊏ DE. ſed KI *f*
e 24. 1. HI + KH (FG;) ergo DE ⊐ HI + FG. S
f 20. 1. mili argumento quævis duæ reliqua majore oſtendentur; & proinde ex iis triangulum *a* conſtitui poteſt. Q. E. D.

PROP.

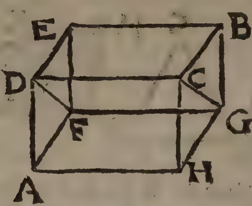
## PROP. XXIII.




*Ex tribus angulis planis* A, B, C, *quorum duo uomodocunque aſſumpti reliquo ſunt majores, ſoli-um angulum* MHIK *conſtituere.* * *Oportet autem llos tres angulos quatuor rectis minores eſſe.* * 21.11.

Fac AD, AE, BE, BF, CF, CG æquales nter ſe. Ex ſubtenſis DE, EF, FG (hoc eſt, x æqualibus HI, IK, KH) *a* fac triang. HKI. a 22. 11. & 22. 1.
irca quod *b* deſcribatur circulus LHKI. * Quo- b 5. 4.
iam vero AD ⊏ HL; *c* ſit ADq = HLq + vid. Clavium.
LMq. *d* ſitque LM recta plano circuli HKI; & c ſcb 47. 1.
ducantur HM, KM, IM. Quoniam igitur ang. d 12. 11.
HLM *e* rectus eſt, *f* erit MHq = HLq + LMq e 3. def. 11. f 47. 1.
= ADq. ergo MH = AD. ſimili argumento g conſtr.
MK, MI, AD (id eſt, AE, EB, &c.) æquantur;
ergo cum HM = AD, & MI = AE; & DE *h* = h conſtr.
HI, *k* erit ang. A = HMI; *k* ſimiliter ang. IMK k 8. 1.
= B. *k* & ang. HMK = C. Factus eſt igitur angulus ſolidus ad M ex tribus planis datis. Q. E. F. Brevitatis cauſa aſſumptum eſt, eſſe AD ⊏ HL, id quod in variis caſibus demonſtratum vide apud Clavium.

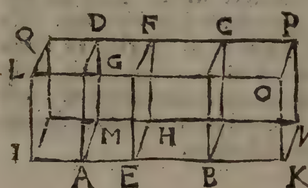
## PROP. XXIV.



*Si ſolidum A parallelis planis con tineatur, adverſ illius plana (AG DB, &c.) parallelo gramma ſunt ſimili & æqualia.*

Planum AC ſe cans plana paral
a 16. 11. lela AG, DB, *a* facit ſectiones AH, DC paralle las. Eadem ratione AD, HC parallelæ ſunt. Er go ADCH eſt parallelogrammum. Similí argu mento reliqua parallelepipedi plana ſunt *b* pa
b 35. def. 1. rallelogramma. Quum igitur AF ad HG, & AD ad HC parallelæ ſint, *c* erit ang. FAD = CHG
c 10. 11. d 34. 1. ergo ob AF *d* = HG, & AD *d* = HC, ac
e 7. 5. propterea AF. AD :: HG. HC, triangula FAD,
g 6. 6. h 4. 1. GAH *g* ſimilia ſunt & *h* æqualia; proinde & pa
k 6. ax. 1. rallelogramma AE, HB ſimilia ſunt & *k* æqua lia. idemque de reliquis oppoſitis planis oſtende tur. ergo, &c.

## PROP. XXV.



*Si ſolidum parallelepipedum ABCD plano EF ſecetur adverſis planis AD, BC parallelo, erit quemadmodum baſis* AH *ad baſim* BH, *ita ſolidum* AHD *ad ſolidum* BHC.

Concipe Ppp. ABCD produci utrinque. ac cipe AI = AE, & BK = EB; & pone plana IQ, KP planis AD, BC parallela. parallelo gramma

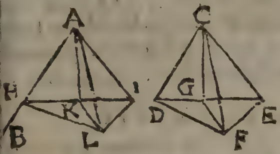
amma IM, AH, *a* & DL, DG, *b* & IQ, AD, a 36. 1. & 1. def. 6.
F, &c. *a* ſimilia ac æqualia ſunt; *c* quare Ppp. b 24. 11.
Q = AF; atque eadem ratione Ppp. BP = c 10. def. 11.
F. ergo ſolida IF, EP ſolidorum AF, EC æ-
uemultiplicia ſunt, ac baſes IH, KH baſium
H, BH. Quod ſi baſes IH ⊏, =, ⊐ KH, *d* e- d 14. 11. & 9. def. 11.
t ſimiliter ſolidum IF ⊏, =, ⊐ EP. eproin- e 6. def. 5.
e AH. BH :: AF. EC. Q. E. D.

*Hæc eadem omni priſmati accommodari poſſunt; nde*

*Coroll.*

Si priſma quodcunque ſecetur plano oppoſitis lanis parallelo, ſectio erit figura æqualis, & ſimilis planis oppoſitis.

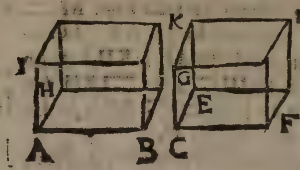
## PROP. XXVI.




*Ad datam rectam lineam* AB, *ejuſque punctum* A, *conſtituere angulum ſolidum* AHIL, *æqualem ſolido angulo dato* CDEF.

A puncto quovis F *a* demitte FG plano DCE a 11. 11. rectam; ducanturque rectæ DF, FE, EG, GD, CG. Fac AH = CD, & ang. HAI = DCE. & AI = CE; atque in plano HAI, fac ang. HAK = DCG, & AK = CG. Tum erige KL rectam plano HAI, & ſit KL = GF. ducaturque AL. erit angulus ſolidus AHIL par dato CDEF. Nam hujus conſtructio illius conſtitutionem penitus æmulatur, ut facile patebit examinanti. ergo factum.

PROP. XXVII.



*A data recta linea* AB, *dato ſolido parallelepipedo* CD *ſimile & ſimiliter poſitum parallelepipedum* AK *deſcribere.*

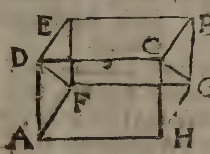
Ex angulis planis BAH, HAI, BAI, qui æquales ſint ipſis FCE, ECG, FCG, *a* fac angulum ſolidum A ſolido C parem. item *b* fac FC. CE :: BA. AH. *b* ac CE. CG :: AH. AI (*c* unde erit ex æquali FC. CG :: BA. AI;) & perficiatur Ppp. AK. erit hoc ſimile dato.

a 26. 11. b 12. 6. c 22. 5.

Nam per conſtr. Pgra *d* BH, FE; *d* & HI, EG; & *d* BI, FG ſimilia ſunt, & *e* horum ideo oppoſita illorum oppoſitis. ergo ſex plana ſolidi AK ſimilia ſunt ſex planis ſolidi CD. *f* proinde AK, CD ſimilia ſolida exiſtunt. Q. E. F.

d 1. def. 6. e 24. 11. f 9. def. 11.

PROP. XXVIII.



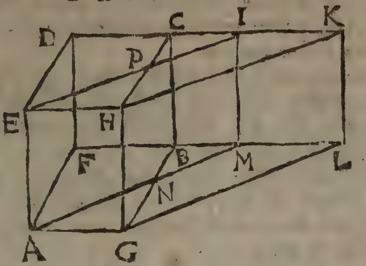
*Si ſolidum parallelepipedum* AB *plano* FGCD *ſecetur per diagonios* DF, CG *adverſorum planorum* AE, HB, *bifariam ſecabitur ſolidum* AB *ab ipſo plano* FGCD.

Nam quia DC, FG *a* æquales & parallelæ ſunt, *b* planum FGCD eſt Pgr. & propter *a* Pgra AE, HB æqualia, & ſimilia, *b* etiam triangula AFD, HGC, CGB, DFE æqualia & ſimilia ſunt. Atqui Pgra AC, AG ipſis FB, FD *a* etiam æqualia & ſimilia ſunt. ergo priſmatis FGCDAH omnia plana æqualia ſunt, & ſimilia planis omnibus priſmatis FGCDEB; & *c* proinde hoc priſma illi æquatur. Q. E. D.

a 24. 11. b 34. 1. c 9. def. 11.

PROP.

PROP. XXIX.

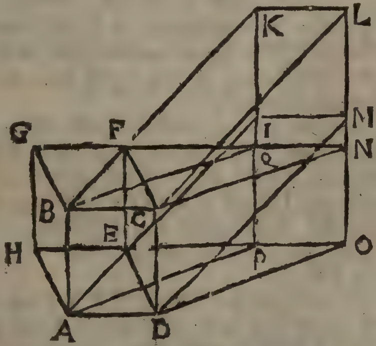



*Solida parallelepipeda* AGHEFBCD, AGHEMLKI *ſuper eandem baſim* AGHE *conſtituta, &* * *in eadem altitudine; quorum inſiſtentes lineæ* AF, AM *in iiſdem collocantur rectis lineis* AG, FL, *ſunt inter ſe æqualia.*

Nam ſi ex a æqualibus priſmatis AFMEDI, GBLHCK commune auferatur priſma NBMPCI, addaturque utrinque ſolidum AGNEHP, b erit Ppp. AGHEFBCD = AGHEMLKI. Q. E. D.

* *Id eſt, inter parallela plana* AGH F, FLKD. & *ſic intellige in ſequent.*
a 10. *def.* 11. & 35. 1.
b 3. & 2. *ax.* 2.

PROP. XXX.




*Solida parallelepipeda* ADBCHEFG, AD-

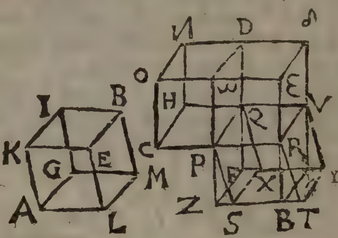
*ADCBIMLK ſuper eandem baſim ADCB conſtituta, & in eadem altitudine, quorum inſiſtentes lineæ AH, AI non in iiſdem collocantur rectis lineis, inter ſe ſunt æqualia.*

a 34. 1. Nam produc rectas HEO, GFN, & LMO, KIP; & duc AP, DO, BQ, CN. a erunt tam DC, AB, HG, EF, PQ, ON; quam AD, HE, GF, BC, KL, IM, QN, PO æquales inter ſeſe

b 29. 11. & parallelæ. b Quare Ppp. ADCBPONQ utrique Pppo. ADCBHEFG, ADCBIMLK æ-

c 1. ax. 1. quale eſt; & c proinde hæc ipſa inter ſe æqualia ſunt. Q. E. D.

## PROP. XXXI.




*Solida parallelepipeda ALEKGMBI, CPωOHQDN ſuper æquales baſes ALEK, CPωO conſtituta, & * in eadem altitudine, æqualia ſunt inter ſe.*

* *Altitudo, eſt perpendicularis à plano baſis ad planum oppoſitum.*

Habeant primo parallelepipeda AB, CD latera ad baſes recta; & ad latus CP productum

a 18 6. a fiat pgr. PRTS æq. & ſimile pgro KELA;

b 27 11. & 10. def. 11. b adeoque Ppp. PRTSQVYX æq. & ſim. Pppo AB. Producantur OωE, NDδ, ωPZ, DQF, ERB, δVγ, TSZ, YXF; & duc Eδ, Bγ, ZF.

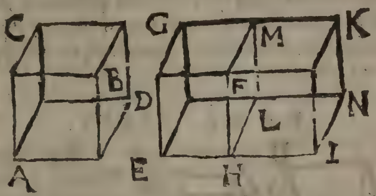
c 30 def. 11. d hyp & 35 1. Plana Oεδ N, CRVH, ZTYF c parallela ſunt inter ſe; d & pgra ALEK, CPωO, PRTS, PRBZ æqualia ſunt. Cum igitur Ppp.

CD.

CD. PV$\delta\omega$ [e] :: pgr. C$\omega$ (PRBZ.) Pte :: Ppp. e 25. 11.
PRBZQV$\gamma$F. PV$\delta\omega$, [f] erit Ppp. CD [f] = f 9. 5.
PRBZQV$\gamma$F [g] = PRVQSTYX [h] = AB. g 19 12. h conſtr.
Q. E. D.

Sin Ppp[a] AB, CD latera baſibus obliqua habeant; ſuper eaſdem baſes, & in eadem altitudine, ponantur parallelepipeda, quorum latera baſibus ſint recta. [k] Ea inter ſe, & obliquis æqualia erunt; [m] proinde & obliqua AB, CD æquantur. Q. E. D. k 19 11. m 1. ax. 1.

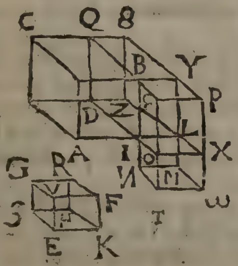
## PROP. XXXII.




*Solida parallelepipeda* ABCD, EFGL *ſub eadem altitudine, inter ſe ſunt ut baſes* AB, EF.

Producta EHI, [a] fac pgr. FI = AB, & [b] comple Ppp. FINM. Liquet eſſe Ppp. FINM. ([c] ABCD.) EFGL [d] :: FI. (AB) EF. Q. E. D. a 45 1. b 31. 1. c 31. 11. d 25. 11.

## PROP. XXXIII.




*Similia ſolida parallelepipeda*, ABCD, EFGH, *inter ſe ſunt in triplicata ratione homologorum laterum* AI, EK.

Producantur rectæ AIL, DIO, BIN. & [a] fiant IL, IO, IN ipſis EK, KH, KF æquales, [b] adeoque a 3. 1. b 17. 11.

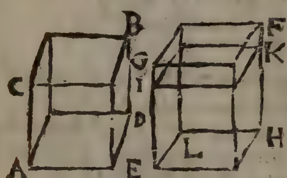
&

& Ppp. IXMT æq. & ſim. Ppp° EFGH.
c31.1. [c] Perficiantur Ppp *a* IXPB, DLYQ. Itaque [d] e-
d hyp. rit AI. IL. (EK) :: DI. IO (HK) :: BI. IN.
e 1.6 (KF;) hoc eſt Pgr. AD. DL :: DL. IX ::
f 32. 11. BO. IT; *f* id eſt Ppp. ABCD. DLQY ::
g conſtr. DLQY. IXBP :: IXBP. IXMT. (*g* EFGH.)
h 10. def. 5. *h* ergo ratio ABCD ad EFGH triplicata eſt ra-
k 1.6. tionis ABCD ad DLQY, [k] vel AI ad EK. Q. E. D.

*Coroll.*

Hinc, ſi fuerint quatuor lineæ rectæ continue proportionales, ut eſt prima ad quartam, ita eſt parallelepipedum ſuper primam deſcriptum ad parallelepipedum ſimile ſimiliterque deſcriptum ſuper ſecundam.

PROP. XXXIV.




*Æqualium ſolidorum parallelepipedorū* ADCB, EHGF *baſes & altitudines reciprocantur* (AD. EH :: EG. AC.) *Et quorum ſolidorum parallelepipedorum* ADCB, EHGF *baſes & altitudines reciprocantur, illa ſunt æqualia.*

Sint primo latera CA, GE ad baſes recta; ſi jam ſolidorum altitudines ſint pares, etiam baſes æquales erunt. & res clara eſt. Sin altitu-
a 3. 1. dines inæquales ſint, à majori EG *a* detrahe EI
b 31. 1. = AC. & per I *b* duc planum IK parallelum
c 32. 11. baſi EH. itaque
d 17. 5.
e 1. 6.
f conſtr.
g 11. 5.
1. *Hyp.* AD. EH *c* :: Ppp. ADCB. EHIK *d* :: Ppp. EHGF. EHIK *e* :: GL. IL *e* :: GE. IE. (*f* AC;) *g* liquet igitur eſſe AD. EH :: GE. AC. Q. E. D.

2. *Hyp.*

2. *Hyp.* ADCB. EHIK *h* :: AD. EH *k* :: EG. EI *l* :: *GL. IL* *m* :: Ppp. EHGF. EHIK, *n* quare Ppp. ADCB = EHGF. Q. E. D. h 32. 11. k hyp. l 1. 6. m 32. 11. n 9. 5.

Sint deinde latera ad baſes obliqua. Erigantur ſuper iiſdem baſibus, in altitudine eadem, parallelepipeda recta. Erunt obliqua parallelepipeda his æqualia. Quare cum hæc per 1. partem reciprocent baſes & altitudines, etiam illa reciprocabunt. Q. E. D.

### *Coroll.*

*Quæ de parallelepipedis demonſtrata ſunt* Prop. 29, 30, 31, 32, 33, 34. *etiam conveniunt priſmatis triangularibus, quæ ſunt dimidia parallelepipeda, ut patet ex* Pr. 28. *Igitur*,

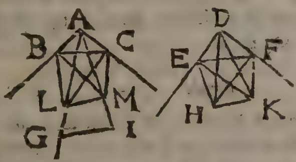
1. Priſmata triangularia æque alta ſunt ut baſes.

2. Si eandem vel æquales habeant baſes, & eandem altitudinem, æqualia ſunt.

3. Si ſimilia fuerint, eorum proportio triplicata eſt proportionis homologorum laterum.

4. Si æqualia ſunt, reciprocant baſes & altitudines. & ſi reciprocant baſes & altitudines, æqualia erunt.

### PROP. XXXV.




*Si fuerint duo plani anguli* BAC, EDF *æquales, quorum verticibus* A, D, *ſublimes rectæ lineæ* AG, DH *inſiſtant, quæ cum lineis primo poſitis angulos contineant æquales, utrumq; utriq;* (ang. GAB = HDE; & GAC = HDF.) *in ſublimibus autem lineis* AG, DH *quælibet ſumpta fuerint puncta* G, H;

&

*& ab his ad plana* BAC, EDF, *in quibus conſi-ſtunt anguli primum poſiti* BAC, EDF, *ductæ fuerint perpendiculares* GI, HK; *à punctis vero* I, K *quæ in planis à perpendicularibus fiunt, ad angulos primum poſitos adjunctæ fuerint rectæ lineæ* AI, DK; *hæ cum ſublimibus* AG, DH *æquales angulos* GAM, HDK *comprehendent,*

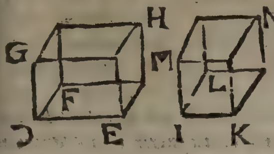
Fiant DH, AL æquales, & GI, LM parallelæ; & MC ad AC, MB ad AB, KF ad DF, KE ad DE perpendiculares, ducanturque rectæ
a 8. 11. BC, LB, LC, atque EF, HF, HE; *a* eſtque LM
b 3. def. 11. recta plano BAC; *b* quare anguli LMC, LMA, LMB; eademque ratione anguli HKF, HKD,
c 47. 1. HKE recti ſunt. Ergo ALq *c* = LMq + AMq *c* = LMq + CMq + ACq *c* = LCq + ACq;
d 48. 1. *d* ergo ang. ACL rectus eſt. Rurſus ALq *e* =
e 47. 1. LMq + MAq *e* = LMq + BMq + BAq *e* = BLq + BAq. *d* ergo ang. ABL etiam rectus eſt. Simili diſcurſu anguli DF*H*, DE*H* recti
f 16. 1. ſunt; *f* ergo AB = DE; *f* & BL = E*H*; *f* & AC = DF; & CL = FH. *g* quare etiam BC
g 4. 1. = EF, *g* & ang. ABC = DEF *g* & ang. ACB
h 3. ax. 1. = DFE. unde reliqui è rectis anguli CBM, BCM reliquis FEK, EFK æquantur. *k* ergo
k 26. 1. CM = FK, *l* ideoque & AM = DK. ergo ſi
l 47. 1. ex LAq *m* = HDq. auferatur AMq = DKq,
m conſtr. *n* remanet LMq = HKq quare trigona LAM,
n 47. 1 & 3. ax. HDK ſibi mutuo æquilatera ſunt. *o* ergo ang.
o 8. 1. LAM = HDK. Q. E. D.

*Coroll.*

Itaque ſi fuerint duo anguli plani æquales; quorum verticibus ſublimes rectæ lineæ æquales inſiſtant, quæ cum lineis primo poſitis angulos contineant æquales, utrumque utrique; erunt à punctis extremis linearum ſublimium ad plana angulorum primo poſitorum demiſſæ perpendiculares inter ſe æquales; nempe LM = HK

PROP.

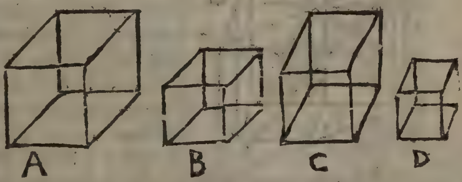
## PROP. XXXVI.




*Si tres rectæ lineæ* DE, DG, DF *proportionales fuerint; quod ex his tribus fit ſolidum parallelepipedum* DH, *æquale eſt deſcripto à media linea* DG (IL) *ſolido parallelepipedo* IN, *quod æquilaterum quidem ſit, æquiangulum vero prædicto* DH.

Quoniam DE. IK *a* :: IL. DF, *b* erit pgr. LK = FE. & propter angulorum planorum ad E & I, ac linearum GD, IM æqualitatem, etiam altitudines parallelepipedorum æquales ſunt, ex coroll. præced. *c* ergo ipſa inter ſe æqualia ſunt. Q. E. D. a *hyp.* b 14. 6. c 31. 11.

## PROP. XXXVII.

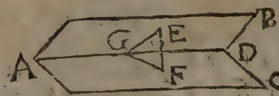



*Si quatuor rectæ lineæ* A, B, C, D *proportionales fuerint, & ſolida parallelepipeda* A, B, C, D *quæ ab ipſis & ſimilia, & ſimiliter deſcribuntur, proportionalia erunt. Et ſi ſolida parallelepipeda, quæ & ſimilia, & ſimiliter deſcribuntur, fuerint proportionalia* (A.B :: C. D.) *& ipſæ rectæ lineæ* A, B, C, D *proportionales erunt.*

Nam rationes parallelepipedorum *a* triplicatæ ſunt rationum, quas habent lineæ. ergo ſi A.B :: C. D. *b* erit Ppp. A. Ppp. B :: Ppp. C. Ppp. D. & vice verſa. a 33. 11. b *ſch* 23. 5.

PROP.

PROP. XXXVIII.

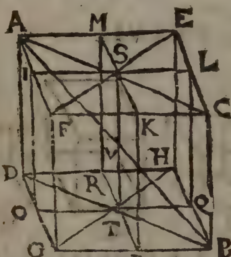



*Si planum* AB *ad planum* AC *rectum fuerit, & ab aliquo puncto* E *eorum, quæ ſunt in uno planorum* (AB) *ad alterum planum* AC *perpendicularis* EF *ducta fuerit, in planorum communem ſectionem* AD *cadet ducta perpendicularis* EF.

Si fieri poteſt, cadat F extra interſectionem AD. In plano AC *a* ducatur FG perpendicularis ad AD, jungaturque EG. Angulus FGE *b* rectus eſt; & EFG rectus ponitur. ergo in triangulo EFG ſunt duo anguli recti. Q. E. A.

a 11. 1.
b 4. & 3. def. 11.
c 17. 1.

PROP. XXXIX.




*Si ſolidi parallelepipedi* AB, *eorum quæ ex adverſo planorum* AC, DB *latera* (AE, FC, AF, EC, & DH, GB, DG, HB) *bifariam ſecta ſint; per ſectiones autem plana* ILQO, PKMR *ſint extenſa; planorum communis ſectio* ST, *& ſolidi parallelepipedi diameter* AB, *bifariam ſe mutuo ſecabunt.*

Ducantur rectæ SA, SC, TD, TB. Propter *a* latera DO, OT lateribus BQ, QT, *b* angulosque alternos TOD, TQB æquales, *c* etiam baſes DT, TB, & anguli DTO, BTQ æquantur. *d* ergo DTB eſt recta linea. eodem modo ASC recta eſt linea. Porro *e* tam AD ad FG, *e* quam FG ad CB; *f* ideoque AD ad CB, *g* ac proinde AC ad DB parallelæ & æquales ſunt.

a 34. 1.
b 29. 1.
c 4. 1.
d ſch. 15. 1.
e 34. 1.
f 9. 11. & 1 ax.
g 33. 1.

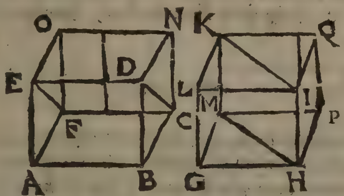
*h* quare

quare A B, & S T in eodem plano ABCD exſi-tunt. Itaque cum anguli AVS, BVT ad verticem, & alterni ASV, BTV æquentur; k & AS = BT; erit AV = BV, l & SV = VT. Q. E. D. h 7. 11. k 7. ax. 1. l 26. 1.

*Coroll.*

Hinc, in omni parallelepipedo diametri omnes ſe mutuo biſecant in uno puncto, V.

P R O P. XL.




*Si fuerint duo priſmata* ABCFED, GHMLIK *equalis altitudinis, quorum hoc quidem habeat baſim* ABCF *parallelogrammum, illud vero* GHM *triangulum; duplum autem fuerit parallelogrammum* ABCF *trianguli* GHM; *æqualia erunt ipſa priſmata* ABCFED, GHMLIK.

Nam ſi perficiantur parallelepipeda AN, GQ, a erunt hæc æqualia ob b baſium AC, GP, & c altitudinum æqualitatem. d ergo etiam priſmata, e horum dimidia, æqualia erunt. Q. E. D. a 31. 11. b 34. 1. & 7. ax. c hyp. d 28. 11. e 7. ax. 1.

*Schol.*

*Ex hactenus demonſtratis habetur dimenſio priſmatum triangularium, & quadrangularium, ſeu parallelepipedorum, ſi nimirum altitudo ducatur in baſim.* Andr. Tacq.

Ut ſi altitudo ſit 10 pedum, baſis vero pedum quadratorum 100 (menſurabitur autem baſis per ch. 35. 1. vel per 41. 1.) multiplica 100 per 10;

T pro-

proveniunt 1000 pedes cubici pro ſoliditate priſmatis dati.

*Vide ſchol. 35. 1.* Nam quemadmodum rectangulum, ita & parallelepipedum rectum producitur ex altitudine ducta in baſim. Ergo quodvis parallelepipedum producitur ex altitudine in baſim ducta, ut patet ex 31. hujus.

Deinde cum totum parallelepipedum producatur ex altitudine in totam baſim, ſemiſſis ejus (hoc eſt priſma triangulare) producetur ex altitudine ducta in dimidiam baſim, nempe triangulum.

*Monitum.*

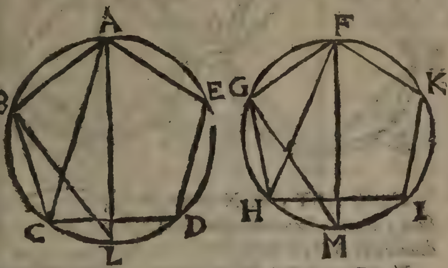
*Nota, litterarum quæ deſignant angulum ſolidum primam eſſe ſemper ad punctum, in quo eſt angulus; litterarum vero quæ denotant pyramidem, ultimam eſſe ad verticem pyramidis.*

Ex. gr. Angulus ſolidus ABCD eſt ad punctum A; pyramidis quoque BCDA vertex eſt ad punctum A, & baſis triangulum BCD.

LIB.

# LIB. XII.

## PROP. I.




*QVæ ſunt in circulis* ABD, FGI *polygona ſimilia* ABCDE, FGHIK, *inter ſe ſunt, ut quadrata à diametris* AL, FM.

Ducantur AC, BL, FH, GM. Quoniam *a* ang. ABC = FGH, *a* atque AB. BC :: FG. GH, *b* erit ang. ACB (*c* ALB) = FHG (*c* FMG.) anguli autem ABL, FGM *d* recti, ac proinde æquales ſunt. *e* ergo triangula ABL, FGM æquiangula ſunt. *f* quare AB. FG :: AL. FM. *g* ergo ABCDE. FGHIK :: ALq. FMq.

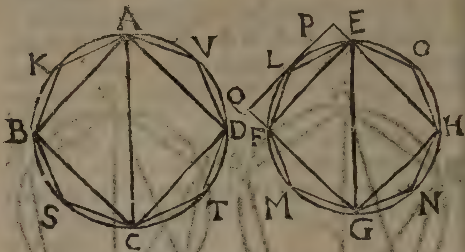
a 1. def. 6.
b 6. 6.
c 21. 3.
d 31. 3.
e 32. 3.
f cor. 4. 6.
g 22. 6.

*Coroll.*

Hinc (quia AB. FG :: AL. FM :: BC. GH, &c.) polygonorum ſimilium circulo inſcriptorum *h* ambitus ſunt ut diametri.

h 1. 12. & 12. 5.

## PROP. II.










*Circuli* ABT, EFN *inter ſe ſunt, quemadmodum quadrata à diametris* AC, EG.

Ponatur ACq. EGq :: circ. ABT. I. Dico I = circ. EFN.

Nam primo, ſi fieri poteſt, ſit I ⊐ circ. EFN, ſitque exceſſus K. Circulo EFN inſcribatur
a ſch. 7. 4. quadratum EFGH, *a* quod dimidium eſt circumſcripti quadrati, adeoque ſemicirculo majus.
b 30. 3. *b* Biſeca arcus EF, FG, GH, HE, & ad puncta biſectionum junge rectas EL, LF, &c. per L
c ſch. 27. 3. duc tangentem PQ (*c* quæ ad EF parallela eſt,) & produc HEP, GFQ; eſtque triangulum
d 41. 1 ELF *d* dimidium parallelogrammi EPQF, adeoque majus dimidio ſegmenti ELF; pariterque reliqua triangula ejuſmodi reliquorum ſegmentorum dimidia ſuperant. Et ſi iterum biſecentur arcus EL, LF, FM, &c. rectæque adjungantur, eodem modo triangula ſegmentorum ſemiſſes excedent. Quare ſi quadratum EFGH è circulo EFN, & è reliquis ſegmentis triangula
e 1. 10. detrahantur, & hoc fiat continuo, tandem *e* reſtabit magnitudo aliqua minor quam K. Eouſque perventum ſit; nempe ad ſegmenta EL, LF, FM, &c. minora quam K, ſimul ſumpta.

ota. ergo I (*f* circ. EFN − K) ⊐ polyg. *f hyp. & 3. ax.*
ELFMGNHO (circ. EFN − ſegm. EL + LF
&c.) Circulo ABT inſcriptum *g* puta ſimile po- *g 30. 3. & 1 poſt. 1.*
lygonum AKBSCTDV. itaque quum
AKBSCTDV. ELFMGNHO *h* :: ACq. *h 1. 12.*
EGq *k* :: circ. ABT. I. ac polyg. AKBSCTDV *k hyp.*
⊐ circ. ABT. *l m* erit polyg. ELFMGNHO *l 9 ax. 1. m 14. 5.*
⊐ I. ſed prius erat I ⊐ ELFMGNHO. quæ
repugnant.

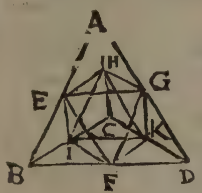
Rurſus, ſi fieri poteſt, ſit I ⊏ circ. EFN. Quoniam igitur ACq. EGq *n* :: circ. ABT. I; *n hyp.* inverſeque I. circ. ABT :: EGq. ACq. pone I. circ. ABT :: circ. EFN. K. *o* ergo circ. ABT *o 14. 5.* ⊏ K. *p* atque EGq. ACq :: circ. EFN. K. Quæ *p 11. 5.* repugnare modo oſtenſum eſt.

Ergo concludendum eſt, quod I = circ. EFN. Q. E. D.

*Coroll.*

Hinc, ut circulus eſt ad circulum, ita polygonum in illo deſcriptum ad ſimile polygonum in hoc deſcriptum.

## PROP. III.




*Omnis pyramis* ABDC *triangularem habens baſim, dividitur in duas pyramides* AEGH, HIKC *æquales & ſimiles inter ſe, triangulares habentes baſes, & ſimiles toti* ABDC; *& in duo priſmata æqualia* BFGEIH, EGDIHK; *quæ duo priſmata majora ſunt dimidio totius pyramidis* ABDC.

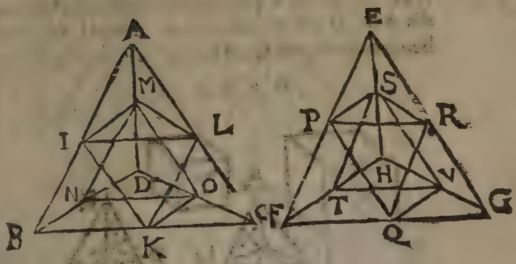
Latera pyramidis biſecentur in punctis E, F, G, H, I, K; junganturque rectæ EF, FG, GE, EI, IF, FK, KG, GH, HE. Quoniam latera

 pyra-

pyramidis proportionaliter ſecta ſunt, *a* erunt
a 2. 6. HI, AB; & GF, AB; & IF, DC; atque HG,
DC, &c. parallelæ; proinde & HI, FG, & GH,
FI parallelæ ſunt. liquet igitur triangula ABD,
b 29. 1. AEG, EBF, FDG, HIK *b* æquiangula eſſe; &
quatuor ultima *c* æquari. eodem modo triangula
c 26. 1. ACB, AHE, EIB, HIC, FGK æquiangula ſunt;
& quatuor poſtrema inter ſe æqualia. ſimiliter
triangula BFI, FDK, IKC, EGH; & denuo
triangula AHG, GDK, HKC, EFI, ſimilia ſunt
& æqualia. Quinetiam triang. HIK ad ADB, &
EGH ad BDC, & EFI ad ADC, & FGK ad
ABC *d* parallela ſunt. Ex quibus perſpicue ſequi-
d 15. 11. tur primo, pyramides AEGH, HIKC æquales
e 10. def. 11. eſſe; totique ABDC, & inter ſe *e* ſimiles. deinde
ſolida BFGEIH, FGDIHK priſmata eſſe, &
quidem æque alta, nempe ſita inter parallela pla-
na ABD, HIK. verum baſis BFGE baſis FDG
f 1. ax. 1. *f* duplex eſt. quare dicta priſmata æqualia ſunt.
g 40. 11. quorum alterum BFGEIH pyramide BEFI, hoc
eſt, AEGH majus eſt, totum ſua parte; proinde
duo priſmata majora ſunt duabus pyramidibus,
totiuſque adeo pyramidis ABDC dimidium ex-
cedunt. Q. E. D.

PROP.

## PROP. IV.




*Si fuerint duæ pyramides* ABCD , EFGH *ejuſ-dem altitudinis , triangulares habentes baſes* ABC, EFG; *ſit autem illarum utraque diviſa & in duas pyramides* (AILM,MNOD; & EPRS , STVH) *æquales inter ſe , & ſimiles toti; & in duo priſmata æqualia* (IBKLMN , KLCNMO ; & PFQRST, QRGTSV;) *ac eodem modo diviſa ſit utraque py-ramidum, quæ ex ſuperiore diviſione natæ ſunt , id-que ſemper fiat; erit ut unius pyramidis baſis ad al-terius pyramidis baſim , ita & omnia , quæ in una pyramide, priſmata ad omnia , quæ in altera pyra-mide priſmata, multitudine æqualia.*

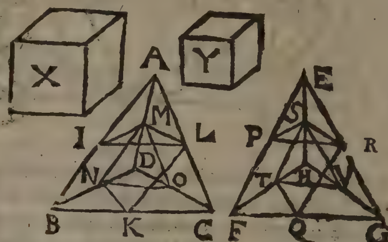
Nam (adhibendo conſtructionem præceden- a 15. 5.
tis) BC. KC *a* :: FG. QG. *b* ergo triang. ABC b 22. 6.
eſt ad *c* ſimile triang. LKC , ut EFG ad ſimile c 2.6.&c.
RQG. ergo permutando ABC. EFG *d* :: LKC. d 16. 5.
RQG *e* :: Priſm. KLCNMO. QRGTSV (nam e *ſch.* 34 11.
hæc æque alta ſunt) *f* :: IBKLMN. PFQRST. f 7. 5.
*g* quare triang. ABC. EFG :: Priſm. KLCNMO g 11. 5.
+ IBKLMN. Priſm. QRGTSV + PFQRST.
Q. E. D.

Sin ulterius ſimili pacto dividantur pyrami-des MNOD, AILM ; & EPRS , STVH , erunt quatuor nova priſmata hic effecta ad quatuor iſthic

iſthic producta, ut baſes MNO & AIL ad baſe STV & EPR, hoc eſt ut LKC ad RQG, vel u ABC ad EFG. *b* quare omnia priſmata pyramidis ABCD ad omuia ipſius EFGH ita ſe habent, ut baſis ABC ad baſim EFG. Q. E. D.

b 12. 5.

P R O P. V.




*Sub eadem altitudine exiſtentes pyramides* ABCD, EFGH, *triangulares habentes baſes* ABC, EFG, *inter ſe ſunt ut baſes* ABC, EFG.

Sit triang. ABC. EFG :: ABCD. X. Dico X = pyr. EFGH. Nam, ſi poſſibile eſt, ſit X ⊐ EFGH; ſitque Y exceſſus. Dividatur pyramis EFGH in priſmata & pyramides, & reliquæ pyramides ſimiliter, *a* donec relictæ pyramides EPRS, STVH minores evadant ſolido Y. Quum igitur pyr. EFGH = X + Y; liquet reliqua priſmata PFQRST, QRGTSV ſolido X majora eſſe. Pyramidem ABCD ſimiliter diviſam concipe; *b* eritque priſm. IBKLMN + KLCNMO. PFQRST + QRGTSV :: ABC. EFG. *c* :: pyr. ABCD. X. *d* ergo X ⊏ priſm. PFQRST + QRGTSV; quod repugnat prius affirmatis.

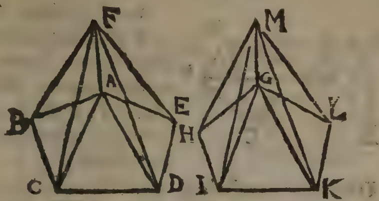
a 1. 10.
b 4. 12.
c hyp.
d 14. 5.

Rurſus, dic X ⊏ pyr. EFGH. pone pyr. EFGH. Y :: X. pyr. ABCD *e* :: EFG. ABC. quia EFGH *f* ⊐ X, *g* erit Y ⊐ pyr. ABCD, quod fieri nequit, ex jam dictis. Concludo igitur, quod X = pyr. EFGH. Q.E.D.

e hyp. & cor. 4 5.
f ſuppoſ.
g 14. 5.

P R O P.

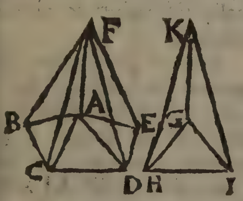
PROP. VI.




*Sub eadem altitudine exiſtentes pyramides ABCDEF, GHIKLM, & polygonas habentes baſes ABCDE, GHIKL, inter ſe ſunt ut baſes ABCDE, GHIKL.*

Duc rectas AC, AD, GI, GK. Eſt baſ. ABC. ACD *a* :: pyr. ABCF. ACDF. *b* ergo compoſite ABCD. ACD :: pyr. ABCDF. ACDF. *a* atqui etiam ACD. ADE :: pyr. ACDF. ADEF. *c* ergo ex æquali ABCD. ADE :: ABCDF. ADEF. *b* ergo componendo ABCDE. ADE :: pyr. ABCDEF. ADEF. porro ADE. GKL *d* :: pyr. ADEF. GKLM; ac, ut prius, atque inverſe GKL. GHIKL :: pyr. GKLM. GHIKLM. *c* ergo iterum ex æqualibus, ABCDE. GHIKL :: Pyr. ABCDEF. GHIKLM. Q. E. D.

a 5. 12.
b 18. 5.
c 22. 5.
d 5. 12.

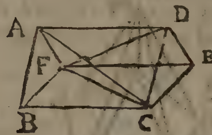



Si baſes non habent latera æque multa, demonſtratio ſic procedet. Baſ. ABC. GHI *e* :: pyr. ABCF. GHIK. *e* atque ACD. GHI :: pyr. ACDF. GHIK. *f* ergo baſ. ABCD. GHI :: pyr. ABCDF. GHIK. *e* Quinetiam baſ. ADE. GHI :: pyr. ADEF. GHIK. *f* ergo baſ. ABCDE. GHI :: pyr. ABCDEF. GHIK.

e 5. 12.
f 24. 5.

PROP.

## PROP. VII.

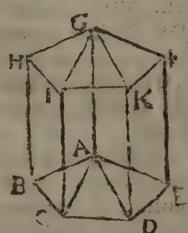



*Omne priſma* ABCDFE *triangularem habens baſim, dividitur in tres pyramides* ACBF, ACDF, CDFE *æquales inter ſe, triangulares baſes habentes.*

Ducantur parallelogrammorum diametri AC, CF, FD. Triang. ACB *a* = ACD. *b* ergo æque altæ pyramides ACBF, ACDF æquantur. eodem modo pyr. DFAC = pyr. DFEC. atqui ACDF, & DFAC una eademque ſunt pyramis. *c* ergo tres pyramides ACBF, ACDF, DFEC, in quos diviſum eſt priſma, inter ſe æquales ſunt. Q. E. D.

a 34. 1.
b 5. 12.
c 1. ax. 1.

*Coroll.*



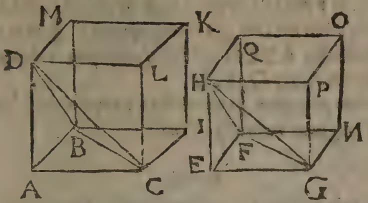

Hinc, quælibet pyramis tertia eſt pars priſmatis eandem cum illa habentis & baſim & altitudinem: ſive, priſma quodlibet triplum eſt pyramidis eandem cum ipſo habentis baſim & altitudinem.

Nam reſolve priſma polygonum ABCDEGHIKF in trigona priſmata, & pyramidem ABCDEH in trigonas pyramides. *a* Erunt ſingulæ partes priſmatis triplæ ſingularum partium pyramidis. *b* proinde totum priſma ABCDEGHIKF totius pyramidis ABCDEH triplum eſt Q. E. D.

a 7. 12.
b 1. 5.

PROB.

PROP. VIII.




*Similes pyramides* ABCD, EFGH, *quæ triangulares habent baſes* ABC, EFG, *in triplicata ſunt ratione homologorum laterum* AC, EG.

*a* Perficiantur parallelepipeda ABICDMKL, EFNGHQOP; quæ *b* ſimilia ſunt & pyramidum ABCD, EFGH *c* ſextupla; *d* ideoque in eadem cum ipſis ratione ad ſe invicem, *e* hoc eſt in triplicata homologorum laterum. Q. E. D.

a 17. 11.
b 9. *def* 11.
c 28. 11. & 7. 12.
d 15. 5.
e 33. 11.

*Coroll.*

Hinc, etiam ſimiles polygonæ pyramides rationem habent laterum homologorum triplicatam; ut facile probabitur reſolvendo has in trigonas pyramides,

PROP. IX.

*Vide Schema præced.*

*Æqualium pyramidum* ABCD, EFGH, *& triangulares baſes* ABC, EFG *habentium, reciprocantur baſes & altitudines. & quarum pyramidum triangulares baſes habentium reciprocantur baſes & altitudines, illæ ſunt æquales.*

1. *Hyp.* Perfecta parallelepipeda ABICDMKL, EFNGHQOP æqualium pyramidum ABCD, EFGH (utrumque utriuſque) *a* ſextupla ſunt, ac æqualia ideo inter ſe, ergo alt. (H.) alt.

a 28. 11. & 7. 12.

b 34. 11. alt. (D) [b] :: ABIC. EFNG [c] :: ABC. EFG.
c 15. 5. Q. E. D.

d hyp. 2. *Hyp.* Alt.(H.) alt. (D) [d] :: ABC.EFG [e] ::
e 15. 5. ABIC. EFNG. [f] ergo parallelepipeda ABIC-
f 34. 11. DMKL, EFNG*H*QOP æquantur; [g] proinde
g 6. ax. 1. & pyramides ABCD, EFG*H*, horum ſubſextu-plæ, pares ſunt. Q. E. D.

*Eadem polygonis pyramidibus conveniunt : nam hæ ad trigonas reduci poſſunt.*

*Coroll.*

*Quæ de pyramidibus demonſtrata ſunt Prop. 6, 8, 9. etiam conveniunt quibuſcunque priſmatis, cum hæc tripla ſint pyramidum eandem baſim & altitudinem habentium. itaque* 1. Priſmatum æque altorum eadem eſt proportio, quæ baſium.

2. Similium priſmatum proportio triplicata eſt proportionis laterum homologorum.

3. Æqualia priſmata reciprocant baſes & altitudines; & quæ reciprocant, ſunt æquales.

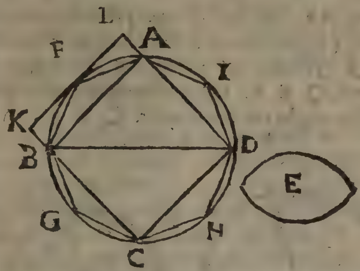
*Schol.*

Ex hactenus demonſtratis elicitur dimenſio quorumcunque priſmatum & pyramidum.

a cor. 1. hujus; & ſch. 40. 11. [a] Priſmatis ſoliditas producitur ex altitudine in baſim ducta; [b] itaque & pyramidis ex tertia
b 7. 12. altitudinis parte ducta in baſin.

PROP.

## PROP. X.




*Omnis conus tertia pars eſt cylindri habentis eandem cum ipſo baſin* ABCD, *& altitudinem æqualem.*

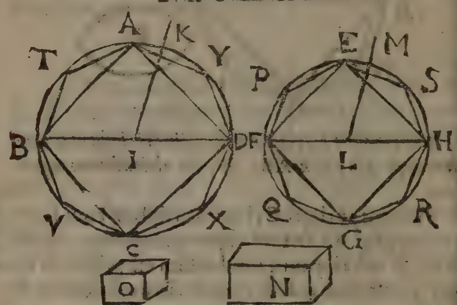
Si negas, primo Cylindrus triplum coni ſuperet exceſſu E. Priſma ſuper quadratum circulo ABCD inſcriptum *a* ſubduplum eſt priſmatis ſuper quadratum eidem circulo circumſcriptum ſibi & cylindro æque alti. ergo priſma ſuper quadratum ABCD ſuperat cylindri ſemiſſem. eodem modo priſma ſuper baſim AFB cylindro æque altum ſegmenti cylindrici AFB *b* dimidio majus eſt. Continuetur biſectio arcuum, & detrahantur priſmata, donec ſegmenta cylindri relicta, nempe ad AF, FB, &c. minora evadant ſolido E. Itaque cylind. — ſegment. AF, FB, &c. (priſma ad baſim AFBGCHDI) *c* majus eſt quam cylind. — E (*d* triplum coni.) ergo pyramis dicti priſmatis *e* pars tertia (ad eandem baſim ſita, ejuſdemque altitudinis) cono æque alto ad baſim ABCD circulum major eſt; pars toto. Q. E. A.

*Vide fig. 2. hujus.*
*a ſch. 7. 4. & cor. 9. 12.*
*b ſch. 27. 3. & cor. 9. 12.*
*c 5 ax. 1.*
*d hyp.*
*e cor. 7. 12.*

Sin conus tertia parte cylindri major dicatur, ſit itidem exceſſus E. Ex cono detrahe pyramides, ut in priori parte priſmata ex cylindro, donec reſtent coni ſegmenta aliqua, puta ad AF, FB,

FB, BG, &c. minora ſolido E. ergo con. — E
f hyp. (f ⅓ cylindr.) ⊐ pyr. AFBGCHDI (con. — ſegment. AF, FB, &c.) ergo priſma pyramidis triplum (æque altum ſcilicet atque ad eandem baſim) cylindro ad baſim ABCD majus eſt, pars toto. Q. E. A. Quare fatendum eſt, quod cylindrus triplo cono æquatur. Q. E. D.

PROP. XI.




*Sub eadem altitudine exiſtentes cylindri, & coni* ABCDK, EFGHM, *inter ſe ſunt ut baſes* ABCD, EFGH.

Sit circ. ABCD. circ. EFGH : : con. ABCDK. N. Dico N = con. EFGHM.

Nam ſi fieri poteſt, ſit N ⊐ con. EFGHM, ſitque exceſſus O. Suppoſita præparatione, & argumentatione præcedentis; erit O majus ſegmentis conicis EP, PF, FQ, &c. ideoque ſolidum N ⊐ pyr. EPFQGRHSM. *a* Fiat in circulo ABCD ſimile polygonum ATBVCXDY.

a 30. 3. & 1. poſt.
b 6 12.
c cor 2. 12.
d hyp.
e 14 5.

Quia pyr. ABVYK. pyr. EFQSM *b* : : polyg. ATBVY. polyg. EPFQS *c* : : circ. ABCD. circ. EFGH *d* : : con. ABCDK. N. *e* erit pyram. EPFQGRHSM ⊐ N. contra modo dicta.

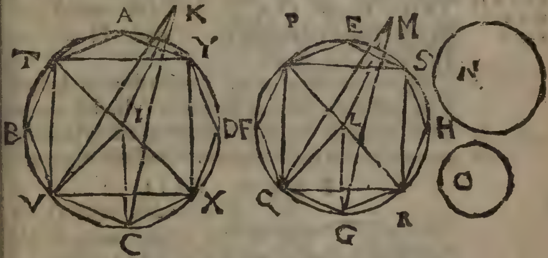
Rurſus dic N ⊏ con. EFGHM. pone con. EFGHM. O : : N. con. ABCDK *f* : : circ. EFGH. ABCD. *g* ergo O ⊐ con. ABCDK, quod

uod abſurdum eſt, ex oſtenſis in priori parte. f hyp. & invertendo.
Itaque potius dic, ABCD. EFGH :: con. g 14. 5.
BCDK. EFGHM. Q. E. D.
Idem demonſtrabitur de cylindris, ſi cononum & pyramidum loco concipiantur cylindri & priſmata. ergo, &c.

S C H O L.

*Ex his habetur dimenſio cylindrorum & conorum quorumcunque.* Cylindri rectæ ſoliditas producitur ex baſe circulari (*a* pro cujus dimenſione conſulendus eſt Archimedes) ducta in altitudinem. *b* igitur & cujuſcunque cylindri. a 1. Prop. de dimenſ. circ. b 11. 12.
*c* Itaque coni ſoliditas producitur ex tertia parte altitudinis ducta in baſim. c 10. 12.

P R O P. XII.




*Similes coni & cylindri* ABCDK, EFGHM, *in triplicata ratione ſunt diametrorum* TX, PR, *quæ in baſibus* ABCD, EFGH.

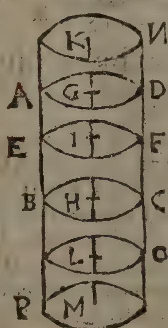
Habeat conus A ad aliquod N rationem triplicatam TX ad PR. dico N = con. EFGHM: Nam ſi fieri poteſt, ſit N ⊐ EFGHM; ſitque exceſſus O. ergo ut in Prioribus, N ⊐ pyr. EPFQGRHSM. Sint axes conorum IK LM, adducanturque rectæ VK, CK, VI, CI; & QM, GM, QL, GL. Quoniam coni ſimiles ſunt, *a* eſt VI. IK :: QL. LM. anguli vero VIK, QLM *b* recti ſunt. *c* ergo trigona VIK, QLM, a 24. def. 11. b 18. def. 11. c 6. 6.

d 4. 6. QLM æquiangula ſunt; *d* unde VC. VI :: QG.
QL. item VI. VK :: QL. QM. ergo ex æ-
e 7. 5. quali V C. VK :: QG.QM. *e* quinetiam V *K*.
C*K* :: QM. M G. ergo rurſus ex æquo V C.
f 5. 6. C*K* :: QG. GM. *f* ergo triangula V *K* C,
Q*M*G ſimilia ſunt; ſimilique argumento reliqua
g 9. def. 11. hujus pyramidis triangula reliquis illius. *g* quare
h cor. 8. 12. pyramides ipſæ ſimiles ſunt. *h* ſunt vero hæ in
k 4. 6. triplicata ratione VC ad QG, *k* hoc eſt VI ad
l 15. 5. RL, *l* vel TX ad PR. *m* ergo Pyr. A I B V C-
m hyp. & 11. 5 XDYK.pyr. EPFQGRHS*M* :: con. ABCD*K*.
n 14. 5. N. *n* unde pyr. EPFQGRHS*M* ⊐ N; quod
repugnat prius dictis.

Rurſus, dic N. ⊏ con. E F G *H M*. ſit con.
o Prius, & inverſe. EFGHM. O : N. con. A B C D *K* *o* :: pyr.
E *P R* M. A T C *K* *p* :: G Q. VC ter :: *q* *PR*.
p cor. 8. 12. TX ter. verum O *r* ⊐ ABCD*K*. quod mo-
q 4. 6.
r 14. 5. do repugnare oſtenſum eſt. Proinde N = con.
EFGH*M*. Q. E. D.

Quoniam vero quam proportionem habent coni, eandem quoque obtinent cylindri, eorum tripli, habebit quoque cylindrus ad cylindrum proportionem diametrorum in baſibus triplicatã.

## PROP. XIII.

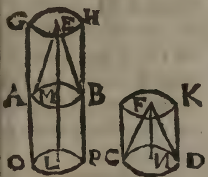


*Si cylindrus* ABCD *plano* EF *ſecetur adverſis planis* BC, AD *parallelo; erit ut cylindrus* AEFD *ad cylindrum* EBCF, *ita axis* GI *ad axem* IH.

a 3. 1. Producto axe, *a* ſume G *K* = GI, & HL = I *H* = L *M*. & concipe per puncta *K*, L, *M*, plana duci circulis AD, BC paral-
b 11. 12. lela. *b* ergo cylind. E D = cyl. AN. & cylin. EC *b* = BO *b* = OP. itaque cylindrus

drus EN cylindri ED æque multiplex eſt, ac axis IK axis IG. pariterque cylindrus FP æque multiplex eſt cylindri BF, ac axis IM axis IH. prout vero IK =, ⊏, ⊐ IM, *c* ſic cylindr. c 11. 12.
EN =, ⊏, ⊐ EP. *d* ergo cyl. AEFD. cyl. d 6. def. 5.
EBCF :: GI. IH. Q. E. D.

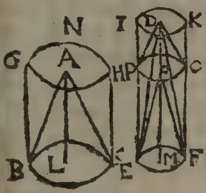
PROP. XIV.




*Super æqualibus baſibus* AB, CD *exiſtentes coni* AEB, CFD, *& cylindri* AH, CK, *inter ſe ſunt ut altitudines* ME, NF.

Productis cylindro HA & axe EM, ſume MI = FN; & per punctum L ducatur planum baſi AB parallelum. *a* erit cyl. AP = CK. *b* atqui cylind. AH. a 11. 12.
AP. (CK) :: ME. ML. (NF.) Q. E. D. b 13. 12.
Idem de conis cylindrorum ſubtriplis dictum puta. * imo de priſmatis & pyramidibus. * Adhibe 9. & 7. 12.

PROP. XV.



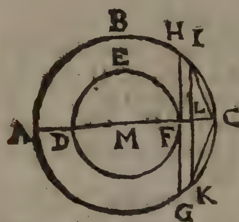

*Æqualium conorum* BAC, EDF, *& cylindrorum* BH, EK, *reciprocantur baſes & altitudines* (BC. EF :: MD. LA:) *& quorum conorum, & cylindrorum reciprocantur baſes & altitudines, illi ſunt æquales.*

Si altitudines pares ſint, etiam baſes pares erunt; & res clara eſt. Sin altitudines ſint impares, aufer MO = LA. a 14. 12.
1. *Hyp.* Eſtque MD. MO (*a* LA) *b* :: cyl. b conſtr.
EK (*c* BH.) EQ *d* :: circ. BC. EF. Q. E. D. c hyp. d 11. 12.

a hyp. f 11. 12. g 11. 5. h 11. 12. k 9. 5. 2. *Hyp.* BC. EF e :: DM. OM (L A) f :: Cyl. EK. EQ g :: BC. EF h :: BH. EQ. k Ergo cylind. EK = BH. Q. E. D.

Simili argumento utere de conis.

## PROP. XVI.




*Duobus circulis* ABCG, DEF *circa idem centrum* M *exiſtentibus, in majori circulo* ABCG *polygonum æquilaterum, & parium laterum inſcribere, quod non tangat minorem circulum* DEF.

Per centrum M extendatur recta A C ſecans circulum D E F in

a 30. 3. F. ex quo erige perpendicularem FH. a Biſeca ſemicirculum ABC, ejuſque ſemiſſem BC, atq; ita continuo, b donec arcus IC minor evadat

b 1. 10. arcu HC. ab I demitte perpendicularem IL. Liquet arcum IC totum circulum metiri, numerumque arcuum eſſe parem, adeoque ſubtenſam

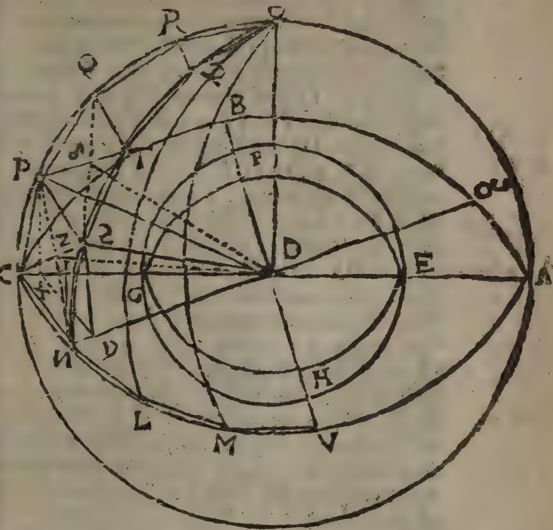
c ſch. 16. 4. IC latus eſſe c polygoni inſcriptibilis, quod circulum DEF minime continget. Nam HG d tan-

d cor. 16. 3. git circulum DEF; e cui parallela eſt IK, ex-

e 28. 1. f 34. def. 1. traque ſita, f quare IK circulum non tangit, multoque magis CI, CK, & reliqua polygoni latera, longius à centro diſtantia, circulum DEF non tangunt. Q. E. F. Coroll. Nota, quod IK non tangit circulum D E F.

PROP.

PROP. XVII.




*Duabus ſphæris* ABCV, EFGH *circa idem centrum* D *exiſtentibus, in majori ſphæra* ABCV *ſolidum polyedrum inſcribere, quod non tangat ſuperficiem minoris ſphæræ* EFGH.

Secentur ambæ ſphæræ plano per centrum faciente circulos EFGH, ABCV. ducanturque diametri A C, BV ſecantes perpendiculariter. Circulo ABCV *a* inſcribatur polygonum æquilaterum VMLNC, &c. circulum E F G H minime tangens. ducta diametro Nα, erectaque DO recta ad planum ABC. per DO, perq; diametros A C, Nα erigi concipiantur plana DOC, DON, quæ ad circulum ABCV *b* recta erunt, ideoque in ſuperficie ſphæræ *c* quadrantes effici-

a 16. 12.

b 18. 11.

c cor. 31. 6.

d 4. 1. efficient *c* DOC, DON. in quibus *d* aptentur rectæ CP, PQ, QR, RO, NS, ST, T$\gamma$, $\gamma$O ipſis CN, NL, &c. pares, & æque multæ. In reliquis quadrantibus OL, OM, &c. inque tota ſphæra eadem conſtructio fiat. Dico factum.

A punctis P, S ad planum A B C V demitte
e 38. 11. perpendiculares PX, SY, *e* quæ in ſectiones AC,
f 12 ax. Na cadent. Quoniam igitur tam *f* anguli recti
g 27. 3. PXC, SYN, *g* quam P C X, S N Y *h* æqualibus peripheriis inſiſtentes, *f* pares ſunt, triangula
h 32. 1. PCX, SNY *h* æquiangula ſunt. Cum igitur PC
k conſtr. *k* = SN, *l* etiam PX = SY, *l* & XC = YN;
l 26. 1.
m 3. ax. 1. *m* quare DX = DY. *n* ergo DX. XC :: DY.
n 7. 5.
o 2. 6. YN. *o* ergo parallelæ ſunt YX, NC. quia vero
p 6. 11. PX, SY pares, & cum eidem plano ABCV re-
q 33. 1. ctæ, etiam *p* parallelæ ſunt, *q* erunt YX,
r 9. 11. S P etiam pares & parallelæ. *r* ergo SP, NC
ſ 7. 11. inter ſe parallelæ ſunt. ergo *ſ* quadrilaterum
t 2. 11. NCPS, eademque ratione S P Q T, T Q R G, ſed & *t* triangulum $\gamma$RO totidem ſunt plana. Eodem modo tota ſphæra ejuſmodi quadrilateris & triangulis repleta oſtendetur. quare inſcriptum eſt polyedrum.

u 11. 11. A centro D *u* duc DZ rectum plano NCPS; & junge ZN, ZC, ZS, ZP. Quoniam D N.
x 4. 6. NC *x* :: DY. YX; eſt NC *y* $\sqsubset$ YX (SP;) pa-
y 14. 5. riterque SP $\sqsubset$ TQ, & TQ $\sqsubset$ $\gamma$R. Et quia
z 3. def. 11. anguli DZC, DZN, DZS, DZP, *z* recti ſunt,
a 15. def. 1. latera vero DC, DN, DS, DP *a* æqualia, &
b 47. 1. DZ commune, *b* erunt ZC, ZN, ZS, ZP æquales inter ſe; proinde circa quadrilaterum
c 15. def. 1. NCPS *c* deſcribi poteſt circulus, in quo (ob NS, NC, CP *d* æquales, & NC $\sqsubset$ SP) NC
d conſtr. *e* pluſquam quadrantem ſubtendit. *f* ergo ang.
e 28. 3. NZC ad centrum obtuſus eſt. *g* ergo NCq $\sqsubset$
f 33. 6. 2 ZCq (ZCq + ZNq.) Sit NI ad AC nor-
g 12. 2. malis. ergo cum ang. A D N (*h* D N C +
h 32. 1.
k 9. ax. 1. DCN) ſit *k* obtuſus, *l* erit ſemiſſis ejus D C N
l 5. 1.
recti

ꝺti ſemiſſe major; proptereaque eo minor eſt
eliquus è recto ang. CNI. [n] unde IN ⊏ IC. n 19. 1.
rgo NCq (NIq + ICq) [o] ⊐ 2 INq. itaque o 47. 1.
N ⊏ ZC.& conſequenter DZ [p] ⊏ DI. atqui p 47. 1.
unctum I eſt [q] extra ſphæram EFGH. ergo q cor.16.12.
unctum Z potiori jure eſt extra ipſam. adeoque
lanum NCPS (cujus [r] proximum centro pun- r 47. 1.
tum eſt Z) ſphæram EFGH non contingit. Et
i ad planum SPQT demittatur perpendicularis
D δ, punctum δ, adeoque & planum SPQT
adhuc ulterius à centro elongatur; idemque eſt
de reliquis polyedri planis. ergo polyedrum
DRQPCN,&c. majori ſphæræ inſcriptum, mi-
norem non contingit. Q. E. F.

*Coroll.*

Hinc ſequitur, *Si in quavis alia ſphæra deſcribatur ſolidum polyedrum, ſimile prædicto ſolido polyedro, proportionem polyedri in una ſphæra ad polyedrum in altera eſſe triplicatam ejus quam habent ſphærarum diametri.*

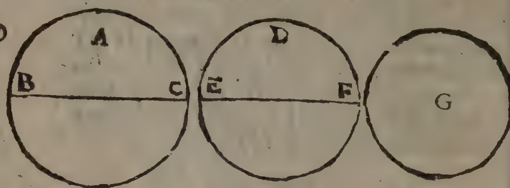
Nam ſi ex centris ſphærarum ad omnes angulos baſium dictorum polyedrorum rectæ lineæ ducantur, diſtribuentur polyedra in pyramides numero æquales & ſimiles, quarum homologa latera ſunt ſemidiametri ſphærarum; ut conſtat, ſi intelligatur harum ſphærarum minor intra majorem circa idem centrum deſcripta. congruent enim ſibi mutuo lineæ rectæ ductæ à centro ſphæræ ad baſium angulos, ob ſimilitudinem baſium, ac propterea pyramides efficientur ſimiles. Quare cum ſingulæ pyramides in una ſphæra, ad ſingulas pyramides illis ſimiles in altera ſphæra [a] habeant proportionem triplicatam laterum homologorum, hoc eſt, ſemidiametrorum ſphærarum; ſint autem [b] ut una pyramis ad unam pyramydem, ita omnes pyramides, hoc eſt, ſolidum polyedrum ex his compoſitum, ad omnes pyramides,

a cor.8. 12.
b 12.15.

mides, id eſt, ad ſolidum polyedrum ex illis conſtitutum; habebit quoque polyedrum unius ſphæræ ad polyedrum alterius ſphæræ proportio-
a 15. 5. nem triplicatam ſemidiametrorum, *c* atque adeo diametrorum.

## PROP. XVIII.

HO




*Sphæræ* BAC, EDF *ſunt in triplicata ratione ſuarum diametrorum* BCEF.

Sit ſphæra BAC ad ſphæram G in triplicata ratione diametri BC ad diametrum EF. Dico G $=$ EDF. Nam ſi fieri poteſt, ſit G $\sqsupset$ EDF. & cogita ſphæram G concentricam eſſe ipſi EDF.
a 17. 12. Sphæræ EDF *a* polyedrum ſphæram G non tan-
b cor. 17. 12. gens, ſphæræque BAC ſimile polyedrum inſcri-
c hyp. batur. *b* Hæc polyedra ſunt in triplicata ratione
d 14. 5. diametrorum BC, EF, *c* id eſt, ſphæræ BAC ad G. *d* Proinde ſphæra G major eſt polyedro ſphæræ EDF inſcripto, pars toto.

Rurſus, ſi fieri poteſt, ſit ſphæra G $\sqsubset$ EDF. Sitque ut ſphæra EDF ad aliam ſphæram H, ita
e hyp. inverſ. G ad BAC, *e* hoc eſt in triplicata ratione dia-
f 14. 5. metri EF ad BC; cum igitur BAC *f* $\sqsubset$ H, incurrimus abſurditatem prioris partis. Quin potius ſphæra G $=$ EDF. Q. E. D.

### *Coroll.*

Hinc, ut ſphæra ad ſphæram, ita eſt polyedrum in illa deſcriptum ad polyedrum ſimile in hac deſcriptum.

# LIB. XIII.

## PROP. I.

*SI recta linea z ſecundum extremam & mediam rationem ſecetur (z.a :: a.e;) majus ſegmentum a aſſumens dimidium totius z, quintuplum poteſt ejus, quod à dimidia totius z deſcribitur, quadrati.*




Dico Q. $a + \frac{1}{2}z = 5$ Q: $\frac{1}{2}z$. ^a hoc eſt $aa + \frac{1}{4}zz + za = zz + \frac{1}{4}zz$. ^b vel $aa + za = zz$. Nam $ze + za$ ^c $= zz$. & $ze$ ^d $= aa$. ^e ergo $aa + za = zz$. Q. E. D.

a 4. 2.
b 3. ax. 1.
c 2. 2.
d hyp. & 16. 6.
e 1. ax. & 1. ax.

## PROP. II.

*Si recta linea* $\frac{1}{2}z + a$ *ſui ipſius ſegmenti* $\frac{1}{2}z$ *quintuplum poſſit, duplæ prædicti ſegmenti* (z) *extrema ac media ratione ſectæ majus ſegmentum eſt* a, *reliqua pars ejus quæ à principio rectæ* $\frac{1}{2}z + a$.

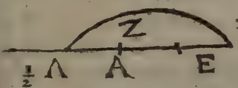
Dico z.a :: a.e. Nam quia per hyp. * $aa + \frac{1}{4}zz + za = zz + \frac{1}{4}zz$; vel $aa + za = zz$ ^a $= ze + za$. ^b erit $aa = ze$. ^c quare z. a :: a. e. Q. E. D.

* 4. 2.
a 2. 2.
b 3 ax. 1.
c 17. 6.

*Vide fig. præced.*

## PROP. III.

*Si recta linea* z *ſecundum extremam ac mediam rationem ſecetur* ( z. a :: a. e; ) *minus ſegmentum* e *aſſumens dimidium majoris ſegmenti* a , *quintuplum poteſt ejus, quod à dimidia majoris ſegmenti* a *deſcribitur, quadrati.*

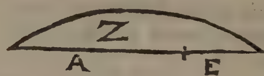



Dico Q: $e + \frac{1}{2}a = 5$ Q: $\frac{1}{2}a$. ^a hoc eſt $ee + \frac{1}{4}aa + ea = aa + \frac{1}{4}aa$. ^b vel $ee + ea = aa$. Nam $ee + ea$ ^c $= ze$ ^d $= aa$. Q. E. D.

a 4 2.
b 3. ax.
c 3 2.
d hyp. & 17. 6.

PROP. IV.

*Si recta linea z ſecundum extremam ac mediam rationem ſecetur* (z. a :: a. e;) *quod à tota* z, *quodque à minori ſegmento* e, *utraque ſimul quadrata, tripla ſunt ejus, quod a majori ſegmento* a *deſcribitur, quadrati.*




Dico zz + ee = 3 aa. *a* vel aa + ee + 2 ae + ee = 3 aa. Nam ae + ee = ze *c* = aa. *d* ergo aa + 2 ae + 2 ee = 3 aa. Q. E. D.

a 4. 2.
b 3. 2.
c 17. 6.
d 2. ax.

PROP. V.

D A C B

---|---------|---------

*Si recta linea* AB *ſecundum extremam & mediam rationem ſecetur in* C, *apponaturque ei* AD *æqualis majori ſegmento* AC; *tota recta linea* DB *ſecundum extremam ac mediam rationem ſecatur, & majus ſegmentum eſt quæ à principio recta linea* AB.

a hyp. Nam quia AB. AD *a* :: AC. CB, invertendoque AD. AB :: CB. AC; erit componendo DB. AB :: AB. AC. (AD.) Q. E. D.

*Schol.*

Quod ſi fuerit BD. BA :: BA. AD. erit BA. AD :: AD. BA − AD. Nam dividendo eſt BD − BA (AD) BA :: BA − AD. AD. ergo inverſe, BA. AD :: AD. BA − AD. Q. E. D.

PROP. VI.

D A C B

---------|---------|---------

*Si recta linea rationalis* AB *extrema ac media ratione ſecetur in* C; *utrumque ſegmentorum* (AC, CB) *irrationalis eſt linea, quæ vocatur apotome.*

a 3. 1.
b 1. 13.
c 6. 10.
d hyp.
e ſch. 12. 10.

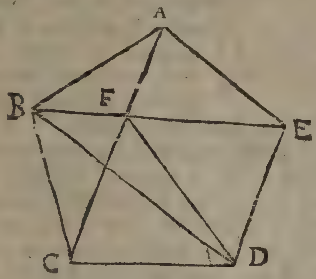
Majori ſegmento AC *a* adde AD = $\frac{1}{2}$ AB; *b* ergo DCq = 5 DAq. *c* ergo DCq ⊤⊥ $^{2}$DAq. proinde cum AB, *e* ideoque ejus ſemiſſis DA ſint $\rho'$, etiam DC eſt $\rho'$. Quia vero 5. 1 :: non

Q.

Q.Q. *f* eſt DC ⌴⊥ DA. *g* ergo DC − AD, id eſt AC eſt apotome. Inſuper quia ACq *h* = AB x BC, & AB eſt ρ', *k* etiam BC eſt apotome. Q.E.D.

f 9. 10.
g 74. 10.
h 17. 6.
k 98. 10.

P R O P. VII.




*Si pentagoni æquilateri* ABCDE *tres anguli, ſive qui deinceps* EAB, ABC, BCD, *ſive* EAB, BCD, CDE *qui non deinceps ſint, æquales fuerint, æquiangulum erit ipſum pentagonum* ABCDE.

Paribus deinceps angulis ſubtendantur rectæ BE, AC, BD.

Quoniam latera EA, AB, BC, CD, angulique incluſi *a* æquantur, *b* erunt baſes BE, AC, BD, *c* angulique AEB, ABE, BAC, BCA pares. *d* quare BF = FA, & *e* proinde FC = FE. ergo triangula FCD, FED ſibi mutuo æquilatera ſunt; *f* unde ang. FCD = FED, *g* proinde ang. AED = BCD. Eodem pacto ang. CDE reliquis æquatur. quare pentagonum æquiangulum eſt. Q.E.D.

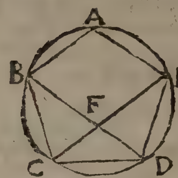
a hyp.
b 4. 1.
c 4. & 5. 1.
d 6. 1.
e 3. ax. 1.
f 8. 1.
g 2. ax. 1.

Sin anguli EAB, BCD, CDE, qui non deinceps, ſtatuantur pares, *h* erit ang. AEB = BDC, & BE = BD, *k* ideoque ang. BED = BDE; *l* totus proinde ang. AED = CDE. ergo propter angulos A, E, D deinceps æquales, ut prius, pentagonum æquiangulum erit. Q.E.D.

h 4. 1.
k 5. 1.
l 2. ax.

P R O P.

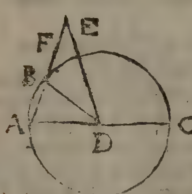
## PROP. VIII.




*Si pentagoni æquilateri & æquianguli* ABCDE *duos angulos* BCD, CDE, *qui deinceps ſint, ſubtendant rectæ lineæ* BD, CE; *hæ extrema ac media ratione ſe mutuo ſecant, & majora ipſarum ſegmenta* BF, *vel* EF *æqualia ſunt pentagoni lateri* BC.

a 14. 4. Circa pentagonum [a] deſcribe circulum ABD.
b 28. 3. [b] Arcus ED = BC. [c] ergo ang. FCD = FDC.
c 27. 3.
d 32. 1. [d] ergo ang. BFC = 2 FCD (FCD + FDC.) Atqui arcus BAE [b] = 2 ED, proinde ang.
e 33. 6. BCF [e] = 2 FCD = BFC. [f] quare BF = BC.
f 6. 1. Q. E. D. Porro quia triangula BCD, FCD
g 27. 3. [g] æquiangula ſunt, [h] erit BD. DC (BF) :: CD
h 4. 6. (BF.) FD. pariterque EC. EF :: EF. FC. Q. E. D.

## PROP. IX.




*Si hexagoni latus* BE, *& decagoni* AB, *in eodem circulo* ABC *deſcriptorum componantur, tota recta linea* AE *extrema ac media ratione ſecatur*, (AE. BE :: BE. AB.) *& majus ejus ſegmentum eſt hexagoni latus* BE.

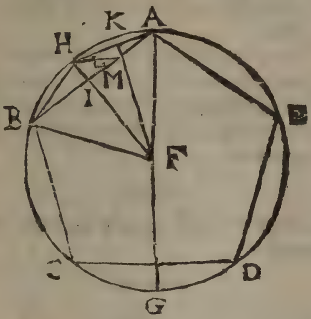
a hyp. & 37. 3. Duc diametrum ADC, & junge rectas DB, DE. Quoniam ang. BDC [a] = 4 BDA, eſtque
b 32. 1. ang. BDC [b] = 2 DBA (DAB + DBA,) erit
c 7 ax. 1. DBA ([b] BDE + BED) [c] = 2 BDA [d] = 2 BDE.
d 5. 1.
e 1. ax. 1. proinde ang. DBA, vel DAB [e] = ADE. Itaque
f 4. 6. trigona ADE, ADB æquiangula ſunt, [f] quare
g cor. 15. 4. AE. AD. ([g] BE) :: AD. (BE.) AB. Q. E. D.

Coroll.

### *Coroll.*

Hinc, ſi latus hexagoni alicujus circuli ſecetur extrema ac media ratione; majus illius ſegmentum erit latus decagoni ejuſdem circuli. *ſch* 5. 13.

### PROP. X.




*Si in circulo* ABCE *pentagonum æquilaterum* ABCDE *deſcribatur; pentagoni latus* AB *poteſt & hexagoni latus* FB, *& decagoni latus* AH, *in eodem circulo deſcriptorum.*

Duc diametrum AG. Biſeca arcum AH in K. Et duc FK, FH, FB, BH, HM.

Semicirc. AG − arc. AC *a* = AG − AD. a 28. 3. & 3. *ax.*
hoc eſt, arc. CG = GD *b* = AH = HB. ergo b *hyp.* & 7. *ax.*
arc. BCG = 2 BHK; *c* adeoque ang. BFG = 2 BFK. *d* ſed ang. BFG = 2 BAG. *e* ergo ang. c 33. 6. d 20. 3.
BFK = BAG. Trigona igitur BFM, FAB *f* æquiangula ſunt. *g* quare AB. BF :: BF. BM. e 1. *ax.* 1. f 32. 1.
*h* ergo AB x BM = BFq. Rurſus ang. AFK *k* = g 4. 6. h 17. 6.
HFK; & FA = FH; *m* quare AL = LH, *m* & k 27. 3.
anguli FLA, FLH pares, ac proinde recti ſunt. m 4. 1.
ergo ang. LHM *m* = LAM *n* = HBA. Trigo- n 27. 3.
na igitur AHB, AMH *o* æquiangula ſunt. *p* qua- o 32. 1. p 4. 6.

re

q 17. 6. re AB. AH :: AH. AM. [q] ergo AB x AM =
r 2. 2. AHq. Quum igitur ABq [r] = AB x BM + AB
ſ 2. ax. x AM, [ſ] erit ABq=BFq + AHq. Q. E. D.

*Coroll.*

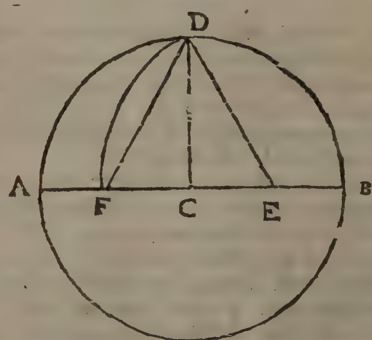
1. Hinc, linea recta (FK) quæ ex centro (F) arcum quempiam (H A) biſecat, etiam rectam (HA) illi arcui ſubtenſam biſecat ad angulos rectos.

2. Diameter circuli (AG) ex angulo quovis (A) pentagoni ducta biſecat & arcum (CD,) quem latus pentagoni illi angulo oppoſitum ſubtendit, & latus ipſum (CD) oppoſitum, idque ad angulos rectos.

*Schol.*

*Hic, ut promiſimus, praxim trademus expeditam problematis* 11. 4.

*Problema.*




*Invenire latus pentagoni circulo* ADB *inſcribendi.*

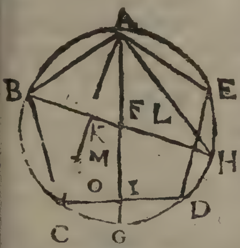
Duc diametrum A B. cui perpendicularem C D

C D ex centro C erige. Biſeca CB in E. Fac EF = ED. Erit DF pentagoni latus.

Nam BF x FC + ECq *a* = EFq *b* = EDq *c* = DCq + ECq. *d* ergo BF x FC = DCq, vel BCq. *e* quare *BF*. BC :: BC. FC. ergo quum BC ſit latus hexagoni, *f* erit F C latus decagoni, proinde DF *h* = √DCq + FCq *g* eſt latus pentagoni. Q. E. F.

a 6. 2.
b *conſtr.*
c 47. 1.
d 3. *ax.*
e 17. 6.
f 9. 13.
g 10. 13.
h 47. 1.

## PROP. XI.




*Si in circulo* ABCD *rationalem habente diametrum* AG, *pentagonum æquilaterum* ABCDE *deſcribatur; pentagoni latus* AB *irrationalis eſt linea, quæ vocatur minor.*

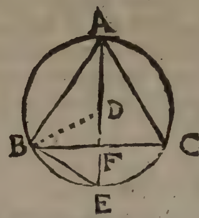
Duc diametrum BFH, rectaſque AC, AH; & * fac FL = $\frac{1}{4}$ radii FH, & CM = $\frac{1}{4}$ CA. * 10. 6.

Ob angulos AKF, AIC *a* rectos, & communem CAI, trigona AKF, AIC *b* æquiangula ſunt; *c* ergo CI. FK *c* :: CA. FA (FB) *d* :: CM. FL. ergo permutando FK. FL :: CI. CM *d* :: CD. CK (2 CM.) *e* componendo igitur CD + CK. CK :: KL. FL. *f* proinde Q: CD + CK (*g* 5 CKq.) CKq :: KLq. FLq. ergo KLq = 5 FLq. Itaque ſi BH (ρ̇) ponatur 8, erit FH 4; FL 1. & FLq. 1. BL 5. & BLq 25. KLq 5. è quibus liquet BL, & KL eſſe ρ̇ *h* ⊓. *k* ideoque BK eſſe Apotomen; cujus congruens KL. cum vero BLq − KLq = 20, *l* erit BL ⊓ √ BLq − KLq. *m* unde BK erit apotome quarta. Quoniam igitur ABq *m* = HB x BK, *n* erit AB minor. Q. E. D.

a *cor.* 10. 13.
b 32. 1.
c 4. 6.
d 15. 5.
e 18. 5.
f 22. 6.
g 1. 13.
h 9. 10.
k 74. 10.
l 9. 10.
m *cor.* 8. 6 & 17. 6.
n 95. 10.

PROP.

PROP. XII.




*Si in circulo ABEC triangulum æquilaterum ABC deſcribatur; trianguli latus AB potentia triplum eſt ejus lineæ AD, quæ ex D centro circuli ducitur.*

Protracta diametro ad E, duc BE. Quoniam arcus BE $a = $ EC, arcus BE ſexta eſt pars circumferentiæ. *b* ergo BE = DE. hinc AEq $c =$ 4 DEq (4 BEq) $d =$ ABq + BEq (+ ADq.) *e* proinde ABq = 3 ADq. Q. E. D.

a cor. 10. 13. b cor. 15 4. c 4 2. d 47. 1. e 3. ax 1.

*Coroll.*

1. AEq. ABq :: 4. 3.

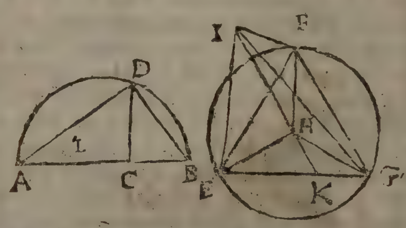
2. ABq. AFq :: 4. 3. *f* Nam ABq. AFq :: AEq. ABq.

3. DF = FE. Nam triang. EBD *g* æquilaterum eſt; *h* & BF ad ED perpendicularis. *h* ergo EF = FD.

4. Hinc AF = DE + DF = 3 DF.

f cor 8. 6. & 21 6. g cor. 15. 4. h cor. 3. 3.

PROP. XIII.




*Pyramidem EGFI conſtituere, & data ſphæra complecti; & demonſtrare quod ſphæræ diameter* AB

AB *potentia ſit ſeſquilatera lateris* E F *ipſius pyramidis* EGFI.

Circa A B deſcribe ſemicirculum A D B. a ſitque AC = 2 CB. ex puncto C erige perpendicularem CD ; & junge AD, DB. Tum radio HE = CD deſcribe circulum H E F G ; cui *b* inſcribe triangulum æquilaterum EFG. ex H *c* erige I H = C A rectum plano E F G, produc IH ad K ; *d* ita ut IK = AB. rectaſque adjunge IE, IF, IG. erit EFGI pyramis expetita. a 10. 6. b cor. 15. 4. c 12. 11. d 3. 1.

Nam quia anguli ACD, IHE, IHF, IHG *e* recti ſunt; & CD, HE, HF, HG *e* pares, *e* atque IH = AC; *f* erunt AD, IE, IF, IG æquales inter ſe. Quia vero A C (2 C B.) C B *g* :: ACq. CDq. erit ACq = 2 CDq. itaque ADq *f* = ACq + CDq *h* = 3 CDq = 3 HEq *k* = EFq. *l* ergo AD, EF, IE, IF, IG pares ſunt, adeoque pyramis EFGI eſt æquilatera. Quod ſi punctum C ſuper H collocetur, & AC ſuper H I, rectæ AB, IH *m* congruent, utpote æquales. quare ſemicirculus ADB axi AB vel IK circumductus *n* tranſibit per puncta, E, F, G, * adeoque pyramis EFGI ſphæræ inſcripta erit. Q. E. F. liquet vero eſſe BAq. ADq *o* :: BA. AC *p* :: 3. 2. Q. E. D. e conſtr. f 47. 1. g 20. 6. h 2. ax. k 12. 13. l 1. ax. 1. m 8 ax. 1. n 15 def 1. * 31. def. 11 o cor. 8 6. p conſtr.

### *Corollaria.*

1. ABq. HEq :: 9. 2. Nam ſi ABq ponatur 9, erit ACq (EFq) 6. *q* proinde HEq erit 2. q 12. 13.

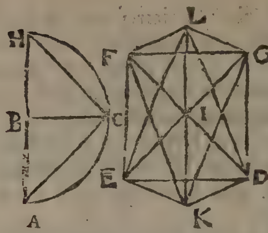
2. Si L centrum fuerit, erit AB. LC :: 6. 1. Nam ſi AB ponatur 6, erit AL, 3; *r* ideoque AC 4; quare LC erit 1. Hinc r conſtr.

3. AB. HI :: 6. 4 :: 3. 2. unde

4. ABq. HIq :: 9. 4.

PROP.

## PROP. XIV.




*Octaedrum* KEFGDL *conſtituere, & data ſphæra complecti, qua & pyramidem; & demonſtrare, quod ſphæræ diameter* AH *potentia ſit dupla lateris* AC *ipſius Octaedri.*

Circa AH deſcribe ſemicirculum ACH. ex centro B erige perpendicularem BC. duc AC,
a 46. 1. HC. Super ED=AC *a* fac quadratum EFGD, cujus diametri DF, EG ſecantes in centro I. ex
b 12. 11. c 3. 1. I duc IL=AB *b* rectam plano EFGD. produc IL, *c* donec IK=IL. Connexis KE, KF, KG, KD, LE, LF, LG, LD; erit KEFGDL octaedrum quæſitum.

Nam AB, BH, FI, IE, &c. æqualium quadra-
d 4. 1. torum ſemidiametri æquales ſunt inter ſe. *d* quare triangulorum rectangulorum LIE, LIF, FIE, &c. baſes LF, LE, FE, &c. æquantur. proinde octo triangula LFE, LFG, LGD, LDE, KEF,
e 27. def. 11. KFG, KGD, KDE æquilatera ſunt, *e* atque octaedrum conſtituunt, quod ſphæræ cujus centrum I, radius IL, vel AB, inſcribi poteſt. (quo-
f conſtr. niam AB, IL, IF, IK, &c. *f* æquales ſunt.) Q. E. F. porro liquet AHq (LKq) *g* = 2 ACq
g 47. 1. (2.LDq.) Q. E. D.

*Corollaria.*

1. Hinc manifeſtum eſt, in Octaedro tres diametros EG, FD, LK ſe mutuo ad angulos rectos ſecare in centro ſphæræ.

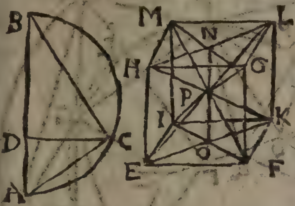
2. Item, tria plana EFGD, LEKG, LFKD eſſe quadrata, ſe mutuo ad angulos rectos ſecantia.

3. Octa-

3. Octaedrum dividitur in duas pyramides ſimiles & æquales EFGDL, & EFGDK, quarum baſis communis eſt quadratum EFGD.

4. Denique, baſes octaedri oppoſitæ, inter ſe parallelæ ſunt. 15. 11.

PROP. XV.



*Cubum EFGHIKLM conſtituere, & ſphæra complecti, qua & priores figuras; & demonſtrare, quod ſphæræ diameter AB potentia ſit tripla lateris EF ipſius cubi.*

Super AB deſcribe ſemicirculum ACB; & *a* fac AB = 3 DA. ex D erige perpendicularem DC, & junge BC ac AC. Tum ſuper EF = AC *b* conſtrue quadratũ EFGH; cujus plano rectæ inſiſtant EI, FK, HM, GL ipſi EF pares, quas connecte rectis IK, KL, LM, IM. Solidum EFGHIKLM cubus eſt, ut ſatis conſtat ex conſtructione. a 10. 6. b 46. 1.

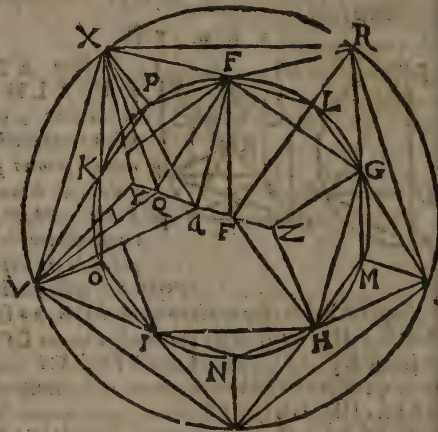
In quadratis oppoſitis EFKI, HGLM duc diametros EK, FI, HL, MG, per quas ducta plana EKLH, FIMG ſe interſecent in recta NO. Hæc diametros cubi EL, FM, GI, HK *c* biſecabit in P, centro cubi. *d* ergo P centrum erit ſphæræ per puncta cubi angularia tranſeuntis. Porro ELq *e* = EKq + KLq *e* = 3 KLq, *f* vel 3 ACq. atqui ABq. ACq *g* :: BA. DA *f* :: 3. 1. *h* ergo AB = EL. Quare cubum fecimus, &c. Q. E. F. c cor. 39. 11. d 15. def. 1. & 14. def. 11. e 47. 1. f conſtr. g cor. 8 6. h 14. 5.

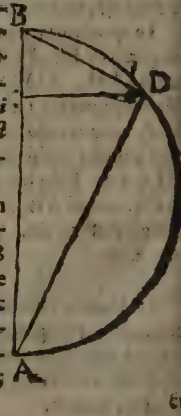
*Coroll.*

1. Hinc, omnes diametri cubi inter ſe æquales ſunt, ſeſeque mutuo in centro ſphæræ biſecant. Eademque ratione rectæ quæ quadratorum oppoſitorum centra conjungunt, biſecantur in eodem centro.

k 47. 1. 2. Diameter ſphæræ poteſt latus tetraedri, & 
l 13. 13. m 15. 13. cubi. nempe ABq $k$ = $l$ BCq + $m$ ACq.

## PROP. XVI.




*Icoſaedrum* ZGHIKFYVXRST *conſtituere, & ſphæra complecti, qua & antedictas figuras; & demonſtrare, quod icoſaedri latus* FG *irrationalis eſt linea, quæ vocatur minor.*

Super AB diametrum ſphæræ deſcribe ſemicirculum ADB; & 
a 10. 6. $a$ fac AB = 5 BC. ex C erige normalem CD, & duc AD ac BD. Ad intervallum EF = BD deſcribe circulum EFKNG;

cui inſcribe pentagonum æquilaterum FKIHG. b 11. 4.
biſeca arcus FG, GH, &c. ac connecte rectas
FL, LG, &c. latera nempe decagoni. Tunc c erige c 12. 11.
EQ, LR, MS, NT, OV, PX ipſi EF æquales,
rectaſque plano FKNG. & connecte RS, ST,
TV, VX, XR; item FX, FR, GR, GS, HS, ST,
HT, IT, IV, KV, KX. Denique producta EQ,
ſume QY = FL; & EZ = FL; rectaſque duci
concipe ZG, ZH, ZI, ZK, ZF; ac YV, YX,
YR, YS, YT. Dico factum.

Nam ob EQ, LR, MS, NT, OV, PX d æquales e & parallelas; etiam quæ illas jungunt, d conſtr. e 6. 11.
EL, QR, EM, QS, EN, QT, EO, QV, EP,
QX f pares & parallelæ ſunt. Item ideo LM f 33. 1.
(vel FG,) RS, MN, ST, &c. æquales ſunt inter ſe. g ergo planum per EL, EM, &c. plano g 15. 11.
per QR, QS, &c. æquidiſtans, h & circulus h 1. def. 3.
QXRSTV è centro Q, circulo EPLMNO æqualis eſt; atque RSTVX eſt pentagonum æquilaterum. Duci vero intellectis EF, EG, EH,
&c. ac QX, QR, QS, &c. quia FRQ k = FLq k 47. 1.
+ LRq; l vel EFq m = FGq, n erunt FR, FG, l conſtr. m 10. 13.
ideoque omnes RS, FG, FR, RG, GS, GH, &c. n ſch. 48. 1.
æquales inter ſe. Proinde 10 triangula RFX, & 1. ax.
RFG, RGS, &c. æquilatera ſunt & æqualia.
Rurſus ob ang. XQY o rectum, erit XYq p = o cor. 14. 11.
QXq + QYq q = VXq vel FGq. quare XY, p 47. 1. q 10. 13.
VX, hiſque ſimiliter YV, YT, YS, YR, ZG, ZH,
&c. æquantur: Ergo alia decem trigona conſtituta
ſunt æquilatera, & æqualia, tam ſibi mutuo,
quam decem prioribus; ac proinde factum eſt
Icoſaedrum.

Porro, biſecta EQ in α, duc rectas αF, αX,
αV; & propter QX r = QV, & commune latus r 15. def. 1.
αQ, anguloſque EQX, EQV rectos; ſ erit αX = ſ 4. 1.
αV. ſimilique argumento omnes, αX, αR, αS,
αT, αV, αF, αG, αH, αI, αK æquantur.

t 9. 13. Quoniam autem ZQ. QE [t] :: QE. ZE; erit
u 3. 13. $Z\alpha q$ [u] $= 5\ E\alpha q$ [x] $= EQq\ (EFq) + E\alpha q$, [y] $= \alpha Fq$.
x 4. 2. ergo $Z\alpha = \alpha F$. [z] pari pacto $\alpha F = Y\alpha$. ergo
y 47. 1. ſphæra, cujus centrum α, radius αF, per 12 puncta icoſaedri angularia tranſibit.

Denique, quia $Z\alpha.\ \alpha E$ :: ZY. QE; [a] ideoque
z 15. 5. $Z\alpha q.\ \alpha Eq$ :: ZYq. QEq. [b] erit $ZYq = 5\ QEq$,
a 22. 6. vel 5 BDq: atqui ABq. BDq [c] :: AB. BC :: 5.
b 14. 5. 1. [d] ergo $ZY = AB$. Q. E. F.
c cor. 8. 6. Itaque ſi AB ponatur $\rho$, [e] erit $EF = \sqrt{ABq}$
d 1. ax. 1. etiam $\rho$; proinde FG pentagoni, idemque Icoſaedri
e ſch. 12. 10. latus, [f] eſt minor. Q. E. D.
f 11. 13

*Coroll.*

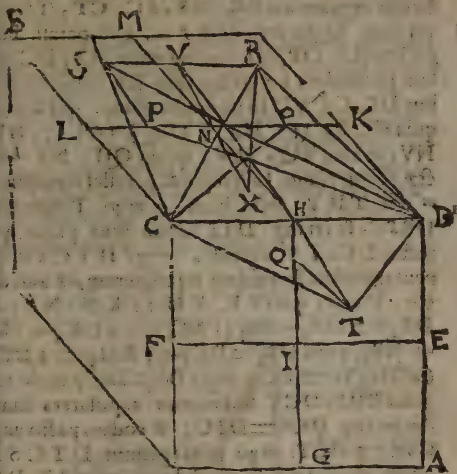
1. Ex dictis infertur, ſphæræ diametrum eſſe potentia quintuplum ſemidiametri circuli quinque latera icoſaedri ambientis.

2. Item manifeſtum eſt, ſphæræ diametrum eſſe compoſitam ex latere hexagoni, hoc eſt, ex ſemidiametro, & duobus lateribus decagoni circuli ambientis quinque latera icoſaedri.

3. Conſtat denique latera icoſaedri oppoſita,
a 33. 1. qualia ſunt RX, HI, eſſe parallela. Nam RX [a] pa-
b ſch. 16. 3. rall. LP. [b] parall. HI.

PROP.

PROP. XVII.




*Dodecaedrum conſtituere, & ſphæra complecti,*
*ua & prædictas figuras; & demonſtrare, quod do-*
*ecaedri latus* RS *irrationalis eſt linea, quæ vocatur*
*potome.*

Sit AB cubus datæ ſphæræ inſcriptus, cujus
itera omnia biſecentur in punctis E, H, F, G,
, L, &c. rectæque adjungantur KL, MH,
IG, EF. *a* Fac HI. IQ :: IQ. QH !; & ſume a 30. 6.
O, NP pares ipſi IQ. Erige OR, PS rectas
lano DB, & QT plano AC. ſintque OR, PS,
QT ipſis IQ, NO, NP æquales. Connexis DR,
S, SC, CT, DT, erit DRSCT pentagonum
Dodecaedri expetiti. Nam duc NV parall. OR,
protracta NV ad occurſum cum cubi centro
I *i*, connecte rectas DS, DO, DP, CR, CP, a 47. 1.
HV, HT, RX. Quia DOq *a* = DKQ (*b* KNq) b 7. ax. 1.
+ KOq *c* = 3 ONq (3 ORq) *d* erit DRq c 4. 13.
X 3 = 4 d 47. 1.

e 4. 2. = 4 ORq e = OPq, vel RSq. ergo DR = RS. Simili argumento DR, RS, SC, CT, TP pares ſunt. Quia vero OR f = g & parall. PS g erunt RS, OP, & h conſequenter RS, DC etiam parallelæ; h ergo hæ cum ſuis conjungentibus DK, CS, VH in uno ſunt plano. quinetiam quia HI. IQ k :: IQ (TQ) QH k :: HN NV; & tam TQ, HN, quam QH, NV k rectæ eidem plano, l adeoque & parallelæ exiſtunt. m erit THV recta linea. n ergo Trapezium DRSC, & triang DTS in uno ſunt plano per rectas DC, TV extenſc. ergo DTCSR eſt pentagonum, & quidem æquilaterum, ex antedictis. Porro, quia PK. KN :: KN. NP; & DSq p = DPq + PSq (PNQ) = p DKq + PKq + NPq, q erit DSq = DKq + 3 KNq = 4 DKq (4DHq) r = DCq. ergo DS = DC; unde trigona DRS, DCT ſibi mutuo æquilatera ſunt. s ergo ang. DRS = DTC; & eodem pacto ang. CSR = DCT. ergo pentagonum DTCSR etiam æquiangulum eſt. Ad hæc, quia AX, DX, CX, &c. ſunt cubi ſemidiametri, t erit XN = IH, vel KN, u adeoque XV = KP. unde ob angulum x rectum RVX, z erit RXq = XVq + RVq (NPq) = KPq + NPq a = 3 KNq b = AXq, vel DXq, &c. ergo RX, AX, DX, & eadem ratione XS, XT, AX æquales ſunt inter ſe. Et ſi eadem methodo, qua conſtructum eſt pentagonum DTCSR, fabricentur 12 ſimilia pentagona tangentia duodecim cubi latera, ea Dodecaedrum conſtituent; ac per eorum puncta angularia tranſiens ſphæra, cujus radius AX, vel RX, Dodecaedrum complectetur. Q. E. F.

f conſtr. 9. 6. 11. — g 33. 1. — h 9. 11. — k 7. 11. — k conſtr. — l 6. 11. — m 32. 6. — n 1. & 2. 11. — o 5. 13. — p 47. 1. — q 1. ax. 2. — r 4. 13. — r 4. 2. — s 8. 1. — t 15. 13. — u 1. ax. 1. — x 29. 1. — z 47. 1. — a 4. 13. — b 15. 13.

Denique, quia KN. NO c :: NO. OK, d erit KL. OP :: OP. OK + PL. Itaque ſi ſphæræ diameter AB ponatur $\dot{\rho}$, erit KL e = $\sqrt{\frac{AB}{3}}$ f etiam $\dot{\rho}$. g unde OP, vel RS latus dodecaedri apotome erit. Q. E. D.

c conſtr. — d 15. 5. — e 15. 13. — f ſch 12. 10. — g 6. 13.

coroll.

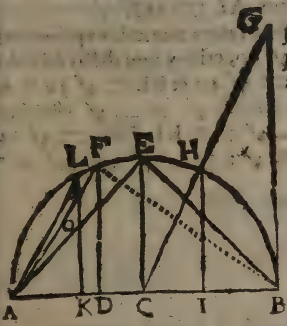
*Coroll.*

1. Hinc, ſi latus cubi ſecetur extrema ac me-
a ratione, majus ſegmentum erit latus dodeca-
ri in eadem ſphæra deſcripti.

2. Si rectæ lineæ ſectæ extrema ac media ra-
one, minus ſegmentum ſit latus dodecaedri,
ajus ſegmentum erit latus cubi ejuſdem ſphęrę.

3. Liquet etiam latus cubi æquale eſſe lineæ
ctæ ſubtendenti angulum pentagoni dodecae-
i eadem ſphæra comprehenſi.

## PROP. XVIII.




*Latera quinq; figurarum exponere, & inter ſe comparare.*

Sit AB diameter ſphæræ, ac AEB ſemicirculus, ſitq; AC $^a$= $\frac{1}{2}$AB, & AD $^b$= $\frac{1}{3}$ AB. Erige perpendiculares CE, DF, & ‸G = AB. junge AF, AE, BE, BF, CG. ex H emitte perpendicularem HI, & ſumpta CK = CI, ex K erige perpendicularem KL, & connecte AL. Denique $^c$ fac AF. AO :: AO. OF.

a 10. 1.
b 10. 6.

Itaque 3. 2 $^d$ :: AB. BD $^e$ :: ABq. BFq, latus Tetraedri. & 2. 1 :: $^a$ AB. AC :: ABq. BEq, $^f$ latus Octaedri.

Item 3. 1 $^d$ :: AB. AD $^e$ :: ABq. AFq, $^g$ latus Hexaedri.

Porro, quia AF. AO $^h$ :: AO. OF. $^k$ erit

c 30. 6.
d conſtr.
e cor. 8. 6.
f 14. 13.
g 15. 13.
h conſtr.
k cor. 17. 13

l 4.6. AO latus Dodecaedri. denique BG (2 BC.) BC [l] :: HI.IC. [m] ergo HI = 2CI [n] = KI. ergo
m 24.5. HIq [o] = 4CIq. proinde CHq = [p] 5 CIq. [q] ergo
n conſtr. ABq = 5 KIq. [r] itaque KI, vel HI, eſt radius circuli circumſcribentis pentagonum icoſaedri; & AK, vel IB, [s] eſt latus decagoni eidem circulo inſcripti. unde AL erit latus pentagoni, [t] idemque Icoſaedri latus. Ex quibus liquet BF, BE, AF eſſe ρ[c] ⊐. & AL, AO eſſe ρ ⊐; atque BF ⊏ BE; & BE ⊏ AF; ac AF ⊏ AO. Quia vero 3 AFq = ABq [u] = 5 KLq. ac AF x AO ⊏ AF x OF, [x] ideoque AF x AO + AF x OF ⊏ 2 AF x OF, [y] hoc eſt AFq ⊏ [z] 2 AOq. [a] erit 3 AFq (5 KLq) ⊏ 6 AOq. proinde KL ⊏ AO; & fortius, AL ⊏ AO.

o 4.2. p 47.1. q 15.5. r cor.16.13. s 16.13. t 16.13. u 1.6. x 4.ax.1. y 1.2. z 17.6. a 47.1.

Jam vero ut hæc latera numeris exprimamus, ſi AB ponatur √ 60, erit ex jam dictis ad calculum exactis, BF = √ 40. & BE = √ 30. & AF = √ 20. item AL = √: 30 − √ 180 (nam AK = √ 15 − √ 3. & KL (HL) = √ 12.) denique AO = √: 30 − √ 500 (√ 25 − √ 5.)

SCHOL.

SCHOL.

*Præter jam dictas figuras nullam dari poſſe figuram ſolidam regularem (nempe quæ figuris planis ordinatis & æqualibus contineatur) admodum perſpicuum eſt.* Nam ad anguli ſolidi conſtitutionem requiruntur ad minimum tres anguli plani; *a* hique omnes ſimul 4 rectis minores eſſe debent. *b* Atqui 6 anguli trigoni æquilateri, 4 quadratici, & 3 hexagonici, ſigillatim 4 rectos exæquant; quatuor vero pentagonici, 3 heptagonici, 3 octagonici, &c. 4 rectos excedunt. ergo ſolummodo ex 3, 4, vel 5 triangulis æquilateris, ex 3 quadratis, vel 3 pentagonis, effici poteſt angulus ſolidus. Proinde, præter quinque prædicta, nulla exiſtere poſſunt corpora regularia.

a 21. 11.

b Vid. ſchol. 32. 1.

Ex P. Herigonio.

*Proportiones ſphæræ, & 5 figurarum regularium eidem inſcriptarum.*

Sit diameter ſphæræ 2. Erunt

Peripheria circuli majoris, 6 |28318.

Superficies circuli majoris, 3 |14159.

Superficies ſphæræ, 12 |56637.

Soliditas ſphæræ, 4 |1879.

Latus tetraedri, 1 |62299.

Latus

Superficies tetraedri, 4 | 6188.

Soliditas tetraedri, 0 | 15132.

Latus hexaedri, 1 | 1547.

Superficies hexaedri, 8.

Soliditas hexaedri, 1 | 5396.

Latus octaedri, 1 | 41421.

Superficies octaedri, 6 | 9282.

Soliditas octaedri, 1 | 33333.

Latus dodecaedri, 0 | 71364.

Superficies dodecaedri, 10 | 51462.

Soliditas dodecaedri, 2 | 78516.

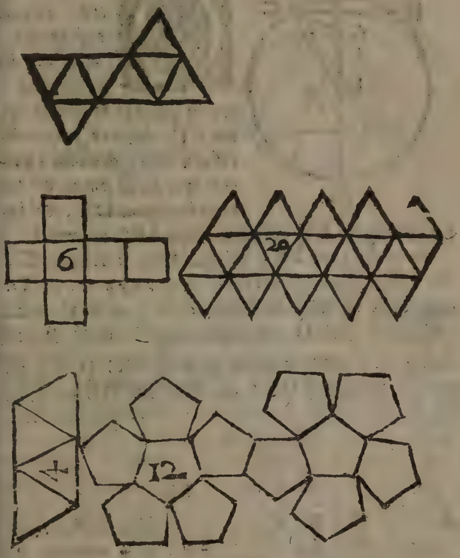
Latus Icoſaedri, 1 | 05146.

Superficies Icoſaedri, 9 | 57454.

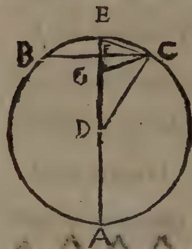
Soliditas Icoſaedri, 2 | 53615.

Quod

*Quod ſi ex charta conficiantur quinque figuræ æquilateræ & æquiangulæ ſimiles his quæ ſunt in ſubjecta figura, componentur quinque figuræ ſolidæ, ſi rite complicentur.*



## PROP. I.

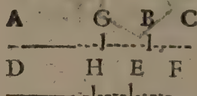



*Quæ ex* D *centro circuli cujuſpiam* ABC *in pentagoni eidem circulo inſcripti latus* BC *ducitur perpendicularis* DF, *dimidia eſt utriuſque lineæ ſimul, & lateris hexagoni* DE, *& lateris decagoni* EC *eidem circulo* ABC *inſcripti.*

a 4. 1.
b 5. 1.
c 32. 1.
d hyp. & 33. 6.
a 20. 13.
f 7. ax.
g 6. 1.

Sume FG = FE, & duc CG. *a* Eſtque CE = CG. ergo ang. CGE *b* = CEG *b* = ECD. ergo ang. ECG *c* = EDC *d* = $\frac{1}{4}$ ADC *e* = $\frac{1}{2}$ CED ( $\frac{1}{2}$ ECD. ) proinde ang. *f* GCD = ECG = EDC. *g* quare DG = GC (CE.) ergo DF = CE (DG) + EF = DE + CE. Q. E. D.

## PROP. II.




*Si binæ rectæ lineæ* AB, DE *extrema ac media ratione ſecentur* ( AB. AG :: AG.GB. *&* DE. DH :: DH. HE;) *ipſæ ſimiliter ſecabuntur, in eaſdem ſcilicet proportiones.* ( AG. GB :: DH. HE. )

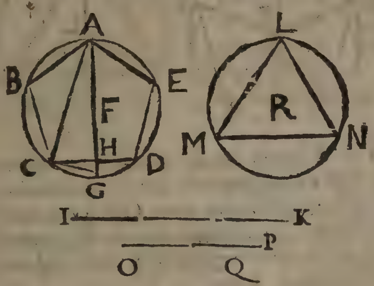
a 17. 6.
b 8. 2.
c 2. ax. 1.
d 22. 5 & 22. 6.

Accipe BC = BG & EF = EH. Eſtque AB x BG *a* = AGq. quare ACq *b* = 4 ABG + AGq *c* = 5 AGq. Similiter erit DFq = 5 DHq. *d* ergo AC. AG :: DF. DH. componendo igitur AC + AG. AG :: DF + DH. DH.

DH,

DH. hoc eſt 2 AB. AG :: 2 DE. DH. *e* pro- e 21. 5.
inde AB. AG :: DE. DH. unde *f* dividendo f 17. 5.
AG. GB :: DH. HE. Q. E. D.

PROP. III.



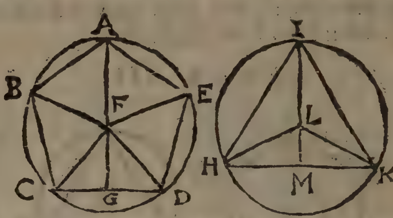

*Idem circulus* ABD *comprehendit & Dodecaedri pentagonum* ABCDE, *& Icoſaedri triangulum* LMN, *eidem ſphæræ inſcriptorum.*

Duc diametrum AG, rectaſque AC, CG. a *ſch.* 47. 1.
Sitque IK diameter ſphæræ, *a* & IKq = 5 OPq. b 30. 6.
*b* fiatque OP. OQ :: OQ. QP. Quia ACq c 47. 1.
+ CGq *c* = AGq *d* = 4 FGq; & ABq *e* = d 4. 2.
FGq = CGq. *f* erit ACq + ABq = 5 FGq. e 10. 13.
porro, quia CA. AB *g* :: AB. CA − AB; ac f 1, & 3. *ax.*
OP. OQ :: OQ. QP. *h* ideoque CA. OP :: g 8. 13.
AB. OQ. *k* erit 3 ACq (*l* IKq.) 5 OPq h 2. 13. & 16. 5.
(*m* IKq) :: 3 ABq. 5 OQq. ergo 3 ABq = 5 k 22 6 & 4. 5
OQq. Verum ob ML *n* latus pentagoni circu- l 15. 13.
lo inſcripti, cujus radius OP, erunt 15 RMq m *conſtr.*
*o* = 5 MLq *p* = 5 OPq + 5 OQq = *3 n *cor.* 16. 13.
ACq + 3 ABq *q* = 15 FGq. *r* ergo RM o 12. 13.
= FG. *ſ* proinde circ. ABD = circ. LMN. p 10. 13.
Q. E. D. q 15. 5.

* *Prius.*
r 1. *ax.* 1. & *ſch.* 48 1.
ſ 1. *def.* 3.

PROP.

PROP. IV.




*Si ex* F *centro circuli pentagonum dodecaedri* ABCDE *circumſcribentis ducatur perpendicularis* FG *ad pentagoni unum latus* CD ; *erit quod ſub dicto latere* CD , *& perpendiculari* FG *comprehenditur rectangulum trigeſies ſumptum , icoſaedri ſuperficiei æquale.* item,

*Si ex centro* L *circuli triangulum icoſaedri* HIK *circumſcribentis , perpendicularis* L M *ducatur ad trianguli unum latus* H K ; *erit quod ſub dicto latere* H K , *& perpendiculari* L M *comprehenditur rectangulum trigeſies ſumptum, icoſaedri ſuperficiei æquale.*

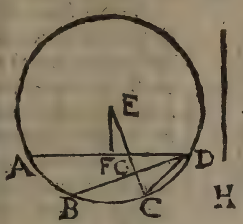
a 8. 1. Duc FA, FB, FC, FD, FE. a Erunt triangula CFD, DFE, EFA, AFB, BFC æqualia. at-
b 41. 1. qui CDxFG b = 2 triang. CFD. ergo 30 CD
c 15. 5. x GF c = 60 CFD d = 12 pentag. ABCDE e =
d 6. ax. ſuperf. dodecaedri. Q. E. D.
e 17. 13. Duc LI, LH, LK. eſtque HK x LM f = 2
f 41. 1. triang. LHK. ergo 30 HK x LM g = 60 HLK
g 15. 5. = 20 HIK h = ſuperfic. icoſaedri. Q. E. D.
h 16. 13.

*Coroll.*

k 15. 5. CDxFG. HKxLM k :: ſuperfic. dodecaed. ad ſuperf. icoſaedri.

PROP.

## PROP. V.




*Superficies dodecaedri ad ſuperficiem icoſaedri in eadem ſphæra deſcripti eandem proportionem habet, quam H latus cubi ad AD latus icoſaedri.*

Circulus ABCD [a] circumſcribat tam dodecaedri pentagonum, quam icoſaedri triangulum; quorum latera BD, AD; ad quæ demittantur ex E centro perpendiculares EF, EGC. & connectatur CD. [a] 3. 14.

Quoniam EC+CD. EC [b] :: EC. CD. erit EG ([c] $\frac{1}{2}$ EC + CD.) EF ([d] $\frac{1}{2}$ EC) [e] :: EF. EG − EF ($\frac{1}{2}$ CD.) atqui H. BD [f] :: BD. H − BD. [g] ergo H. BD :: EG. EF. proinde H x EF = BD x EG. quum igitur H. AD [h] :: H x EF. AD x EF. erit H. AD :: BD x EG. AD x EF :: [l] ſuperfic. dodecaedri ad ſuperfic. icoſaedri. Q. E. D. b 9. 13. c 1. 14. d cor. 12. 13 e 15. 5. f cor. 17. 13. g 2. 14. h 1. 6. k 7. 5. l cor. 4. 14.

PROP.

PROP. VI.




*Si recta linea AB ſecetur extrema ac media ratione; erit ut recta BF potens id, quod à tota AB, & id quod à majori ſegmento AC, ad rectam E, potentem id quod à tota AB, & id quod à minori ſegmento BC; ita latus cubi BG ad latus icoſaedri BK eidem ſphæræ cum cubo inſcripti.*

Circulo, cujus ſemidiameter AB, inſcribantur dodecaedri pentagonum BFGHI, & icoſaedri triangulum BKL. a quare BG latus cubi erit eidem ſphæræ inſcripti. igitur BKq b = 3 ABq; & Eq c = 3 ACq. ergo BKq. Eq d :: ABq. ACq e :: BGq. BFq. permutando igitur BGq. BKq :: BFq. Eq. f unde BG. BK :: BF. E. Q. E. D.

a cor. 17. 13.
b 12. 13.
c 4. 13.
d 15. 5.
e 2. 14.
f 22. 6

PROP. VII.

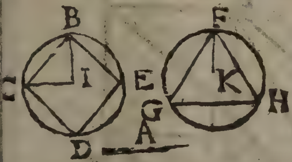
*Dodecaedrum eſt ad Icoſaedrum, ut cubi latus ad latus Icoſaedri, in una eademque ſphæra inſcripti.*

Quoniam a idem circulus comprehendit & dodecaedri pentagonum & icoſaedri triangulum, b erunt perpendiculares à centro ſphæræ ad plana pentagoni & trianguli ductæ inter ſe æquales. itaque ſi dodecaedrum & icoſaedrum intelligantur eſſe diviſa in pyramides, ductis rectis à centro ſphæræ ad omnes angulos, omnium pyramidum altitudines erunt inter ſe æquales. Cum igitur pyramides æque altæ c ſint ut baſes, & ſuperficies dodecaedri ſit æqualis 12 pentagonis, ſuperficies vero icoſaedri 20 triangulis; erit

a 3. 14.
b 47. 1.
c 5, & 6. 12.

rit dodecaedrum ad icoſaedrum, ut ſuperficies
lodecaedri ad ſuperficiem icoſaedri, *d* hoc eſt, ut d 5. 14.
atus cubi ad latus icoſaedri.

## PROP. VIII.

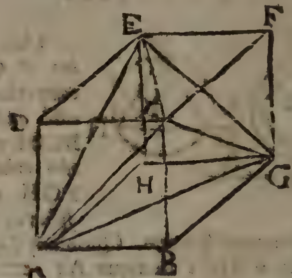



*Idem circulus* B C D E *comprehendit & cubi quadratum* BCDE *& octaedri triangulum* FGH, *ejuſdem ſphæræ.*

Sit A diameter ſphæræ. Quoniam Aq *a* = 3 a 15. 13.
BCq *b* = 6 BIq ; itemque Aq *c* = 2 GFq b 47. 1.
= 6 KFq; erit BI = KF. *e* ergo circulus CBED c 14. 13.
= GFH. Q. E. D. d 12. 13.
e 2. def. 1.

## LIB. XV.

### PROP. I.




IN *dato cubo* ABGHDCFE *pyramidem* AGEC *deſcribere.*

Ab angulo C duc diametros CA, CG, CE; Eaſque connecte diametris AG, GE, EA. Hæ omnes inter ſe *a* æquales ſunt, utpote æqualium quadratorum diametri. ergo triangula CAG, CGE, CEA, EAG æquilatera ſunt, ac æqualia: proinde AGEC eſt pyramis, quæ cubi angulis inſiſtit, eiqueidcirco *b* inſcribitur. Q. E. F.

a 47. 1.

b 31. *def.* 11.

## PROP. II.

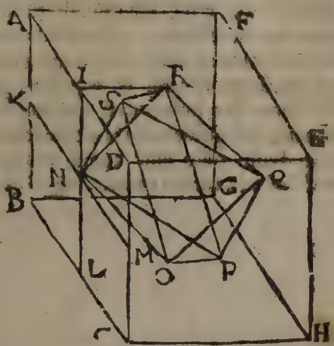



*In data pyramide* AB-DC *octaedrum* EGKIFH *deſcribere.*

*a* Biſeca latera pyramidis in punctis E, I, F, K, G, H; quæ connecte 12 rectis EF, FG, GE, &c. Hæ omnes *b* æquales ſunt inter ſe. proinde 8 triangula EHI, HK, &c. æquilatera ſunt & æqualia, adeoque conſtituunt *c* octaedrum *d* in data pyramide deſcriptum. Q. E. F.

a 10. 1.
b 4. 1.
c 27. def. 11.
d 31. def. 11.

## PROP. III.

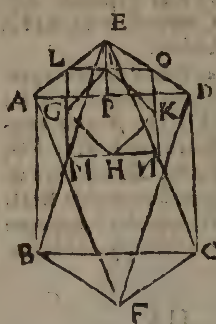



*In dato cubo* CHGBDEFA *octaedrum* NPQSOR *deſcribere.*

Connecte quadratorum * centra N, P, Q, S, O, R, 12 rectis NP, PQ, QS, &c. quæ *a* æqualia ſunt inter ſe, ideoque 8 triangula efficiunt æquilatera & æqualia. proinde *b* inſcriptum eſt cubo Octaedrum NPQSOR. Q. E. F.

* 8. 4.
a 4. 1.
b 31. & 27. def. 11.

PROP. IV.




*In dato octaedro* ABCDEF *cubum inſcribere.*

Latera pyramidis EABCD, cujus baſis quadratum ABCD, biſecentur rectis LM, MN, NO, OL quæ *a* æquales ſunt & *b* parallelæ lateribus quadrati ABCD. *c* ergo quadrilaterum LMNO eſt quadratum.

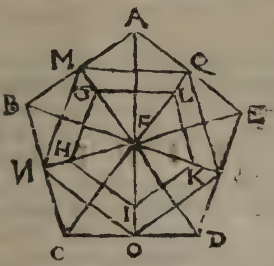
a 4. 1.
b 2. 6.
c 29. def. 1.

Eodem modo, ſi latera quadrati LMNO biſecentur in punctis G, H, K, I, & connectantur GH, HK, KI, IG, erit GHKI quadratum. Quod ſi eadem arte in reliquis 5 pyramidibus octaedri centra triangulorum rectis conjungantur, deſcribentur quadrata ſimilia & æqualia quadrato GHKI. quare ſex hujuſmodi quadrata cubum conſtituent, qui quidem intra octaedrum deſcriptus erit, *d* cum octo ejus anguli tangant octo octaedri baſes in earum centris. Q. E. F.

d 31. def. 11.

PROP.

PROP. V.




*In dato Icoſaedro Dodecaedrum inſcribere.*

Sit A B C D E F pyramis Icoſaedri, cujus
aſis pentagonum ABCDE; centra autem tri-
ngulorum G, H, I, K, L; quæ con- a 5. 4.
ectantur rectis GH, HI, IK, KL, LG.
rit GHIKL pentagonum dodecaedri inſcri-
endi.

Nam rectæ FM, FN, FO, FP, FQ,
er centra triangulorum tranſeuntes, a biſe- a cor. 3. 3.
ant baſes. b ergo rectæ MN, NO, OP, b 4. 1.
PQ, QM æquales ſunt inter ſe. quinetiam
FM, FN, FO, FP, FQ c pares ſunt.
d ergo anguli MFN, NFO, OFP, c 4. 1. d 8. 1.
PFQ, QFM æquantur. pentagonum igi-
tur GHIKL æquiangulum eſt; e proinde & e 4. 1.
æquilaterum, cum FG, FH, FI, FK, FL f pares f 12. 13.
int. Quod ſi eadem arte in reliquis undecim
pyramidibus icoſaedri, centra triangulorum re-
tis lineis connectantur, deſcribentur pentagona
æqualia & ſimilia pentagono GHIKL. quam-
obrem 12 hujuſmodi pentagona dodecaedrum

conſtituent; quod quidem in icoſaedro erit deſcriptum, cum viginti anguli dodecaedri in centris viginti baſium icoſaedri conſiſtant. Quapropter in dato icoſaedro dodecaedrum deſcripſimus. Q. E. F.

F I N I S.

*nnotationes in* Elementa Euclidis *nu-*
*per edita, in quibus obſcura illuſtrantur, errata emendantur, plurimaque quæ conducant ad Geometriæ rudimenta facilius percipienda adjiciuntur.*

*pag.*13.*lin.*5. ſcribe, Rurſus ang. ACD e = DC; & ang. BCD e = BDC f ergo ang. ACD = BDC, id eſt ang. ADC = BDC. Q. F. N. e 5. 1. f 9 ax.

*p.*17 *l.*ult. ſcribe, conjunganturque FC, IC, & roducatur ACG.

*p.*18. *l.*3. ſcribe, ſimili argumento ang. ICH = BH. ergo totus ACD, f (BCG) g major eſt u-roque CAB, & ABC. Q. E. D.

*p.*21. apponantur figuræ quæ deſunt.

*p.*40. *lin.*18. ſcribe, *Schol.*

*Imo ſi fuerint duæ rectæ, ſecenturque ambæ in uotcunque partes, idem provenit ex ductu totius in tum, & partium in partes.*

Nam ſit Z = A + B + C, & Y = D + E; quia ) Z a = DA + DB + DC, & E Z a = EA + EB + EC, & YZ a = DZ + EZ, b erit ZY = DA + DB + DC + EA + EB + EC. Q. E. D. a 1. 2. b 2. ax.

*Hinc patet ratio ducendi rectas compoſitas in mpoſitas. Nam omnia partium rectangula accipere portet, & habetur rectangulum ex totis.*

Sin linearum in ſe ducendarum ſignis + adiſceantur ſigna −, etiam ſignorum ratio habena eſt. Quippe ex + in − provenit −; at ex − in − rovenit +. Nam ſit + A ducenda in B − C. & uoniam + A non affirmatur de toto B, ſed de jus parte tantum, qua ſuperat C, debet AC maere negata. quare prodibit AB − AC. Vel ſic; uia B conſtat partibus C, & B − C, * erit AB = AC + A in B − C; aufer utrinque AC, erit AB − AC = A in B − C. Similiter ſi − A ducenda t in B − C, quoniam ex vi ſigni − non nega- * 1. 2.

 tur

tur A de toto B; ſed de ejus ſolummodo exceſſu ſupra C, debet AC manere affirmata. proveniet ergo − AB + AC. Vel ſic; quia AB * = AC + A in B − C; tolle utrinque omnia, erit − AB = AC − A − in B + C; adde AC utrinque, eritq; − AB + AC = A in B − C.

Atque ex his rite perſpectis, quæ ſubſequuntur 9. propoſitiones, aliæque ejuſmodi innumeræ, ex linearum in ſe ductarum comparatione emergentes (quas apud Vietam, & alios Analyſtas in numerato habes) nullo negotio demonſtrantur, rem plerumque quaſi ad ſimplicem calculum exigendo.

* 19. ax. Porro, * liquet productum ex quapiam magnitudine in numeri cujuſlibet partes æquari producto ex eadem in totum numerum. Ut 5 A + 7 A = 12 A. & 4 A in 5 A + 4 A in 7 A = 4 A in 12 A. quare quæ in hoc loco de rectarum in ſe ductu dicta ſunt, eadem de numerorum in ſe multiplicatione intelligi poſſunt. proinde etiam quæ in 9 ſequentibus theorematis de lineis affirmantur, eadem valent de numeris accepta; quippe cum iſtæ omnes ab hac prima immediate dependeant, & deducantur.

*p.* 42. inter demonſtr. & Schol. propoſitionis quintæ, ſcribe.

*Hoc theorema paulo aliter effertur, & facilius demonſtratur, ſic; Rectangulum ex ſumma & differentia duarum rectarum* A, E, *æquatur differentiæ ex ipſis.*

* ſch. 1. 2. Nam ſi A + E ducatur in A − E, * provenit Aq − AE + EA − Eq = Aq − Eq. Q. E. D.

*p.* 44. poſt demonſtrationem prop. 9. ſcribe,

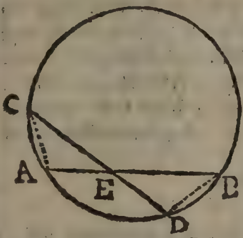
Aliter effertur & facilius demonſtratur, ſic;

*Aggregatum quadratorum ex ſumma, & differentia duarum rectarum* A, E, *æquatur duplo quadratorum ex ipſis.*

a 4. 2. b ſch. 7. 2. Nam Q: A + E a = Aq + Eq + 2 AE. & Q: A − E b = Aq + Eq − 2 AE. Hæc collecta faciunt 2 Aq + 2 Eq. Q. E. D.

*p.* 67.

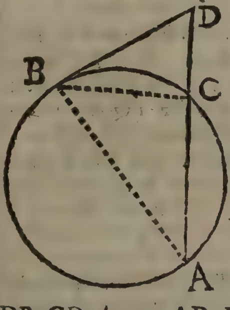
*p*.67. poſt demonſtrationem prop.28 ſcribe;
Quod ſi ſubtenſa AC $\sqsubset$ vel $\sqsupset$ DF, erit ſimili modo arcus AC $\sqsubset$, vel $\sqsupset$ DF.




*p*.71. poſt demonſtrationem prop. 35. ſcribe, Facilius ſic, & univerſaliter; connecte AC & BD. atque ob angulos *a* CEA, DEB, *b* ipſoſque C, B (ſuper eodem arcu AD) pares; trigona CEA, BED, *c* æquiangula ſunt. *d* ergo CE. EA :: EB. ED. *e* proinde CE×ED = EA×EB. Q. E. D.

a 15 1.
b 21. 3.
c cor. 32 1.
d 4 6
e 16. 6.

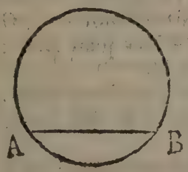
Quæ ex 6. lib. citantur, tam hic quam in ſeq. ab hac minime pendent; quare iis uti licuit.




*p*. 71. Inter demonſtr. & coroll. prop. 36. ſcribe, Facilius ac univerſalius ſic;

Duc AB, & BC. ac ob angulos A, DBC *a* pares, & D communem, triangula BDC, ADB *b* æquiangula ſunt. *c* ergo AD. DB :: DB. CD. *d* quare AD×DC = DBq. Q. E. D.

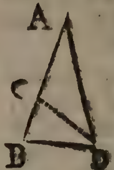
a 32. 3.
b 32. 1.
c 4 6.
d 17 6.




*p*.76. ad def. 7. 4. ſubſtitue figuram hanc.

*pag*.82. poſt demonſtrationem propoſ. 10. 4. ſcribe ſic.

Hcc



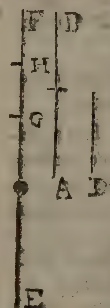

Hæc conſtructio Analytice indagatur ſic; Factum ſit; & angulum BDA biſecet recta DC. [a] ergo DA. DB :: CA. CB. item ob ang. CDA [b] $=\frac{1}{2}$ ADB [c] = A, [d] eſt CA = DC. ac ob ang. DCB [e] = A + CDA = 2 A [c] = B, [d] erit DB = DC. [f] ergo DB = CA. proinde DA. ([c] BA.) CA :: CA. CB. [g] unde BA x CB = CAq.

[a] 3 6. [b] conſtr. [c] hyp. [d] 6. 1. [e] 32 1. [f] 2. ax. [g] 17. 6.

*p.* 98. ſcribe Prop. 8. 5. ſic.

P R O P. 8.

*Inæqualium magnitudinum* AB, AC, *major* AB *ad eandem* D *majorem habet rationem, quam minor* AC: *& eadem* D *ad minorem* AC *majorem rationem habet, quam ad majorem* AB.




Sume EF, EG, ipſarum AB, AC æquemultiplices, ita ut EH ipſius D multiplex, major ſit quam EG, at minor quam EF. (Quod facile continget, ſi utraque EG, GF majores accipiantur ipſa D.) Liquet juxta 8 def. 5. fore $\frac{AB}{D} \sqsubset \frac{AC}{D}$; ac $\frac{D}{AB} \sqsupset \frac{D}{AC}$. Quæ E. D.

*p.* 100 *lin.* ult. poſt B, D, F. ſcribe, Porro ob A. B [b] :: C. D [b] :: E. F, [c] ſi G ⊏, =, ⊐ K, erit ſimiliter H ⊏, =, ⊐ L; & I ⊏, =, ⊐ M. ac proinde ſi G ⊏, =, ⊐ K, erit ſimili modo G + H + I ⊏, =, ⊐ K + L + M. [c] quare A. B :: A + C + E. B + D + F. Q. E. D.

[b] hyp. [c] 6 def. 5.

*pag.* 102. circa 23 *lin.* poſt (æquatur) ſcribe, Ergo, quum AG. DH :: C. F :: GB. HE. erit, &c. ut ſequitur ibi.

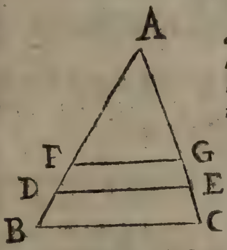
*p.* 104. *lin.* 1. poſt KO ſcribe, Itaque ablatis hinc inde communibus HL, KM, &c. ut ibi ſequitur.

*p*.111. *l*.12. dele, Hujuſce demonſtratio, &c. & ſcribe, Intellige G⊏DE. *a* ergo B⊏G. *b* ergo  a 10. 5.
C  b 8. 5.
A⊏A. Rurſus concipe H ⊏ E. *c* ergo H ⊐ A.  c ſch. 13 5.
G B G F G G
*a* quare A⊏H. *b* proinde A⊏H *d* vel D. Q. E. D.  d 13 5.
C C F

*p*.114. circa 25. *lin*. dele, cum igitur, & ſcribe, Verum ſi HC, &c. ut ſequitur.

*p*.116. *l*.2. dele Imo ſi plures, &c. & ſcribe ſic.

*Schol.*




*Imo ſi plures* DE, FG, *ad unum latus* BC *parallelæ fuerint, erunt omnia laterum ſegmenta proportionalia.*

Nam DF. FA *a* :: EG. GA; & componendo, invertendoque FA. DA :: GA. EA; *a* ac DA. DB :: EA. EC. ergo ex æquo DF. DB :: EG. EC. Q. E. D.  a 2. 6.

*Coroll.*

Si DF. DB :: EG. EC; *a* erunt BC, DE, FG parallelæ.

*p*.119. Prop. 8. demonſtretur ſic.

Nam ob angulos BAC, ADB *a* rectos; *b* ideoque æquales, & B communem, trigona BAC, ADB *c* ſimilia ſunt. Simili diſcurſu, ſimilia ſunt triangula BAC, ADC. *d* proinde ADB, ADC ſimilia erunt. Q. E. D.  a hyp. b 12. ax. c 32 & 4 6. d vid. 21. 6.

*Coroll.* &c. ut ſequitur.

*pag*. 121. *lin. antepen*. ſcribe, Vel ſic; Datæ ſint AB, BC; ex quibus fac angulum rectum ABC. duc AC, & huic normalem CD, cui occurrat AB protracta in D. *a* eſtque AB. BC :: BC. BD.  a cor. 8. 6.

*pag*. 122. dele figuram iſtam furciferam.

*ibid.*

*ibid. lin.* 6. dele, vel ita; $CD = CB$. & quæ ſeq. cum ſua figura.

*pag.* 123. poſt *lin.* 3. ſcribe, Vel (in eadem figura) ſint AB, BF duæ datæ, *b* liquet eſſe A B. BF :: BF. BE.

*p.* 136. Propoſ. 31. *a* demonſtretur ſic.

a cor. 8. 6. b cor 20. 6. c 24 5. d ſch. 14 5. Ab angulo recto BAC demitte perpendicularem AD. Quoniam DC. CA :: *a* CA. CB, *b* erit AL. BF :: DC. CB. Item ob DB. BA :: *a* BA. BC, *b* erit BG. BF :: DB. BC. *c* ergo $AL + BG$. BF :: $DC + DB$ (BC.) BC. ergo $AL + BG = BF$. Q. E. D.

*pag.* 146. *lin.* penult. ſcribe, vel ſic, ſit $a = \frac{x}{2}$, & $b = \frac{y}{2}$. quare $2a = x$, & $2b = y$. ergo $2a + 2b = x + y$. ergo $a + b = \frac{x + y}{2}$.

*p.* 147. *lin.* 17. ſcribe, Vel ſic, ſit $a = \frac{2x}{3}$, & $b = \frac{2y}{3}$, & $x + y = g$. ob $3a = 2x$, & $3b = 2y$, eſt $3a + 3b = 2x + 2y = 2g$. ergo $a + b = \frac{2}{3}g = \frac{2}{3} : x + y$.

*p.* 149. *l.* 9. ſcribe, Vel ſic; ſit $a = \frac{b}{3}$, & $c = \frac{d}{3}$, vel $3a = b$, & $3c = d$, eſtque $\frac{c}{a}$ * $= \frac{3c}{3a} = \frac{d}{b}$. *5. 15.

*ibid. lin.* 27. dele, Applicare potes, &c. & ſcribe, Vel ſic; ſit $a = \frac{2b}{3}$, & $c = \frac{2d}{3}$, vel $3a = 2b$, & $3c = 2d$. Eſt $\frac{c}{a} = \frac{3c}{3a} = \frac{2d}{2b} = \frac{d}{b}$.

*LEMMA.*

| | | | | |
|---|---|---|---|---|
| AE, | BF, | CG, | DH, | *Si proportionales numeri* A, B, C, D *proportionales* * *numeros* AE, BF, CG, DH |
| A, | B, | C, | D, | |
| E, | F, | G, | H. | |

DH *metiantur per numeros* E, F, G, H, *erunt* [E, F, G, H] *proportionales.*

Nam ob AEDH $\overset{a}{=}$ BFCG, $^{a}$ & AD = BC, $^{b}$ erit $\frac{AEDH}{AD} = \frac{BFCG}{BC}$, $^{c}$ hoc eſt EH = FG. $^{a}$ ergo E. F :: G. H. Q. E. D. a 19. 7. b 1. ax. 7. c 9. ax. 7.

*Coroll.*

Hinc $\frac{Bq}{Aq} = \frac{B}{A}$ in $\frac{B}{A}$. $^{d}$ Nam 1. $\frac{B}{A}$ :: B. Bq. $^{d}$ & 1. A :: A. Aq. $^{e}$ ergo 1. $\frac{B}{A}$ :: $\frac{B}{A}$. $\frac{Bq}{Aq}$. $^{d}$ ergo $\frac{Bq}{Aq} = \frac{B}{A} \times \frac{B}{A}$. Similiter $\frac{B}{Ac}$ in $\frac{Bq}{Ac} = \frac{BC}{Acc}$. & ſic de reliquis. d 15. def. 7. e lem. prae.

P R O P. 22.

Aq, B, C. Si tres numeri, Aq, B, C
4, 8, 16. deinceps ſint proportionales, primus autem Aq ſit quadratus; *& tertius* C *quadratus erit.*

Nam ob AqC $\overset{a}{=}$ Bq, $^{b}$ erit C = $\frac{Bq}{Aq}$ $\overset{c}{=}$ Q. $\frac{B}{A}$. Liquet vero $\frac{B}{A}$ eſſe numerum, $^{d}$ ob $\frac{Bq}{Aq}$, vel C numerum. ergo ſi tres, &c. a 20. 7. b 7. ax. 7. c cor. lem. prae. d hyp. & 14. 8.

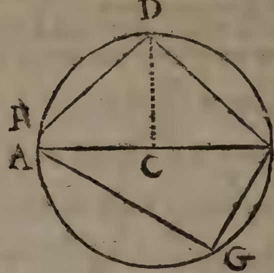
P R O P. 23.

Ac, B, C, D. *Si quatuor numeri* Ac,
8, 12, 18, 27. B, C, D *deinceps ſint proportionales, primus autem* Ac *ſit cubus; & quartus* D *cubus erit.*

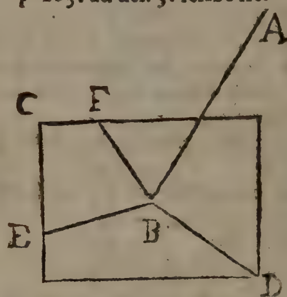
Nam quia AcD $\overset{a}{=}$ BC, $^{b}$ erit D = $\frac{BC}{Ac}$ $\overset{c}{=}$ $\frac{B}{Ac} \times$ C; hoc eſt (ob AcC = $^{d}$ Bq, & $^{b}$ proinde C = $\frac{Bq}{Ac}$) D = $\frac{B}{Ac} \times \frac{Bq}{Ac}$ $\overset{e}{=}$ $\frac{BC}{Acc}$ $\overset{e}{=}$ C: $\frac{B}{Ac}$. $^{e}$ liquet vero ipſum $\frac{B}{Ac}$ eſſe numerum, quia $\frac{BC}{Acc}$, vel D numerus ponitur; ergo ſi quatuor numeri, &c. a 19. 7. b 7. ax. 7. c cor. lem. prae. d 20. 7. e 15. 8

p. 192.

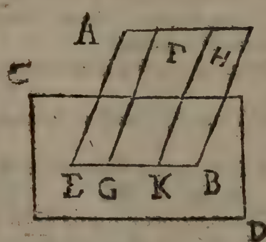
*p.*192. ſubſtitue hanc figuram.




*p.* 263. ad def. 3. ſcribe ſic.

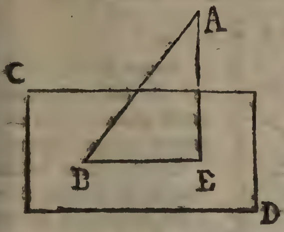



3. Linea recta AB eſt ad planum CD recta, cum ad rectas omnes lineas BD, BE, BF, à quibus illa tangitur, quæque in propoſito ſunt plano, rectos efficit angulos ABD, ABE, ABF.

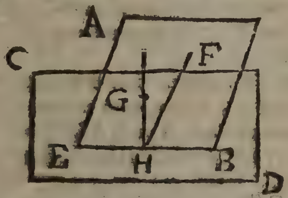



4. Planum AB ad planum CD rectum eſt, cum rectæ lineæ FG, HK, quæ communi planorum ſectioni EB ad rectos angulos in uno plano AB ducuntur, alteri plano CD ad rectos ſunt angulos.

5. Rectæ



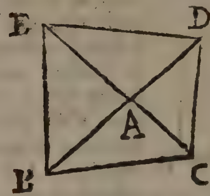

5. Rectæ lineæ A B ad planum C D inclinatio eſt, cum à ſublimi termino A rectæ alius lineæ AB ad planum C D deducta fuerit perpendicularis A E; atque à puncto E, quod perpendicularis AE in ipſo plano CD fecerit, ad propoſitæ illius lineæ extremum B, quod in eodem eſt plano, altera recta linea EB fuerit adjuncta: eſt, inquam, angulus acutus ABE inſiſtente linea AB, & adjuncta EB comprehenſus.




6. Plani AB ad planum CD inclinatio, eſt angulus acutus FHG rectis lineis FH, GH contentus, quæ in utroque planorum AB, CD ad idem communis ſectionis BE punctum H ductæ, rectos cum ſectione BE efficiunt angulos FHB, GHB.

*pag.* 275. Propoſitio 21 ſcribatur ſic.

PROP. 21.



*Omnis ſolidus angulus A ſub minoribus quam quatuor rectis angulis planis continetur.*

Latera enim ſolidi anguli A ſecans planum utcunque faciat figuram multilateram BCDE, & totidem triangula ABC, ACD, ADE, AEB. Omnes angulos polygoni voco X; & ſummam angulorum ad trigonorum baſes voco Y: quare X + 4 Rect. *a* = Y + A. Quia vero (ex angulis ad B) *b* eſt ang. ABE + ABC ⊏ CBE; idemque verum ſit de angulis ad C, ad D, ad E. *c* liquet fore Y ⊏ X. proinde erit A ⊐ 4 Rect. Q. E. D.

a 32. 1. & ſch. 32. 1.
b 20. ſ 11.
c 5. ax. 1.

*p.* 277. *lin.* antepen. dele Brevitatis cauſa aſs. &c. & ſcribe ſic; Aſſumptum eſt fore AD ⊏ HL. Hoc autem conſtat. Nam ſi AD = vel ⊐ HL, erit ang. A *a* =, *b* vel ⊏ HLI. Eodem modo erit B =, vel ⊏ HLK, & C =, vel ⊏ KLI. quare A + B + C * quatuor rectos aut exæquabunt, aut excedent, contra hypoth. quin potius ſit AD ⊏ HL. Q. E. D.

a conſtr.
b 8. 1.
c 21. 1.
* 4 cor. 32. 1.

*FINIS.*

# EUCLIDIS DATA

*ſuccincte demonſtrata;*

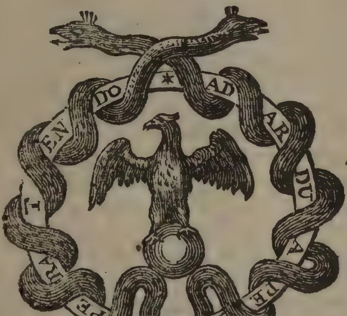
Una cum Emendationibus quibuſdam & Additionibus ad ELEMENTA.

EUCLIDIS

nuper edita.

*Opera*

Mri. Is. BARROW, *Cantabrigienſis*, Coll. *Trin.* Soc.




*LONDINI*
Excudebat *R. Daniel*, 1659.

# Ornatiſſimo viro
# D. IACOBO STOCK,
## amico ſuo & patrono ſingulari.

*NEc publica, nec tui nominis luce dignum cenſeo hunc paucorum dierum partum puſillum & præmaturum. Qui quidem quod ſe mundo, quodque Tibi, ſpectandum obtulerit, duplici nomine arrogantiæ ſpeciem incurrit. Sed utrinque parata eſt excuſatio qualiſcunque. Nam amico obtemperatum oportuit jubenti mitterem hunc libellum Euclidæis (quæ cognatione proxima attingit) Elementis ſubjungendum. In eum quicquid eſt in publicum aut peccati aut meriti protinus rejicio, facti cujus author fuit, rationem redditurum. In Te autem delictum quod maxime aggravat, idem potenter extenuat, Tibi tantum debere. Nam cum iis, qui Diis ipſis ſacrificia, ac modica magnis Regibus donaria offerre non dubitarunt, ſatius eſſe credo, etiam pro immenſis beneficiis parum, quam nihil rependere. Sufficiat igitur regeſſiſſe, me Tibi multis magniſque nominibus obſtrictum fore; vices, quas potuero maximas, referre debere; ultra vota & grates nihil poſſe; illa privatim, has publice perſolutas præcellere; quibus agendis, quam jamdiu ſpe & ſtudio aucuper, occaſionem nondum comparere; præſtare hanc*

*oblatam præhendere, quamvis exilem, quam elapſam nequicquam pœnitentia proſequi. Eſto igitur hæ oblatio pignus quoddam & præludium futuræ am plioris, in qua meritorum in me Tuorum hiſtoria u berior ac diſtinctior commemoranda occurret. Qu ſimpliciter agnoſcere, non aut fuſe deſcribere, au digne prædicare, præſentis eſt inſtituti. Ac rever jam brevis ſum ἑκὼν ἀέκοντί γε θυμῷ, neceſſitate po tius coactus, quam inductus conſilio. Nam me vel ventis turgentia alio avocant; ac vereor ne hæc p ne currenti calamo exequentem, quæ hæc ad te perf ret, amica manus, importuna patientia præſtoletu Quid ſupereſt igitur, niſi ut te domi ſtudiis ac reb honeſtis animum intendentem ſalutari præſentia tu tetur, eum exorem venerandi ac ἀῤῥήτου nominis quem tantæ beneficentiæ benignum remuneratore jugibus votis exopto; idemque me extemplo ſup Tyrrhenos, Ionios, Ægeoſque fluctus longinqua profectionem ſuſcepturum comitetur. Obteſtor auten ne tenuis opellæ patrocinium reſpuas, quod ultro in pertiire dignatus es*

Tibi devinctiſſimo

& obſequentiſſimo.

*I. B.*

# *EVCLIDIS* Data.

## *Definitiones.*

DAta magnitudine dicuntur ſpatia, lineæ, anguli, quibus æqualia poſſumus invenire.

II. Ratio dari dicitur, cui poſſumus eandem invenire.

III. Rectilineæ figuræ ſpecie dari dicuntur, [illegible]arum & ſinguli anguli dati ſunt, & laterum ra[illegible]nes ad invicem datæ ſunt.

Hinc, datæ ſunt ſpecie figuræ, quibus ſimiles [illegible]veniri poſſunt.

IV. Poſitione dari dicuntur puncta, lineæ, [illegible]gulique, quæ eundem ſitum ſemper obtinent.

V. Circulus magnitudine dari dicitur, cujus [illegible] quæ ex centro datur magnitudine.

VI. Poſitione & magnitudine dari dicitur [illegible]rculus, cujus datur centrum poſitione, & ea [illegible]æ ex centro magnitudine.

VII. Circuli ſegmenta magnitudine dari [illegible]icuntur, in quibus dati ſunt magnitudine angu[illegible] & ſegmentorum baſes.

VIII. Poſitione & magnitudine dari dicun[illegible]ur circuli ſegmenta, in quibus anguli magnitu[illegible]ine dati ſunt, & ſegmentorum baſes poſitione [illegible] magnitudine.

IX. Magnitudo magnitudine major eſt data, quando ablata data, reliqua eidem æqualis eſt.

X. Magnitudo magnitudine minor eſt data, quando adjuncta data, tota eidem æqualis eſt.

Ut ſi A data ſit, erit $A + B \sqsubset B$ data. At $B \sqsupset A + B$ data.

XI. Magnitudo magnitudine major eſt data quam in ratione, quando ablata data, reliqua ad eandem habet rationem datam.

XII. Magnitudo magnitudine minor eſt data quam in ratione, quando adjuncta data tota ad eandem rationem habet datam.

Ut ſi A data ſit, & B detur, erit A+B ⊏ C, data q. in r. ſin A+B detur, erit B ⊐ C data q. in r.

P R O P. 1.

A. B. *Datarum magnitudinum A, B,*
a. b. *ad invicem datur ratio.*

* hyp. a 1. def. b ſch. 7. 5. c 1. def.

Nam quia A * datur, a inveniri poteſt aliqua a = A. Eodem jure ſume b = B. b eſtque a. b :: A. B. c quare ratio A/B data eſt. Q. E. D.

P R O P. 2.

A. B. *Si data magnitudo A ad aliam*
a. b. *aliquam B habeat rationem datam datur etiam hæc alia magnitudine*

* hyp. a 1. def. d. b 2. def. d. c 9. 5.

Nam ob A * datam, a ſume a = A; ac ob A/B * datam, b ſit a/b = A/B. c ergo b = B. a quare B datur. Q. E. D.

P R O P. 3.

A. B. *Si quotlibet datæ magnitudine*
a. b. *A, B componantur, etiam ea A+B quæ ex his componitur, data erit.*

a 1. def. b 2. ax. 1.

Nam a cape a = A, & b = B; b eſtque a + b = A+B. a quare A+B datur. Q. E. D

P R O P. 4.

A. B. *Si à data magnitudine A auferatur data magnitudo B, etiam reliqua A−B dabitur.*
a. b.

a 1. def. d. b 3. ax. 1.

a Sint enim a = A, & b = B. ergo A−B = a−b. a proinde A−B datur. Q. E. D.

PROP

PROP. 5.

B. *Si magnitudo* A *ad ſui-ipſius aliquam partem* B *habeat rationem*
D. *datam, etiam ad reliquam* A−B *habebit rationem datam.*

Nam, quia $\frac{A}{B}$ a data eſt, b ſit A. B :: C. D. ergo A. A − B :: C. C − D. b proinde $\frac{A}{A-B}$ datur. Q. E. D. a hyp. b 2. def d. c cor. 9. 5.

PROP. 6.

A. B. *Si componantur duæ magnitudi-*
C. D. *nes* A, B, *habentes ad invicem rationem datam, etiam quæ ex his componitur magnitudo* A+B, *habebit ad utramque* A *&* B *rationem datam.*

Nam a ſit A. B :: C. D. b ergo A + B. B :: C + D. D. c quare $\frac{A+B}{B}$ datur. Similiter $\frac{B+A}{A}$ datur. Q. E. D. a 2. def. d. b 18 5. c 2. def. d.

PROP. 7.

A. B. *Si data magnitudo* A + B *data ratione ſecetur, utrumque ſegmentorum* A, *&* B *datum eſt.*

Nam ob $\frac{A}{B}$ * datam, a erit $\frac{A+B}{A}$ data. b ergo A datur. Eodem modo B datur. Q. E. D. * hyp. a 6 dat. b 2. dat.

PROP. 8.

A. C. B. *Quæ* A, B *ad idem* C *rationem*
D. E. F. *habent datam, habebunt ad invicem rationem datam.*

Nam a ſit A. C :: D. E. a & C. B :: E. F. quare ex æquali A. B :: D. F. a ergo $\frac{A}{B}$ datur. Q. E. D. a 1. def. d.

*Coroll.*

Rationes ex datis rationibus compoſitæ, datæ ſunt. Ut $\frac{A}{B}$ ſit ex $\frac{A}{C}$, & $\frac{C}{B}$ datis.

PROP. 9.

A. B. C. *Si duæ, pluresve magnitudines*
D. E. F. *A, B, C ad invicem habeant rationem datam, habeant autem illæ magnitudines A, B, C ad alias quaſdam D, E, F rationes datas, etſi non eaſdem; illæ aliæ magnitudines D, E, F etiam ad invicem habent rationes datas.*

a 20. def 5. b hyp. c cor. 8. dat.

Nam ratio $\frac{D}{E}$ [a] ſit ex [b] datis $\frac{D}{A}$, $\frac{A}{B}$, $\frac{B}{E}$; [c] ergo $\frac{D}{E}$ datur. Eadem de cauſa datur $\frac{E}{F}$. Q. E. D.

PROP. 10.

A. B. C. *Si magnitudo magnitudine major fuerit data, quam in ratione; & ſimul utraque illa eadem major erit data quam in ratione. Sin autem ſimul utraq; magnitudo eadem magnitudine major fuerit data, quam in ratione; & reliqua illa eadem major erit data quam in ratione; aut reliqua data eſt cum conſequente, ad quam habet altera magnitudo rationem datam.*

a 6. dat. b 11. def. d. c 17. 5. d 5. d.

1. Sint A, & $\frac{B}{C}$ datæ. [a] erit $\frac{B+C}{C}$ data. [b] ergo $A+B+C \sqsubset C$ data q. in r. Q. E. D.

2. Sint A, & $\frac{B+C}{C}$ datæ: [c] ergo $\frac{B}{C}$ datur. proinde $A+B \sqsubset C$ data q. in r. Q. E. D.

3. Sint $A+B$, & $\frac{C}{B+C}$ datæ. [d] Liquet $\frac{B}{B+C}$ dari. Q. E. D.

PROP. 11.

A. B. C. *Si magnitudo magnitudine major ſit data quam in ratione, eadem ſimul utraque major erit data quam in ratione. Et ſi eadem ſimul utraque major ſit data quam in ratione, eadem reliqua magnitudine major erit data quam in ratione.*

1. A

1. A, & $\frac{B}{C}$ dantur. *a* ergo $\frac{B}{B+C}$ datur. proinde a 6. *dat.*
*b* A + B ⊏ B + C data q. in r. Q. E. D. b 11. *def. d.*
2. A, & $\frac{B}{B+C}$ dantur. *c* ergo $\frac{B}{C}$ datur, proinde c 5. *dat.*
*b* A + B ⊏ C data q. in r. Q. E. D.

PROP. 12.

A. B. C. *Si fuerint tres magnitudines A, B, C, & prima cum ſecunda (A + B) data ſit, ſecunda quoque cum tertia (B + C) data ſit; aut prima A tertiæ C æqualis eſt, aut altera altera major data.*

Nam ſi A + B, & B + C pares ſint, [b] liquet a 4 *ax.* 1.
A & C æquari; ſin iſtæ impares fuerint, *b* liquet b 4 *dat.*
exceſſum A — C, vel C — A dari. Q. E. D.

PROP. 13.

D, A + B, C.
E.

*Si fuerint tres magnitudines D, A + B, C, & earum prima D ad ſecundam A + B habeat rationem datam; ſecunda autem A + B tertia C major ſit data quam in ratione; prima quoque D major erit tertia C data quam in ratione.*

Sint A, & $\frac{B}{C}$, ac $\frac{D}{A+B}$ datæ; *a* ſitque A + B. a 2. *def. d.*
D :: A. E *b* :: B. D − E. ergo *c* E, *d* & $\frac{B}{D-E}$ b 19. 5. c 2. *dat.* d 2. *def. d.* e 8. *dat.* f 11. *def. d.*
& (ob $\frac{B}{C}$ datam) *e* $\frac{C}{D-E}$ dantur. *f* quare D (E +: D − E) ⊏ C data q. in r. Q. E. D.

PROP. 14.

A. C.
B. D.
E.

*Si duæ magnitudines A & C ad invicem habeant rationem datam, utrique autem illarum adjiciatur data magnitudo B & D; totæ A + B, C + D, aut habent rationem datam, aut altera A + B altera C + D major erit data quam in ratione.*

Nam

a 12. 5. b hyp. c 2. def. d. Nam ſi A. C :: B. D $^{a}$:: A + B. C + D ob $\frac{A}{C}$ $^{b}$ datam, $^{c}$ liquet $\frac{A+B}{C+D}$ dari.

d 2. def. d. e 2. dat. f 4. dat. g 11. def. d. Saltem $^{d}$ ſit A. C :: E. D. $^{a}$ :: A + E. C + D. Ergo $^{c}$ $\frac{A+E}{C+D}$, ac $^{e}$ E, $^{f}$ ideoque B – E dantur. $^{g}$ proinde A + B ( A + E : + B – E ) ⊏ C + D data q. in r. Q. E. D.

P R O P. 15.

A. C. B. D. E.

*Si duæ magnitudines* A & C *habeant ad invicem rationem datam, & ab utraque harum auferatur data magnitudo* B & D; *reliquæ magnitudines* A – B, C – D *ad invicem habebunt aut rationem datam, aut altera* A – B, *altera* C – D *major erit data quam in ratione.*

a 19. 5. b hyp. c 2 def. d. d 2 def. 2. e 2. dat. f 4 dat. g 11. def. d. $^{b}$ Nam ſi A. C :: B. D $^{a}$ :: A – B. C – D. ob $\frac{A}{C}$ datam, $^{c}$ liquet $\frac{A-B}{A-C}$ dari.

Saltem $^{d}$ ſit A. C :: E. D $^{a}$ :: A – E. C – D. Ergo $^{e}$ $\frac{A-E}{C-D}$, & $^{e}$ E, ac $^{f}$ ideo E – B dantur. $^{g}$ proinde A – B ( A – E : + E – B ) ⊏ C – D data q. in r. Q. E. D.

P R O P. 16.

B. C. A. D. E.

*Si duæ magnitudines* B, C *habeant rationem datam, & ab una quidem illarum* C *auferatur data magnitudo* D, *alteri autem* B *adjiciatur data magnitudo* A; *tota* A + B *reſidua* C – D *major erit data quam in ratione.*

a 2. def. d. b 19. 5. c 2. def. d. d 2. dat. e 3 dat. f 11. def. d. Sit enim C. B $^{a}$ :: D. E $^{b}$ :: C – D. B – E. ergo $^{c}$ $\frac{C-D}{B-E}$, & $^{d}$ E, ac $^{e}$ ideo E + A dantur. $^{f}$ proinde B + A ( E + A : + B – E ) ⊏ C – D data q. in r. Q. E. D.

PROP.

PROP. 17.

A+B. D+E. C. *Si fuerint tres magnitudines* A+B, C, D+E; *&* *prima quidem* A+B *ſecunda* C *major ſit data quam in ratione, tertia quoque* D+E *eadem ſecunda* C *major ſit data quam in ratione; prima* A+B *ad tertiam* D+E *aut rationem habebit datam, aut altera altera major erit data quam in ratione.*

Nam ob A, D, & B E a datas, b erit B data. a hyp. b 8. dat.
C, C E

ergo per 14. hujus.

PROP. 18.

A+C. E. G. B+D. F. H. *Si fuerint tres magnitudines, atque ex his una utraque reliquarum major ſit data quam in ratione; reliquæ duæ aut datam rationem habebunt ad invicem, aut altera altera major erit data quam in ratione.*

Datæ ſint A, B, C D ac ſit A+C=B+D.
E, F;

Sitque C.E a :: A.G b :: C+A. E+G. itemque a 1. def. d. b 12 5
D. F a :: B. H b :: D+B: F+H. c ergo c 1. def. d.
C+A d hoc eſt B+D, c & B+D, ac e idcirco d 7. 5. e 8. 5.
E+G, E+G, F+H f 2 dat.
E+G quin & G ac H f dantur. ergo per 15.
F+H; (hujus.

PROP. 19.

A+B. E. C+D. F. *Si fuerint tres magnitudines, & prima quidem magnitudo ſecunda magnitudine major ſit data quam in ratione, ſit quoque ſecunda major tertia data quam in ratione; prima magnitudo tertia magnitudine major erit data quam in ratione.*

Sint A, C, & C+D, D datæ; dico A+B
B E

⊏ E data q. in r.

Nam

a 2. def. d. b 19. 5. c 2. def. d. d 2. dat. e 3. dat. f 8. dat. g 11. def. d.

Nam ſit C+D.B a :: C.F b :: D. B − F. ergo c $\frac{C}{F}$ & d F, ac e ideo F + A, & c $\frac{D}{B-F}$ f ideoque $\frac{E}{B-F}$ dantur. g proinde A+B (F+A :+B − F) ⊏ E data q. in r. Q. E. D.

P R O P. 20.

A. C. E.
B. D.

*Si datæ fuerint duæ magnitudines* A, C; *& auferantur ab ipſis magnitudines* B, D *habentes ad invicem rationem datam; reſiduæ magnitudines* A − B, C − D *aut habebunt ad invicem rationem datam, aut altera* A − B *altera* C − D *major erit data quam in ratione.*

a 19. 5. b 2. def. d.

Nam ſi A. C :: B. D a :: A − B. C − D; b liquet $\frac{A-B}{C-D}$ dari.

c 2. dat. d 4. dat. e 11. def. d.

Saltem ſit D. B b :: C. E a :: C − D. E − B. ergo b $\frac{C}{E}$ & c E, ac d propterea A − E, b itemque $\frac{C-D}{E-B}$ datæ ſunt. e ergo A − B (A − E : + E − B) ⊏ C − D data q. in r. Q. E. D.

P R O P. 21.

A. C. E.
B. D.

*Si datæ fuerint duæ magnitudines* A, C; *& adjiciantur ipſis aliæ magnitudines* B, D *habentes ad invicem rationem datam; totæ* A + B C + D *aut habebunt ad invicem rationem datam, aut altera* A + B *altera* C + D *major erit data quam in ratione.*

a 12. 5. b 2. def. d.

Nam ſi B. D :: A. C a :: A + B. C + D, b liquet $\frac{A+B}{C-D}$ dari.

c 2. dat. d 4. dat.

Saltem ſit B. D b :: E. C a :: B + E. D + C. ergo c E, d ideoque A − E, & b $\frac{B+E}{D+C}$ dantur.

e ergo

e ergo A+B ( B+E :: +A−E ) ⊏ C+D data q. in r. Q. E. D. [e 11. def.]

PROP. 22.

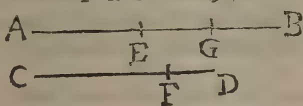
A. B. C.

*Si duæ magnitudines* A, B *ad aliam aliquam magnitudinem* C *habeant rationem datam, & ſimul utraque* A + B *ad eandem* C *habebit rationem datam.*

Nam ob $\frac{A}{C},\frac{B}{C}$ a datas, b erit $\frac{A}{B}$ data. c quare $\frac{A+B}{B}$, b ideoque $\frac{A+B}{C}$ data eſt. Q. E. D.

[a hyp. b 8. d. c 6 d.]

PROP. 23.

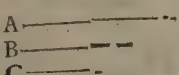



*Si totum* AB *ad totum* CD *habeat rationem datam, habeant autem & partes* AE, EB *ad partes* CF, FD *rationes datas ( etſi non eaſdem ; ) habebunt omnia ad omnia rationes datas.*

Nam ſit AE. CF a :: AG. CD b :: GE. FD. a ergo $\frac{GE}{FD}$ datur. quare ( ob $\frac{EB}{FD}$ c datam ) d erit $\frac{GE}{EB}$, ac e ideo $\frac{EB}{GB}$ data. ergo quum c $\frac{AB}{CD}$, & a $\frac{AG}{CD}$, d ideoque $\frac{AB}{AG}$, ac proinde e $\frac{AB}{GB}$, dentur, d erit $\frac{EB}{AB}$, data. Quare e $\frac{AB}{AE}$, & d $\frac{AE}{EB}$, & e $\frac{EB}{CF}$, dantur. Q.E.D.

[a def. d. b 19. 5. c hyp. d 8. dat. e 5. dat.]

PROP. 24.

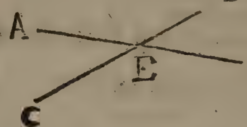



*Si tres rectæ lineæ, A, B, C, proportionales fuerint ; prima autem A ad tertiam C habeat rationem datam; & ad ſecundam B habebit rationem datam.*

Nam

a cor. 20. 6. Nam A. C a :: Aq. Bq. b ergo $\frac{Aq}{Bq}$ data eſt.
b 2. def. d.
c 1. d. proinde $\frac{A}{E}$ c datur. Q. E. D.

PROP. 25.



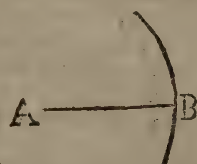

*Si duæ rectæ lineæ, AB, CD poſitione datæ ſe mutuo ſecuerint, punctum E, in quo ſe invicem ſecant, poſitione datum eſt.*

a 4. def. d. a Nam hæ lineæ alibi quam in E, neutrius ſitu mutuo, ſeſe interſecare nequeunt.

*Schol.*

a Idem patet de quibuſcunque lineis poſitione datis, ſeque in unico puncto interſecantibus: ut de circuli arcu, & recta, &c.

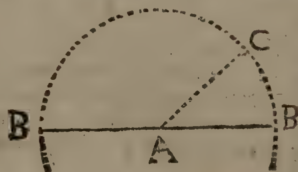
PROP. 26.




*Si rectæ lineæ AB extremitates A, B, poſitione datæ ſint, recta AB poſitione & magnitudine data eſt.*

a 14 ax. Poſitione quidem, a quia inter eoſdem terminos unica recta duci poteſt: &
b 1. def. d. magnitudine, b quia ſi centro A per B ducatur circulus, hujus omnes radii ipſi AB æquantur.

PROP. 27.




*Si rectæ lineæ AB poſitione & magnitudine datæ, data fuerit una extremitas A; & altera extremitas B data erit.*

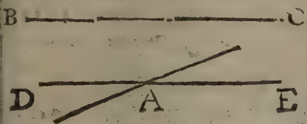
Nam

Nam ſi centro A, ſpatio AC *a* = AB *b* duca- a 1. *def. d.*
tur circulus, cui data recta *c* occurrat in B, *d* erit b 3 *poſt.* c 2. *poſt.*
extremitas B data. d *cor.* 25.

*Schol.*

Vides partes puncti B determinandas eſſe.

P R O P. 28.

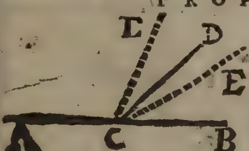



*Si per datum punctum A contra datam poſitione rectam B C agatur recta linea DE, acta recta DE poſitione data eſt,*

Nam *a* dic alteram per A ad BC fore paralle- a 4. *def. d.*
lam. Hæc idcirco ad DE *b* parallela erit. *c* Quod b 30. 1.
repugnat. c 34. *def.* 1.

*Nota*, Vocabulum *contra* in hoc libro paralleliſmum ſignificare.

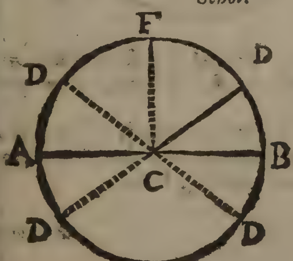
P R O P. 29.




*Si ad poſitione datam rectam* AB *, datumque in ea punctum* C *, agatur recta linea* C D, *quæ faciat angulum* DCB *datum ; acta recta* C D *poſitione data erit.*

*a* Nam quævis alia C E angulum *b* efficiet a 4. *def. d.*
majorem, vel minorum dato BCD. b 9. *ax.* 1.

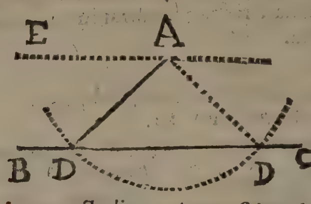
*Schol.*




Determinari debet ſitus anguli dati tam reſpectuperpendicularis C F, quam ipſius AB, ut cernis in appoſita figura.

P R O P.

PROP. 30.



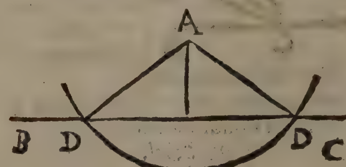

*Si à dato puncto A in datam poſitione rectam* B C *agatur recta linea* A D, *quæ faciat angulum* ADC *datum, acta linea* A D *poſitione data eſt.*

a 28. dat. b 1. def. d. c 29 dat. Nam per A duc A E ad BC parallelam. *a* Hęc poſitione datur. Item ang. DAE par dato alterno ADC *b* datus eſt. *c* ergo recta *AD* poſitione data eſt. Q. E. D.

*Schol.*

Hinc praxim diſcimus à dato puncto ducendi rectam, quæ cum data poſitione recta datum angulum effici'.

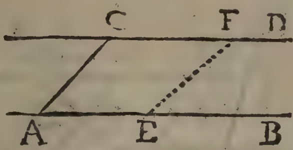
PROP. 31.




*Si à dato puncto A in datam poſitione rectam* BC *data magnitudine recta A* D *ducatur, poſitione quoque data erit.*

a 1. def. d. b ſch. 25. d. c 16. d. Nam puncta D, per quæ tranſit circulus centro *A*, *a* ſpatio *A* D deſcriptus, *b* data ſunt. *c* ergo *A* D poſitione data eſt. Q. E. D.

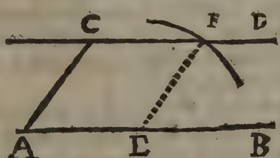
## PROP. 32.




*Si in datas poſitione parallelas rectas* AB, CD *agatur recta linea* AC, *quæ faciat angulos datos* BAC, ACD, *acta recta* AC *magnitudine data eſt.*

Nam ad E (quodvis punctum in AB) fac
ang. BEF = *a* BAC. liquet rectas EF, AC *b* pa- a 1. def. d.
rallelas, & *c* pares fore. *d* quare AC data eſt. b 29. 1. c 34. 1.
Q. E. D. d 2. def. d.

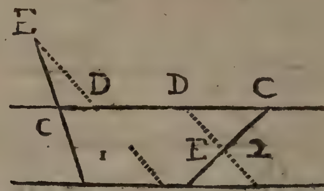
## PROP. 33.




*Si in datas poſitione parallelas rectas* AB, CD *agatur magnitudine data recta* AC, *faciet angulos* BAC, ACD *datos.*

Nam ex quovis puncto E in AB, ſpatio EF
*a* = AC deſcribe circulum occurrentem rectæ a 1. def. d.
CD in F. *b* Liquet EF, & AC parallelas eſſe b 34. 1.
poſſe. *c* ergo. c 29. 1.

PROP. 34.




*Si in datas poſitione parallelas rectas* AB, CD *à dato puncto* E *agatur recta linea* ECA, *ſecabitur data ratione.*

Nam ab E duc rectam E B utcunque parallelis occurrentem in D, & B. *a* liquet eſſe EC. CA :: ED. DB. *b* quare FC/CA datur. Q. E. D.

a 2. 6.
b 2. def. d.

PROP. 35.

*Si à dato puncto* E *in datam poſitione rectam* AB *agatur recta linea* EA, *ſeceturque data ratione; agatur autem per punctum ſectionis* C *contra datam poſitione rectam* A B *recta linea* C D; *acta linea* CD *poſitione data eſt.*

Recta enim E B ducta ab E utcunque in A B, *a* ſecetur ſic ut ED. DB :: EC. CA. ob punctum D datum, *b* erit CD poſitione data. Q. E. D.

a 10. 6.
b 28. dat.

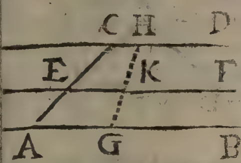
PROP. 36.

*Si à dato puncto* E *in datam poſitione rectam lineam* AB *agatur recta linea* EA; *adjiciatur autem ipſi aliqua recta* EC, *quæ ad illam* (EA) *habeat rationem datam; per extremitatem autem* C *adjectæ lineæ* EC *agatur contra datam poſitione rectam* AB *recta linea* CD; *acta linea* CD *poſitione data eſt.*

Demonſtratio parum differt à præcedenti. *Vide fig.* 2.

PROP.

PROP. 37.

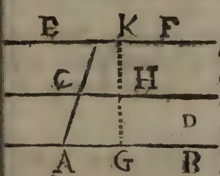



*Si in datas poſitione parallelas rectas* AB, CD, *agatur recta linea* AC, *& ſecetur ratione data; agatur autem per ſectionis punctum* E *contra datas poſitione rectas* AB, CD *linea recta* EF; *acta recta* EF *poſitione data eſt.*

Nam duc rectam GH utcunque occurrentem parallelis. Hæc [a] ſecta ſit in K ita ut GK. KH :: AE. EC. [b] Punctum K parallelæ (EF) ſitum determinat. Q. E. F.

a 2. def. d
b 28. dat.
& ſch. 2. 6.

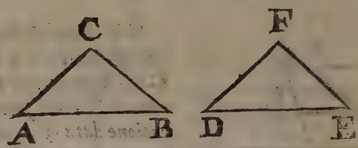
PROP. 38.




*Si in datas poſitione rectas parallelas* AB, CD *agatur recta linea* AC; *adjiciatur autem ipſi quædam recta* CE, *quæ ad actam* AE *habeat rationem datam; per extremitatem autem* E *adjectæ* CE *agatur contra datas poſitione parallelas* AB, CD *recta linea* EF; *acta recta linea* EF *eſt data poſitione.*

Demonſtratio perſimilis eſt præcedenti. Cerne & compara figuras.

PROP. 74.




*Si trianguli* ABC *ſingula latera* AB,BC,AC *magnitudine data ſint, triangulum* ABC *ſpecie datum eſt.*

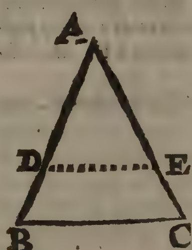
a 22. 1. Nam *a* fac triang. D E F ipſi A B C æquilaterum.
b 5. 6. Hoc eidem *b* æquiangulum erit. *c* ergo ABC
c 3. def. d. ſpecie datum eſt. Q. E. D.

PROP. 40.

*Si trianguli* A B C *ſinguli anguli,* A, B, C *magnitudine dati ſint, triangulum* ABC *ſpecie datum eſt.*

a 23. 1. Nam ad quamvis DE *a* fac triang. DEF ipſi
b 4. 6. ABC æquiangulum. *b* Hoc eidem ſimile erit.
c 3. def. d. *c* proinde trigonum A B C ſpecie datum eſt. Q. E. D.

PROP. 41.




*Si triangulum* A B C *unum angulum* A *datum habeat; circa datum autem angulum* A *duo latera* AB, AC *ad invicem habeant rationem datam; triangulum* A B C *ſpecie datum eſt.*

Nam in uno latere dati anguli ſume quampiam AD; & *a* ſit A B. A C :: AD.AE. & duc DE. *b* Liquet trigonum A D E ipſi A B C ſimile fore. *c* Quare ABC ſpecie datum eſt. Q. E. D.

a 1. def. d.
b 6. 6.
c 3. def. d.

PROP.

PROP. 42.

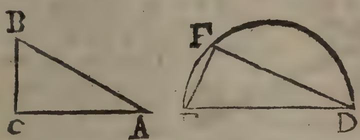
*Si trianguli* ABC *latera ad invicem habeant rationem datam, triangulum* ABC *ſpecie datum eſt.*

Nam *a* fac AB. BC :: DE. EF. *a* & BC. CA :: EF. FD. *b* Liquet trigonum DEF trigono ABC aſſimilari. *c* quare ABC ſpecie datum eſt. Q. E. D.

a 12. 6.
b 5. 6.
c 3. *def. d.*

*Vide fig.* 39.

PROP. 43.

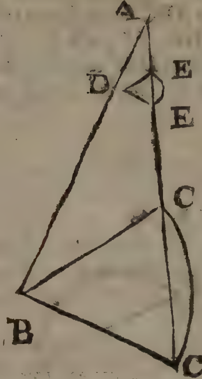



*Si trianguli rectanguli* ACB *circa unum acutorum angulorum* A *latera* AB, AC *ad invicem rationem habeant datam, triangulum* ACB *ſpecie datum eſt.*

Nam eſto DEF ſemicirculus utcunque; & *a* fac AB. AC :: DE. DF. inventamque DF *b* adapta in ſemicirculo; & duc EF. *c* Liquet triang. DFE ipſi ACB aſſimilari; & *d* proinde ipſum ACB ſpecie dari. Q. E. D.

a 12. 5.
b 1. 4.
c 32. 1. & 4. 6.
d 3. *def. d.*

PROP. 44.

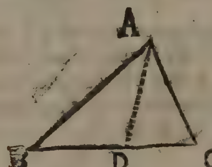



*Si triangulum* ABC *habeat unum angulum* A *datum ; circa alium autem angulum* ABC *latera* AB, BC *ad invicem habeant rationem datam ; triangulum* ABC *ſpecie datum eſt.*

Nam in crure dati anguli ſume quamlibet A D. & *a* fac A B. BC :: AD. DE. centro D ſpatio D E deſcribe circulum, qui ſecet alterum dati anguli latus in E. *b* Eritque triang. ADE ipſi ABC ſimile. *c* quare datur ſpecie triang. ABC. Q.E.D.

*a* 2. def. d.
*b* 7. 6.
*c* 3. def. d.

PROP. 45.




*Si triangulum* B A C *unum angulum* BAC *datum habeat ; circa datum autem angulum* BAC *latera ſimul utraque tanquam unum* (BA+AC) *ad reliquum latus* (B C) *rationem habeant datam ; triangulum* BAC *ſpecie datum eſt.*

Datum angulum B A C *a* biſecet recta A D. *b* ergo B A. A C :: B D. D C. & componendo BA + AC. AC :: BC. DC. permutando igitur BA + AC. BC :: AC. DC. ergo ob $\frac{BA + AC}{BC}$ *c* datam, *d* erit $\frac{AC}{DC}$ data. item ang. DAC ſub-

*a* 9. 1.
*b* 3. 6.
*c* hyp.
*d* 2. def. d.

duplus

duplus dati B A C *e* datur. *f* ergo ang. C datur. *e* 2.*dat.*
*g* proinde trigonum ABC ſpecie datum eſt. *f* 44.*dat.* *g* 40.*dat.*

*Coroll.*

Hinc in triangulo, datis uno latere AB, uno angulo BAC, & ratione aggregati laterum ad baſim (R ad S;) datur triangulum. Nam datum angulum biſeca, & fac R.S :: AB. BD. & centro B ſpatio B D duc circulum occurrentem rectæ biſecanti in D; & produc BDC. habes triangulum.

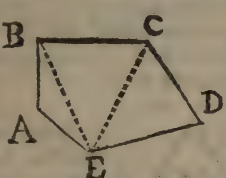
## PROP. 46.

*Si triangulum* B A C *unum angulum* C *datum habeat; circa alium autem angulum* BAC *latera ſimul utraque tanquam unum* (BA+AC) *habeant ad reliquum* (B C) *rationem datam; triangulum* BAC *ſpecie datum eſt.*

Nam biſecto angulo BAC, erit (ut in præcedenti) AC data. item ang. C *a* datus eſt. ergo *a hyp.*
$\overline{DC}$
ang. D A C, *b* proinde & duplus B A C datur. b 2.*dat.*
*c* quare triang. BAC ſpecie datur. Q. E. D. c 40.*dat.*

*Deducetur ab hac corollarium ſimile præcedenti.*

## PROP. 47.

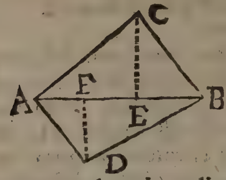



*Data ſpecie rectilinea* ABCDE *in data ſpecie triangula* BAE, CDE BCE *dividuntur.*

Nam ob ang. B, & BA *a* dat. *b* erit triang. $\overline{AE}$ BAE ſpecie datum. Similí diſcurſu triang. CDE ſpecie datur. *c* quare ang. DCE datus eſt; Hunc deme ex dato BCD, *d* eſtque reliquus BCE datus. Similiter ang. CBE datur. *e* ergo triang. BCE etiam ſpecie datum eſt. Q. E. D.

a *hyp.* & 3.*def.d*
b 41.*dat.*
c 3.*def.d;*
d 4 *dat.*
e 40.*dat.*

PROP. 48.

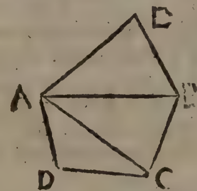



*Si ab eadem recta AB deſcribantur triangula ACB, ADB data ſpecie, habebunt ad invicem rationem datam.*

Duc enim perpendiculares CE, DF. Liquet angulos trianguli rectanguli CEB, a proinde & $\frac{CE}{CB}$ dari. ergo (quum $\frac{AB}{CB}$ b data ſit) c erit $\frac{CE}{AB}$ data. Simili diſcurſu datur $\frac{DF}{AB}$; c quare $\frac{CE}{DF}$, d hoc eſt triang. $\frac{ACB}{ADB}$ datur. Q. E. D.

a 40. dat.
b hyp.
c 8. dat.
d ſch. 1. 6.

PROP. 49.

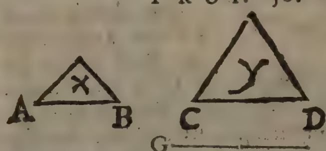



*Si ab eadem recta linea AB duo rectilinea quælibet ABCD, AEB data ſpecie deſcribantur, habebunt ad invicem rationem datam.*

Nam rectilineum ABCD reſolvatur in triangula. a hæc ſpecie data ſunt. ergo ob communem baſim AC, b ratio ADC ad ACB & c proinde totius ABCD ad ACB datur. b item ratio AEB ad ACB. d proinde & ABCD ad AEB datur. Q. E. D.

a 47. dat.
b 48. dat.
c 8. dat.
d 8. dat.

PROP. 50.



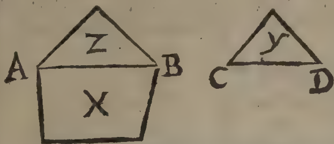

*Si duæ rectæ lineæ AB CD ad invicem habeant rationem datam; & ab illis ſimilia, ſimiliterque deſcripta rectilinea X, Y habebunt ad invicem rationem datam.*

Nam

Nam ſit AB. CD :: [a] CD. G. [b] liquet AB ad G, [c] hoc eſt X ad Y dari. Q. E. D.

a 11. 6.
b 8. *dat.*
c *cor* 20. 6.

PROP. 51.

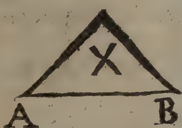



*Si duæ rectæ lineæ AB, CD habeant ad invicem rationem datam; & ab illis rectilinea quæcunque X, Y ſpecie data deſcribantur; habebunt ad invicem rationem datam.*

Nam [a] fac Z ſimile ipſi Y. Ac ob [b] $\frac{Z}{X}$, [c] & $\frac{Z}{Y}$ datas, [d] liquet $\frac{X}{Y}$ dari. Q. E. D.

a 18. 6.
b 49. *dat.*
c 50. *dat.*
d 8. *dat.*

PROP. 52.

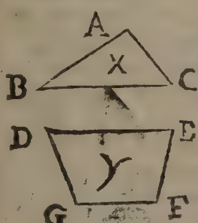



*Si à data magnitudine recta AB figura X ſpecie data deſcribatur, deſcripta figura X magnitudine data eſt.*

Nam ABq [a] datur ſpecie, & magnitudine; & [b] $\frac{ABq}{X}$ datur. [c] ergo X datur.

a 3. & 1. *def d.*
b 49. *dat.*
c 2. *dat.*

PROP. 53.



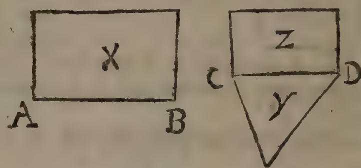

*Si duæ figuræ X, Y ſpecie datæ fuerint; & unum latus unius BC ad unum latus alterius DE habuerit rationem datam; reliqua quoque latera AB ad reliqua EG habebunt rationem datam.*

Nam

a 3.def.d.
b hyp.

Nam { a AB, b $\overline{BC}$, a $\overline{DE}$, $\overline{EF}$, a $\overline{FG}$ } dantur. &c. ergo per 8. dat.

PROP. 54.

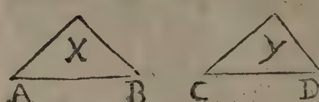



*Si duæ figuræ* X, Y *ſpecie datæ ad invicem habuerint rationem datam, etiam latera* (AB, CD, &c.) *habebunt ad invicem rationem datam.*

a 18. 6. b 3. def. d. c 49. dat. d hyp. e 8. dat. f cor 20. 6 & 24. dat.

Nam ad CD [a] fiat Z ipſi X ſimilis.[b] Hæc ſpecie datur.[c] ergo $\frac{Y}{Z}$ datur. Proinde ob $\frac{Y}{X}$ [d] datam, [e] datur $\frac{X}{Z}$. [f] ergo $\frac{AB}{CD}$ datur. ergo per præcedentem.

PROP. 55.



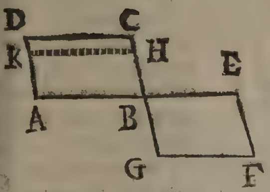

*Si ſpatium* X *magnitudine & ſpecie datum fuerit, ejus latera* (AB &c.) *magnitudine data erunt.*

a 18. 6. b 1. dat. c 54. dat. d 2. dat.

Nam ad quamvis CD [a] fac Y ſimile ipſi X. hoc ſpecie & magnitudine datur. [b] ergo $\frac{Y}{X}$ datur. [c] quare $\frac{CD}{AB}$ datur. [d] ergo AB data eſt. Q. E. D.

PROP.

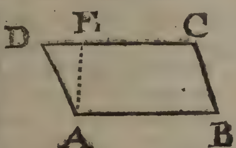
PROP. 56.




*Si duo æquiangula parallelogramma* AC, BF *habuerint ad invicem rationem datam; eſt ut primi latus* AB *ad ſecundi latus* BE, *ita reliquum ſecundi latus* BG *ad eam* BH, *ad quam alterum primi latus* BC *habet rationem datam, quam habet parallelogrammum* AC *ad parallelogrammum* BF.

Nam duc HK parall. AB. Liquet eſſe BC. BH *a* :: AC. AH *b* :: AC. BF. Q. E. D. a 1. 6. b 14. 6. & 7. 5.

PROP. 57.




*Si datum ſpatium* AC *ad datam rectam* AB *applicatum fuerit, in angulo* BAD *dato, datur applicationis altitudo* AD.

*a* Erige perpendicularem AE. eſtque AB. AE *b* :: ABq. AB x AE *c* :: ABq. pgr. AC. *d* ergo AE datur. quare per E duc parallelam DC, *e* hæc abſcindet quæſitam AD. Q. E. F. a 11. 1. b 1. 6. c 35. 1. d 1. & 2. dat. e 28. & 25. dat.

PROP. 58.

*Si datum ad datam rectam applicetur, deficiens data ſpecie figura, latitudines defectus datæ ſunt.*

Non differt à vigeſima octava ſextæ.

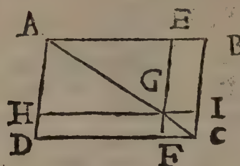
PROP. 59.

*Si datum ad datam rectam applicetur, excedens data ſpecie figura, latitudines exceſſus datæ ſunt.*

Eadem eſt cum vigeſima nona ſextæ.

PROP

## PROP. 60.



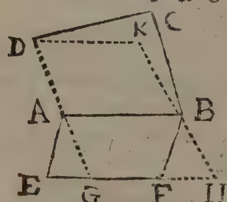

*Si datum ſpecie parallelogrammum (HE, vel DB) dato gnomone HCE augeatur, vel minuatur; latitudines gnomonis HD, EB datæ ſunt.*

a 3. dat. b 24. 6. c 55. dat. d hyp. e 4. dat.

1. *Hyp.* Liquet totum DB tam *a* magnitudine, quam *b* ſpecie dari, *c* proinde & latitudines AB, AD; è quibus aufer *d* datas AE, AH, *e* manent EB, HD datæ. Q. E. D.

2. *Hyp.* Liquet HE *b* ſpecie, & *a* magn. *c* dari, *c* quare & latera AE, AH; hæc deme ex *d* datis AB, AD: *e* remanent EB, HD datæ. Q. E. D.

## PROP. 61.



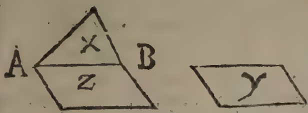

*Si ad datæ ſpecie figuræ ABCD unum latus AB applicetur parallelogrammum ſpatium AF in angulo BAE dato; habeat autem data figura AC ad parallelogrammum AF rationem datam; parallelogrammum AF ſpecie datum eſt.*

a 3. def. d. b 49. dat. c 8. dat. d 35. 1. e 1. 6. f hyp. & 4. dat. g 40. dat. h 3 def. d.

Ad DAG protractam duc (per B) parallelam, cui occurrant EFH, & DK parall. AB. Ac ob $\frac{AD}{AB}$, & ang. BAD *a* dat. *a* liquet pgr. AK ſpecie dari. *b* ergo $\frac{AK}{AC}$ & *c* proinde $\frac{AK}{AF}$, *d* vel $\frac{AK}{AH}$, *e* hoc eſt $\frac{AD}{AG}$ dantur. *e* ergo $\frac{AB}{AG}$ datur. Item ob angulos E, & GAE *f* notos, *g* datur $\frac{AE}{AG}$; *c* ergo $\frac{AB}{AE}$ datur. *h* unde pgr. AF ſpecie datur. Q.E.D.

PROP.

PROP. 62.




*Si duæ rectæ AB, CD ad invicem habeant rationem datam; & ab una quidem data ſpecie figura X deſcripta ſit, ab altera autem ſpatium parallelogrammum Y in angulo dato; habeat autem figura X ad parallelogrammum Y rationem datam; parallelogrammum Y ſpecie datum eſt.*

Nam ad AB ſit pgr. Z ſimile ipſi Y. *a* Hujus ratio ad Y, & *b* proinde ad X datur. *c* ejuſque anguli dantur. *d* ergo Z ſpecie datur. *e* proinde & Y. Q. E. D.

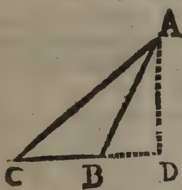
a 50. dat.
b 8. dat.
c hyp
d 61. dat.
e 3 def. d.

PROP. 63.

*Si triangulum ſpecie datum ſit, quod ab unoquoq; laterum deſcribitur quadratum, ad triangulum habebit rationem datam.*

Sequitur ex 49. hujus.

PROP. 64.




*Si triangulum ABC angulum obtuſum ABC datum habeat; illud ſpatium, quo latus AC obtuſum angulum ſubtendens magis poteſt quam latera AB, CB obtuſum angulum ABC ambientia, ad triangulum ABC habebit rationem datam.*

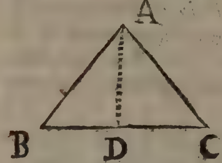
Nam demittatur AD perpendicularis productæ CBD. atque ob angulos *a* ABD, & D datos, *b* datur $\frac{BD}{AD}$, *c* hoc eſt $\frac{BD \times CB}{AD \times CB}$. *d* ergo

a 4. dat.
b 49 dat.
c 1. 6
d 8. dat.

2 BD

e 12. 2. f 41. 1. $\frac{2\,BD \times CB}{\frac{1}{2}\,AD \times CB}$, hoc eſt, e $\frac{ACq - ABq - CBq}{f\ \text{triang. } ABC}$ datur. Q.E.D.

## PROP. 65.




*Si triangulum* ACB *angulum acutum* C *datum habeat; illud ſpatium, quo latus* AB *angulum* C *ſubtendens minus poteſt, quam latera* AC, CB *angulum acutum* C *ambientia, habebit ad triangulum* ACB *rationem datam.*

a 40 dat. Nam duc perpendicularem AD. Datur *a* $\frac{CD}{AD}$,

b 6. c 8. dat. *b* hoc eſt $\frac{CD \times BC}{AD \times BC}$. *c* ergo $\frac{2\,CD \times BC}{\frac{1}{2}\,AD \times BC}$, hoc

d 13. 2. e 41. 1. eſt *d* $\frac{ACq + BCq - ABq}{e\ \text{triang. } ACB}$ datur. Q.E.D.

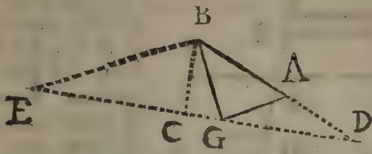
## PROP. 66.

*Si triangulum* ACB *habuerit angulum* C *datum; quod ſub rectis* AC, CB *datum angulum* C *comprehendentibus, continetur rectangulum, habebit ad triangulum* ACB *rationem datam.*

a 40 dat. b 6 c 41. 1. d 8 dat. Nam in figura præcedentis, eſt *a* $\frac{AC}{AD}$, *b* hoc eſt, *b* $\frac{AC \times BC}{AD \times BC}$, *c* hoc eſt $\frac{AC \times BC}{2\ \text{triang. } ACB}$ data. *d* ergo $\frac{AC \times BC}{\text{triang. } ACB}$ datur. Q.E.D.

PROP.

## PROP. 67.




*Si triangulum* ABG *habuerit datum angulum* BAG; *illud ſpatium, quo duo datum angulum* BAG *comprehendentia latera tanquam una recta* BA + AG, *plus poſſunt, quam quadratum à reliquo latere* BG, *ad triangulum* ABG *habebit rationem datam.*

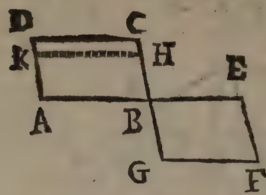
Produc BA ita ut AD = AG. per B duc BE parall. AG; cui occurrat DGE. denique duc normalem BC.

Liquet ang. D [a] = AGD [b] = E. [c] quare BE = BD, ideoque EC = CD. [e] ergo EG x GD + CGq = CDq. proinde BDq [f] (CDq + BCq) [g] = EG x GD + CGq + BCq = EG x GD [*] + BGq. Iam ob angulos AGD, & D [h] ſubduplos dati BAG, liquet [k] $\frac{AD}{DG}$, ideoq; $\frac{ADq}{DGq}$ dari. Cum igitur BA x AD. ADq [l] :: BA. AD [m] :: EG. GD :: [l] EGxGD. GDq, & permutando BAxAD. EGxGD :: ADq. GDq; [n] erit $\frac{BAxAD}{EGxGD}$; [o] hoc eſt $\frac{BAxAG}{EGxGD}$ data. [p] Atqui $\frac{BAxAG}{triang.\ AGB}$ datur; [q] ergo $\frac{EG\ x\ GD}{triang.\ AGB}$ datur. Q. E. D.

a 5. 1.
b 29. 1.
c 6. 1.
d cor. 3. 3.
e 5. 2.
f 47. 1.
g 2. ax. 1.
* 47. 1.
h 32. 1.
k 40. dat.
l 1. 6.
m 2. 6.
n 2. def. d.
o conſtr.
p 66 dat.
q 8. dat.

PROP.

PROP. 68.

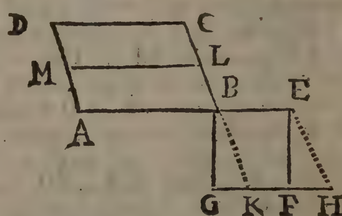



*Si duo parallelogramma æquiangula* AC, BF *habeant ad invicem rationem datam, & unum latus* AB *ad unum latus* BE *habeat rationem datam; & reliquum latus* BC *ad reliquum latus* BG *habebit rationem datam.*

a 2 def. d.
b 56. dat.
c 8. dat.

Nam ſit AB. BE :: BG. BH. a ergo $\frac{BG}{BH}$ datur. b item $\frac{BC}{BH}$ datur. c ergo $\frac{BC}{BG}$ datur.

PROP. 69.




*Si duo parallelogramma* AC, BF *datos angulos habeant, & ad invicem rationem datam; habeat autem & unum latus* AB *ad unum latus* BE *rationem datam; & reliquum latus* B C *ad reliquum latus* BG *habebit rationem datam.*

Latera AB, BE jaceant in directum. produc C B K, ac G F H ad occurſum cum EH parall. CK.

a hyp.

Ob a ang. KBE (ABC) & pgr. a $\frac{AC}{BF}$, vel

AC

$\frac{AC}{BH}$ & *a* $\frac{AB}{BE}$ datas, *c* liquet $\frac{KB}{BC}$ dari. item ob ang. G, & GBK *d* datos, *e* datur $\frac{KB}{BG}$. *f* quare $\frac{BC}{BG}$ datur. Q. E. D.

b 35. 1.
c 68. *dat.*
d *hyp.* & 4. *dat.*
e 40. *dat.*
f 8. *dat.*

P R O P. 70.

*Si duorum parallelogrammorum* (AC, BH, *vel* BF) *circà æquales angulos* (ABC, KBE) *aut circa inæquales quidem* (ABC, GBE) *datos tamen, latera* (AB, BE, & BC, BK, & BC, BG) *ad invicem habeant rationem datam; & ipſa parallelogramma* (AC, BH, & AC, BF) *habebunt ad invicem rationem datam.*

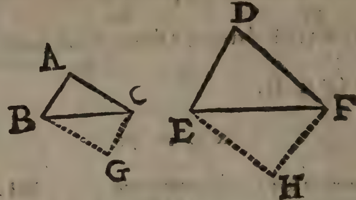
Nam (in fig. præced.) ſit AB. BE :: KB. BL. & duc LM parall. BA.

Primo, Quia *a* $\frac{AB}{BE}$ *b* id eſt $\frac{KB}{BL}$, *a* ac $\frac{KB}{CB}$ datæ ſunt, *c* erit $\frac{CB}{BL}$, *d* hoc eſt $\frac{AC}{AL}$, *e* vel pgr. $\frac{AC}{BH}$ data. Q. E. D.

Secundò, Ob angulos G, & GBK *f* datos, *g* datur $\frac{BK}{BG}$; item *h* $\frac{CB}{BG}$ data eſt. *c* ergo $\frac{CB}{BK}$ datur. proinde, ut prius, $\frac{AC}{BH}$, hoc eſt pgr. $\frac{AC}{BF}$ datur. Q. E. D.

a *hyp.*
b *conſtr.*
c 8. *dat.*
d 1. 6.
e 14. 6.
f *hyp.* & 4. *dat.*
g 40. *d.*
h 35. 1.

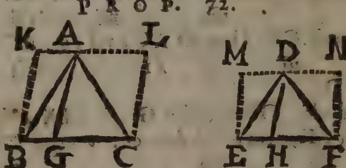
PROP. 71.




*Si duorum triangulorum* ABC, DEF, *circa æquales angulos, aut circa inæquales quidem, datos tamen* (A, & D) *latera* AB, DE, & AC, DF *ad invicem habeant rationem datam; & ipſa triangula* ABC, DEF *habebunt ad invicem rationem datam.*

a 70. dat. b 15. 5. c 34. 1. Nam compleantur pgra. AG, DH. *a* hæc datam habent rationem, *b* proinde & trigona ABC, DEF illorum *c* ſubdupla. Q. E. D.

PROP. 72.

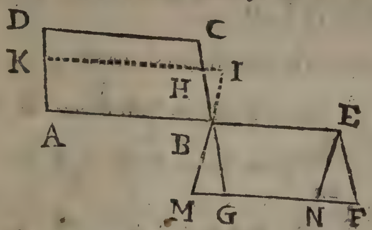



*Si duorum triangulorum* ABC, DEF *& baſes* BC, EF *fuerint in ratione data, & actæ ab angulis ad baſes* (AG, DH,) *quæ faciant ang.* AGC, DHF *æquales, aut inæquales quidem, ſed tamen datos, habeant ad invicem rationem datam; & ipſa triangula* ABC, DEF *habebunt ad invicem rationem datam.*

Nam duc BK ad AG, ac EM ad DH parallelas, & comple pgra. CK, FM. Hæc ſe habent juxta 70. hujus; quare triangula eorum * 34. 1. * ſubdupla ABC, DEF rationem habent datam. Q. E. D.

PROP.

P R O P. 73.




*Si duorum parallelogrammorum* (AC, BF, *vel* AC, BN) *circa æquales angulos, aut circa inæquales quidem, ſed tamen datos, latera ad invicem ita ſe habeant, ut ſit quemadmodum primi latus* AB *ad ſecundi latus* B E, *ita reliquum ſecundi latus* (BG, *vel* BM) *ad aliam aliquam rectam* (BH, *vel* BI;) *habeat autem & reliquum primi latus* BC *ad eandem rectam* (BH *vel* BI) *rationem datam; & ipſa parallelogramma* (AC, BF, *vel* AC,BN) *habebunt ad invicem rationem datam.*

Nam 1. Hyp. liquet *a* CB *b* id eſt AC da-
$\overline{BH}$, $\overline{AH}$ *c*($\overline{BF}$) — a *hyp.* b 2. 6.
ri. Q. E. D. — c 14. 6.

2. Hyp. Duc parallelam IHK. *a* Liquet angulos I B H (G B M) & B H I (A B H) dari. — a *hyp & 4 dat.*
*b* ergo BH datur. item CB *a* data eſt. *c* proinde — b 40. *dat.* c 8 *dat.*
$\overline{BI}$ $\overline{BI}$ — d 35. 1.
CB, hoc eſt pgr. AC *d* vel AC datur. Q. E. D.
$\overline{BH}$ $\overline{BF}$ $\overline{BN}$,

P R O P. 74.

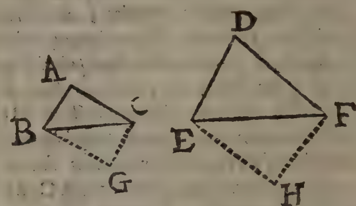
*Si duo parallelogramma datam rationem habeant, aut in æqualibus angulis (ut* AC, BF) *aut inæqualibus quidem, ſed tamen datis (ut* AC, BN;) *erit ut primi latus* AB *ad ſecundi latus* BE, *ita alterum ſecundi latus* (BG, *vel* BM) *ad eam* (BH, *vel* BI) *ad quam reliquum primi latus* B C *rationem habet datam.*

a 56. dat. Nam in fig. præcedentis. 1. Hyp. a Liquet $\frac{CB}{BH}$ dari. Q. E. D.

2. Hyp. ut in præcedenti, datur $\frac{BI}{BH}$, ac ex hyp. $\frac{AC}{BF(BN)}$ item AB. BE :: a *MB. BI b :: GB. BH. a quare $\frac{CB}{BH}$ etiam datur. c ergo $\frac{CB}{BI}$ data eſt Q. E. D.

* hyp.
b 4 6.
c 8. dat.

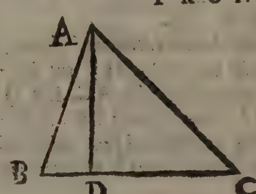
PROP. 75.




*Si duo triangula* ABC, DEF *ad invicem habeant rationem datam, aut in angulis* (A, D) *æqualibus, aut inæqualibus quidem ſed tamen datis, erit ut primi latus* A B *ad ſecundi latus* D E, *ita alterum ſecundi latus* DF *ad eam rectam, ad quam reliquum primi latus* AC *habet rationem datam.*

Nam compleantur pgr. AG, DH. Ergo per præcedentem.

PROP. 76.

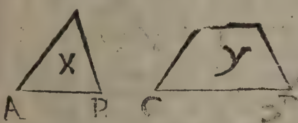



*Si à trianguli* ABC *ſpecie dati vertice* A *linea perpendicularis* AD *agatur ad baſim* BC, *acta linea* AD *ad baſim* BC *habebit rationem datam.*

Nam

Nam ob angulos, * B, & ADB datos, a datur $\frac{AB}{AD}$; a item $\frac{AB}{BC}$ datur. b Ergo $\frac{AD}{BC}$ datur. Q. E. D. * *hyp. & 3. def. d.* a 40. *dat.* b 8. *dat.*

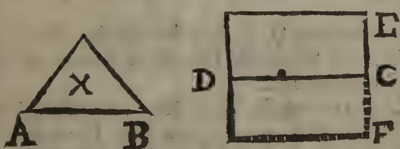
PROP. 77.




*Si datæ figuræ ſpecie* X, Y *ad invicem habeant rationem datam, & quodlibet latus unius* AB *ad quodlibet alterius latus* CD *habebit rationem datam.*

Nam a $\frac{ABq}{X}$, & b $\frac{Y}{X}$, ac c proinde $\frac{ABq}{Y}$ datur; item $\frac{CDq}{Y}$ datur. c ergo $\frac{ABq}{CDq}$, ac ideo $\frac{AB}{CD}$ datur. Q. E. D. a 49. *dat.* b *hyp.* c 8. *dat.*

PROP. 78.

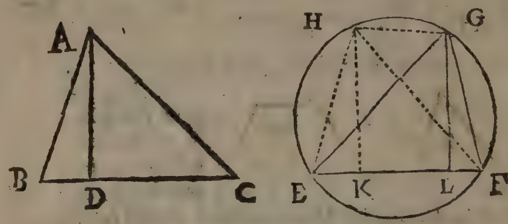



*Si data figura ſpecie* X *ad aliquod rectangulum* DCE *habeat rationem datam; habeat autem & unum latus* AB *ad unum latus* DC *rationem datam; rectangulum* DCE *ſpecie datum eſt.*

Sit DC. AB :: AB. CF. a ergo $\frac{DC}{CF}$ datur. a 8. *dat.*
Item ob b $\frac{X}{ABq}$, & c $\frac{X}{DCE}$ datas. e erit $\frac{ABq}{DCE}$, d hoc eſt $\frac{DC \times CF}{D\,CE}$, vel e $\frac{CF}{CE}$ data. proinde e $\frac{DC}{CE}$ datur. f quare rectang. DCE ſpecie datur. Q. E. D. b 49. *dat.* c *hyp.* d 17. 6. e 1. 6. f 3. *def. d.*

 PROP.

PROP. 79.




*Si duo triangula* ABC, GEF *unum angulum* BAC *uni angulo* EGF *æqualem habeant; ab æqualibus autem angulis* BAC, EGF *ad baſes* BC, EF *perpendiculares agantur* AD, GL; *ſitque ut primi trianguli baſis ad perpendicularem, ita & alterius trianguli baſis ad perpendicularem* (BC.AD :: EF. GL;) *illa triangula* ABC, EGF *æquiangula ſunt.*

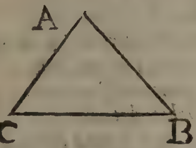
Circa triang. GEF deſcribe circulum. Fac ang. FEH = B. Connecte HE, HG; & demitte perpendicularem HK.

Liquet triangula ABC, HEF, & ABD, HEK, ac ACD, HFK æquiangula fore. Proinde EK. KH :: BD. DA. *a* & FK. KH :: CD. DA. *b* quare EF. KH :: BC. DA :: *c* EF. LG. *d* quare KH = LG. *e* ergo HG parall. KL. *f* unde ang. EGH = GEF. *g* ergo arcus EH, FG, *h* ideoque anguli EFH, GEF æquantur. *k* Item ang. EHF = EGF. *l* ergo trigona EHF, EGF; *m* proinde & trigona EGF, ABC ſibi mutuo æquiangula ſunt. Q. E. D.

a 4. 6.
b 24. 5.
c *hyp.*
d 9. 5.
e 33. 1.
f 29. 1.
g 26. 3.
h 27. 3.
k 21. 3.
l 32. 1.
m 21. 6.

PROP.

### PROP. 80.




*Si triangulum* ABC *unum angulum* A *datum habuerit; quod autem ſub lateribus* AB, AC *datum angulum comprehendentibus continetur rectangulũ, habeat ad quadratum reliqui lateris* BC *rationem datam; triangulum* ABC *ſpecie datum eſt.*

Nam Q: AC + AB: − CBq vocetur X. [a] ergo $\frac{X}{\text{triang.}ABC}$; [b] & $\frac{AC \times AB}{\text{triang.}ABC}$; & [c] propterea $\frac{X}{AC \times AB}$ data eſt. [d] item $\frac{AC \times AB}{CBq}$ datur. [e] ergo $\frac{X}{CBq}$, ideoq; $\frac{X + CBq}{CBq}$, [f] hoc eſt $\frac{Q: AC + AB}{CBq}$, datur. [g] proinde triang. ABC ſpecie datur. Q.E.D.

a 67. *dat.*
b 66. *dat.*
c 8. *dat.*
d *hyp.*
e 6. *dat.*
f *hyp.*
g 46. *dat.*

### PROP. 81.

A. D.
B. E.
C. F.

*Si tres rectæ proportionales* A, B, C *tribus rectis proportionalibus* D, E, F *extremas* A, D, *&* C, F *habuerint in ratione data; medias quoque* B, E *habebunt in ratione data. Et ſi extrema* A *ad extremam* D, *& media* B *ad mediam* E *habeat rationem datam; & reliqua* C *ad reliquam* F *habebit rationem datam.*

Nam primo, ob $\frac{A}{D}$ & $\frac{C}{F}$ datas, [a] datur $\frac{AC}{DF}$ [b] hoc eſt, $\frac{Bq}{Eq}$. ergo $\frac{B}{E}$ datur. Q. E. D.

Secundo, ob [c] $\frac{Bq}{Eq}$, [b] hoc eſt $\frac{AC}{DF}$ datam, & [c] $\frac{A}{D}$ datam, [d] datur $\frac{C}{F}$. Q. E. D.

a 70. *dat.*
b 17. 6.
c *hyp.*
d 68. *dat.*

## Prop. 82.

A. B :: D. E.
B. C :: E. F.

*Si quatuor rectæ proportionales fuerint* (A. B :: D. E) *erit ut prima* A *ad eam* C, *ad quam ſecunda* B *rationem habet datam, ita tertia* D *ad eam* F, *ad quam quarta* E *rationem habet datam.*

a hyp. b 2. def. d. Nam quia B. C :: a E. F. & a $\frac{B}{C}$ data eſt; b erit $\frac{E}{F}$ data. atqui ex æquali A. C :: D. F. ergo, &c.

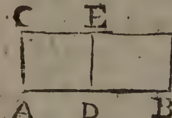
## Prop. 83.

A. B. C. D.
F. E.

*Si quatuor rectæ* A, B, C, D *ita ad invicem ſe habeant, ut tribus ex iis, quibuſcunque ſumptis* A, B, C, *& quarta ipſis proportionali accepta* E, *ad quam reliqua* D *ex quatuor rectis proportionem habet datam; erit ut quarta* D *ad tertiam* C, *ita ſecunda* B *ad eam* F, *ad quam habet prima* A *rationem datam.*

a 16. 6. b hyp. c 1. 6. d 7. 5. Nam AE a = BC b = DF. & datur b $\frac{D}{F}$, c hoc eſt $\frac{AD}{AE}$, d vel $\frac{AD}{DF}$, e vel $\frac{A}{F}$. ergo, &c.

## Prop. 84.




*Si duæ rectæ* AB, AC *datum ſpatium comprehendant in angulo* A *dato; ſit autem altera* AB *altera* AC *major data* DB; *etiam unaquæque ipſarum* AB, AC *data erit.*

a 3. def. d. b hyp. c 59. d. d 3. dat. Nam comple quadratum AE. a Hoc ſpecie datum eſt. b item pgr. CB, & recta DB dantur. c ergo AC, vel AD, & tota d proinde AB datur. Q. E. D.

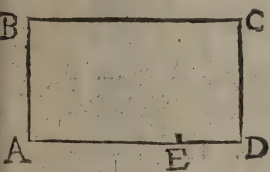
## PROP. 85.

*Si duæ rectæ* BD, DE *datum ſpatium comprehendant in angulo* BDE *dato, ſit autem ſimul utraq;* (BD+DE) *data; & earum quoque unaquæque* BD, *&* DE *data erit.*

Nam ſume DA=DE, & comple quad. DC. Hoc ſpecie datur; item pgr. BE, & recta BA *a* dantur. *b* ergo AD (DE) & *c* reliqua DB dantur. Q. E. D.

a *hyp.*
b 58. *dat.*
c 4. *dat.*

## PROP. 86.




*Si duæ rectæ* AB, AD *datum ſpatium* BD *comprehendant in angulo dato; quadratum autem unius* AD *quadrato alterius* AB *majus ſit dato quam in ratione* (nempe ut ſit ADxAE datum, & * reliqui ADxED ad ABq ratio data;) *& utraque ipſarum* AB, AD *data erit.* * 2. 2.

Nam ob BD, & DAxAE *a* data, *b* datur $\frac{BD}{DA \times AE}$. *c* ergo $\frac{AB}{AE}$ *d* ideoque $\frac{ABq}{AEq}$ datur. *e* item $\frac{ABq}{AD \times ED}$, datur. *f* ergo $\frac{AEq}{AD \times ED}$, ideoque $\frac{AEq}{4AD \times ED}$, *g* & $\frac{AEq}{4AD \times ED + AEq}$ *h* hoc eſt $\frac{AEq}{Q{:}AD + ED}$ datur. *k* ergo $\frac{AE}{AD\;\;ED}$; & *l* componendo $\frac{AE}{2AD}$, *ideoq; $\frac{AE}{AD}$, *m* hoc eſt $\frac{AEq}{AD \times AE}$ datur. denique igitur ob *e* datum ADxAE, *n* erit AEq data. *o* ergo AE, & *p* proinde AD, ac AB datæ ſunt. Q.E.D.

a *hyp.*
b 1. *dat.*
c 69. *dat.*
d 51. *dat.*
e *hyp.*
f 8. *dat.*
g 6. *dat.*
h 8. 2.
k 54 *dat.*
l 6 *dat.*
* 8. *dat.*
m 1. 6.
n 2 *dat.*
o 55. *dat.*
p 57 *dat.*

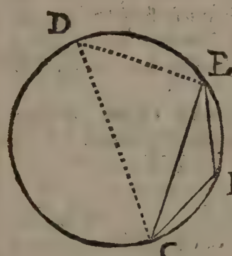
### PROP. 87.

*Si duæ rectæ* AB, AD *datum ſpatium comprehendant in angulo dato, quadratum autem unius* AD *quadrato alterius* AB *majus ſit dato* ( AD x AE; ) *earum utraque* AB, AD *data erit.*

a 2. dat.
b 69. dat.
c hyp. & 2. 2.
d 8. & 6. dat.
e 8. 2.
d 6 dat.
e 1 6.
f hyp.
g 2. dat.
h 55. dat.
k 57. dat.

Nam ob $\frac{BA\text{x}AE}{BD}$ *a* datum, *b* erit $\frac{AE}{AB}$, ideoq; $\frac{AEq}{ABq}$, *c* hoc eſt $\frac{AEq.}{AD\text{x}ED}$, *d* ac idcirco $\frac{AEq}{AEq+4AD\text{x}ED}$, *e* hoc eſt $\frac{AEq}{Q:AD+ED}$, ac proinde $\frac{AE}{AD+ED}$, & *d* componendo $\frac{AE}{2AD}$, *e* ac ideo $\frac{AE}{AD}$, *e* hoc eſt $\frac{AEq}{ADA\text{x}E}$ data. ergo ob ADxAE *f* datum, dantur *g* AEq, & *h* AE, ac *k* ideo AD, ac AB. Q. E. D.

### PROP. 88.




*Si in circulum* CFED *magnitudine datum acta ſit recta linea* CE, *quæ ſegmentum auferat, quod datum angulum* F *comprehendat; acta recta linea* CE *magnitudine data eſt.*

a hyp
b 4. dat.
c 40. dat.
d hyp & 5. def d.
e 2. dat.

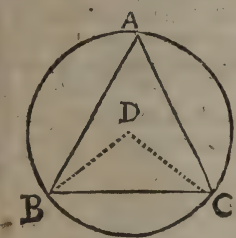
Nam ducatur diameter CD; & connectatur ED. Ac ob ang. F *a* datum, *b* erit ang. D ( reliquus è 2 rectis ) datus. item rectus CED datur. *c* quare $\frac{CE}{CD}$ datur. ergo ob *d* datam CD, *e* erit CE data. Q. E. D.

PROP. 89.

*Si in datum magnitudine circulum* CFED *data magnitudine recta* CE *acta fuerit, auferet ſegmentum quod angulum* (CFE) *datum comprehendet.*

Nam (in fig. præcedentis) quia $\frac{CE}{\overline{CD}}$, & ang. CED dantur, *a* erit ang. D datus. *b* ergo ang. F *c* (1 Rect. — D) datus erit. Q. E. D. a 43. dat. b 4. dat. c 32. 3.

PROP. 90.



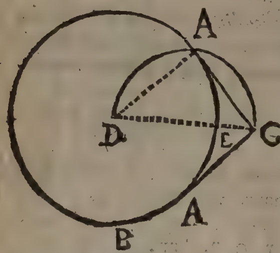

*Si in circuli poſitione dati circumferentia* BAC *datum fuerit punctum* B, *ab eo autem puncto* B *ad circumferentiam circuli inflexa fuerit recta* BAC *quæ datum angulum* A *efficiat; inflexa rectæ altera extremitas* C *data erit.*

Ad *a* centrum D duc BD, & CD; *b* datuſque eſt ang. D dati A *c* duplus. quare ob BD *d* datam, *e* erit DC data. *f* ergo punctum C datum eſt. Q. E. D. a 1. 3. b 1. dat. c 20. 3. d 26. dat. e 29. dat. f ſch. 25. dat.

Si ang. A obtuſus fuerit; ſume reliquum è 2 rectis acutum; ejus ſubſidio punctum C invenies, juxta dicta.

PROP.

PROP. 91.



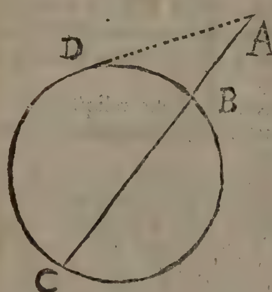

*Si à dato punſto G aſta fuerit recta GA, quæ datum poſitione circulum* B E A *contingat; aſta linea* GA *poſitione & magnitudine data eſt.*

Nam centrum D & punctum G connectat recta D G. ſuper qua deſcriptus ſit ſemicirculus D A G circulo priori occurrens in A. Ob ang. D A G *a* rectum; GA circulum *b* tangit. *c* ergo G A ſitu & magnitudine datur. Q. E. D.

a 31. 3.
b cor. 16. 3.
c 26. dat.

Hinc modus diſcitur à dato puncto tangentem ducendi, eo nonnunquam expeditior qui habetur ad 17. 3.

PROP. 92.

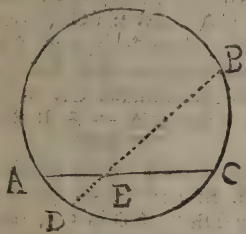



*Si extra circulum poſitione datum* BCD *accipiatur aliquod punctum* A, *à dato autem puncto* A *in circulum producatur quædam recta* AC; *datum eſt id quod ſub acta linea* AC, *& ea* A B, *quæ inter punctum* A *& convexam peripheriam* B *comprehenditur rectangulum* CAB.

Nam

*a* Nam duc tangentem A D, *b* eritque ADq (hoc eſt C A x AB) datum. Q. E. D. *a* 91. *dat.* *b* 36. 3

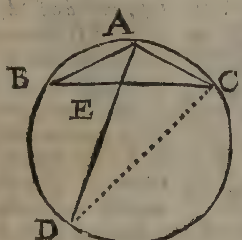
PROP. 93.




*Si intra datum poſitione circulum* ABCD *ſumatur aliquod punctum* E; *per punctum autem* E *agatur in circulum aliqua recta* AFC; *quod ſub ſegmentis* AE, EC *actæ rectæ lineæ comprehenditur rectangulum, datum eſt.*

Nam per E duc rectam DEB utcunque occurrentem circulo in B, & D. eſtque rectang. DEB = *a* AEC. *b* ergo AEC datur. Q. E. D. *a* 35. 1. *b* 2. *def. d.*

PROP. 94.




*Si in circulum* BACD *magnitudine datum agatur recta linea* BC, *quæ ſegmentum auferat, quod angulum* BAC *datum comprehendat; angulus autem* BAC, *qui in ſegmento conſiſtit, bifariam ſecetur; ſimul utraque rectarum* BA, AC *quæ angulum datum* BAC *comprehendunt, ad lineam* AD, *quæ angulum bifariam ſecat, habebit rationem datam: & quod ſub ſimul utriſque* BA, AC, *quæ datum angulum* BAC *comprehendunt, rectis: & inferne abſciſſa* (ED) *ab ea* AD, *quæ angulum* BAC *in circumferentia datum bifariam ſecat, rectangulum datum erit.*

Duc

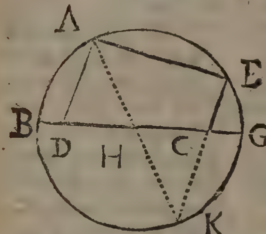
a 88. dat. * 1. dat. b 3. 6. c 12. 5. * 4 6. d 2. def. d.

Duc CD; & primo ob angulos BAC, CAD datos, [a] dantur ſubtenſæ BC, CD,* ideoque $\frac{CB}{DC}$ datur. Cum igitur CA. AB :: [b] CE. EB, & permutando CA. CE :: AB. EB :: (CA + AB. CB ::) * AD. DC. (Nam * ob ang. BAE = CAD; & D = BD; trigona ABE, ADC ſimilia ſunt) ac rurſus permutando CA + AB. AD :: CB. DC. [d] erit $\frac{CA + AB}{AD}$ data. Q. E. D.

e 21. 3. b 4 6. c prius. d 16. 6. e 52. dat. f 1. def. d.

Secundo, ob triangula AEB, DEC [e] ſimilia; [b] erit CD, DE :: AB. BE [c] :: CA + AB. CB. [d] ergo CA + AB in DE = CD in CB. atqui CD x CB [e] datur [f] ergo CA + AB in DE datum eſt. Q. E. D.

PROP. 95.




*Si in circuli* BAG *poſitione dati diametro* BG *ſumatur datum punctum* D; *à puncto autem* D *in circulum producatur quædam recta* DA, *&* *agatur à ectione* A *ad rectos angulos in productam rectam* DA *linea* AE; *per punctum autem* E, *in quo linea* AE, *quæ ad rectos angulos conſiſtit, occurrit circumferentiæ circuli*, *agatur parallela* (ECK) *productæ rectæ* DA; *datum eſt illud punctum* C, *in quo parallela* EK *occurrit ipſi diametro* BG; *&* *quod ſub parallelis lineis* AD, EC *comprehenditur rectangulum, datum eſt.*

a 31. 3.

Nam connectatur AK. [a] eſtque AB (ob angulum E, vel DAE rectum) diameter. ergo in-

interſectio H eſt centrum. *b* ergo DH datur. At- b 26. *dat.*
qui ob KH. HA *c* :: CH. HD, *d* eſt CH = HD. c 4. 6.
*e* ergo CH datur. *f* ergo punctum C datur. d 9. 5.
Q. E. D. *g* ergo KC x CE, hoc eſt *d* AD x CE e 1. *def. d.*
datur. Q. E. D. f 27. *dat.* g 93. *dat.*

F I N I S.